Imperial College London

60006 Computer Vision (Term 2)

Tutorial 3: Interest Point Detection

1. Suppose we use the Harris detector for interest point detection. At each pixel, we calculate a 2x2 matrix M,

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

At one particular pixel, after calculation, we get the following matrix,

$$M = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

1.1 Please derive the eigenvalues for this matrix.

You can use the following linear algebra knowledge. For eigenvalues λ and eigenvectors $\boldsymbol{u}$ associated with matrix M, we have

$$M\boldsymbol{u} = \lambda \boldsymbol{u}$$

which can be re-written as,

$$(M - \lambda I)\boldsymbol{u} = 0$$

This means that the matrix $M - \lambda I$ is singular and its determinant $|M - \lambda I|$ is zero. For a 2x2 matrix, its determinant is defined by,

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

1.2 Please compute the Harris detector response $R = \lambda_1 \lambda_2 - k(\lambda_1 + \lambda_2)^2$ with $k = 0.05$.

1.3 We would like to compare the Harris detector response between different scales. We decide to use the scale-adapted Harris detector, which calculates the matrix in this way,

$$M = \sum_{x,y} w(x,y) \sigma^2 \begin{bmatrix} I_x^2(\sigma) & I_x(\sigma) I_y(\sigma) \\ I_x(\sigma) I_y(\sigma) & I_y^2(\sigma) \end{bmatrix}$$

Suppose we get the following matrix at a pixel,

$$M = \sigma^2 \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

where $\sigma = 2$. Please derive the eigenvalues for this matrix and the corresponding Harris detector response.

1.4 For the above matrix

$$M = \sigma^2 \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Directly calculate the scale-adapted Harris detector using equation $R = \det(M) - k(\mathrm{tr}(M))^2$, with $k = 0.05$ without performing eigen-decomposition. Check whether the result is the same.

2. SIFT uses the difference of Gaussian (DoG) filter to calculate the detector response. The DoG approximates the scale-normalised Laplacian of Gaussian (LoG) filter. Let us denote the Gaussian filter as,

$$G(x,y,\sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

2.1 Please prove the following equation holds (also known as the heat diffusion equation),

$$\frac{\delta G}{\delta \sigma} = \sigma \nabla^2 G$$

where ∇^2 is the Laplacian operator and $\nabla^2 G = \frac{\delta^2 G}{\delta x^2} + \frac{\delta^2 G}{\delta y^2}$.

2.2 The difference of Gaussian is defined as $DoG(x, y, \sigma) = G(k\sigma) - G(\sigma)$. Given that we can approximate the derivative $\frac{\delta G}{\delta \sigma}$ using the finite difference,

$$\frac{\delta G}{\delta \sigma} \approx \frac{G(k\sigma) - G(\sigma)}{k\sigma - \sigma}$$

please derive the relationship between $DoG(x, y, \sigma)$ and the scale-normalised Laplacian of Gaussian $\sigma^2 \nabla^2 G$.