60006 - Tutorial 3
Interest Point Detection

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Question 1

Question 1.1: Please derive the eigenvalues for this matrix.

$$\det\left(\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix} - \lambda\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right) : \lambda^2 - 6\lambda + 8$$
$$\lambda = 4 \quad \lambda = 2$$

Question 1.2: Please compute the Harris detector response.

$$\begin{aligned} R &= \lambda_1\lambda_2 - k(\lambda_1 + \lambda_2)^2 \\ &= 4 * 2 - 0.05(4 + 2)^2 \\ &= 6.2 \end{aligned}$$

Question 1.3: Please derive the eigenvalues for this matrix and the corresponding Harris detector response.

$$M = \begin{bmatrix}12 & 4\\ 4 & 12\end{bmatrix}$$
$$\lambda_1 = 16 \quad \lambda_2 = 8$$
$$R = 16 * 8 - 0.05(16 + 8)^2 = 99.2$$

Question 1.4: Directly calculate the scale-adapted Harris detector using trace and determinant instead of eigen decomposition.

$$\det(M) = 128$$
$$\mathrm{tr}(M) = 24$$
$$R = \det(M) - k(\mathrm{tr}(M))^2 = 128 - 0.05(24)^2 = 99.2$$

Question 2

Question 2.1: Please prove the following equation holds (also known as the heat diffusion equation).

$$G(x, y, \sigma) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$
\begin{aligned}
\frac{\partial G}{\partial \sigma} &= \frac{(y^2+x^2)\,\mathrm{e}^{-\frac{y^2+x^2}{2\sigma^2}}}{2\pi\sigma^5} - \frac{\mathrm{e}^{-\frac{y^2+x^2}{2\sigma^2}}}{\pi\sigma^3} \\
&= -\frac{(2\sigma^2 - y^2 - x^2)}{2\pi\sigma^5}\mathrm{e}^{-\frac{y^2+x^2}{2\sigma^2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 G}{\partial x^2} &= \frac{(x^2-\sigma^2)\,\mathrm{e}^{-\frac{x^2+y^2}{2\sigma^2}}}{2\pi\sigma^6} \\
\frac{\partial^2 G}{\partial y^2} &= \frac{(y^2-\sigma^2)\,\mathrm{e}^{-\frac{y^2+x^2}{2\sigma^2}}}{2\pi\sigma^6}
\end{aligned}
$$

Question 2.2:

$$
\frac{\partial G}{\partial \sigma} \approx \sigma \nabla^2 G \quad \text{and} \quad \frac{\partial G}{\partial \sigma} \approx \frac{G(k\sigma - G(\sigma))}{k\sigma - \sigma}
$$

$$
DoG(x,y,\sigma) = G(k\sigma) - G(\sigma) \approx (k-1)\sigma^2 \nabla^2 G
$$