Imperial College London

60006 Computer Vision (Term 2)

Tutorial 2: Edge Detection, Hough Transform

1. This is a region of an image f:

3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52
3	4	8	15	25	45	50	52

1.1 If we use zero padding at the boundary pixels, what is gradient magnitude of the second row after being convolved by 3x3 Prewitt filter kernels?

$$h_x = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}, h_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Note: to perform convolution, we first flip the filter kernel (for 2D kernel, flip both horizontally and vertically), then move the centre of the kernel across all pixels in the image, at each pixel calculate the product and sum.

1.2 If we use replicate padding (padding using the same value as the boundary pixel), what is the gradient magnitude of the second row?

1.3 If we use replicate padding and the 3x3 Sobel filter, what is the result? The Sobel filter kernels are,

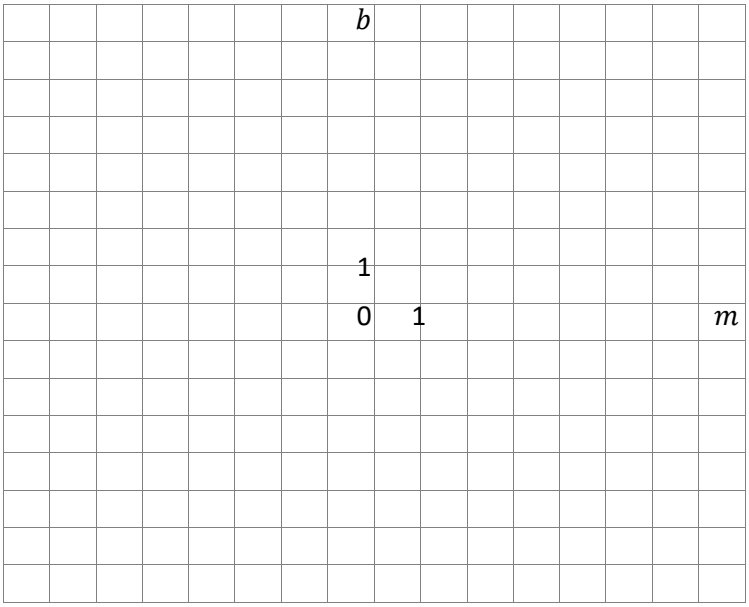
$$\mathrm{h_x} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}, h_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

2. For a 3x3 patch from a region of an image

7	12	9
6	7	8
3	6	9

Calculate the gradient direction for the pixel in the centre using the Prewitt filters.

3. The following points appear in an edge map: (-2, -3), (-1, -1), (0, 1), (1, 3), (4, 9).
We would like to use the Hough transform to estimate a line model from the points. Please sketch the corresponding lines in the parameter space using the slope intercept form for the line model $y = mx + b$, i.e. $b = y - mx$. Find the line model parameters for these points. You can use the figure below (abscissa: m, ordinate: b) to do this.



4. Suppose we have a set of points (x1, y1), (x2, y2), … and we would like to fit a line model $y = mx + b$ to these points. Another way to find the model parameters is to minimise the sum of squared difference for the fitting error, i.e. $\min_{m,b} \sum_i [y_i - (mx_i + b)]^2$. We can write the loss function using the matrix form,

$$\min_{\beta} E(\beta) = (Y - X\beta)^T \cdot (Y - X\beta)$$

where

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \end{pmatrix}, X = \begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \end{pmatrix}, \beta = \begin{pmatrix} m \\ b \end{pmatrix}$$

Please calculate the derivative $\frac{\partial E}{\partial \beta}$ and estimate the optimal β that minimises the loss function $E(\beta)$.