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Image Filtering I

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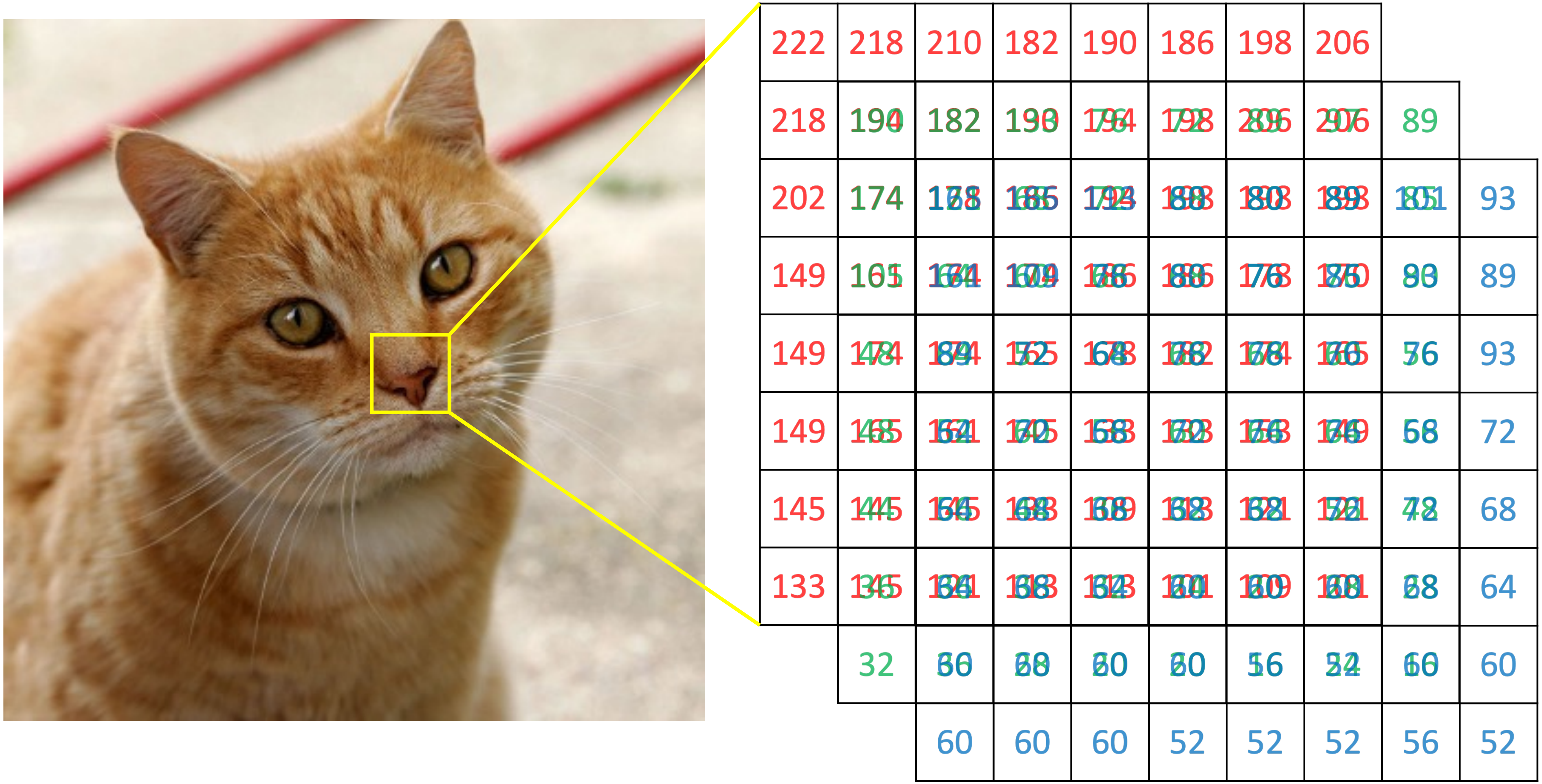
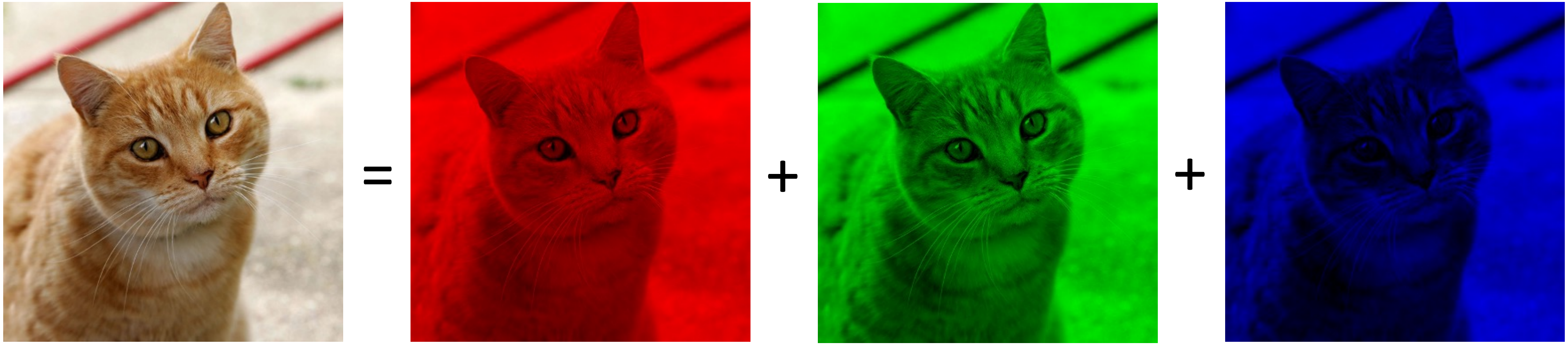
What is image filtering?

- Image representation
- Image filters and applications

Image representation

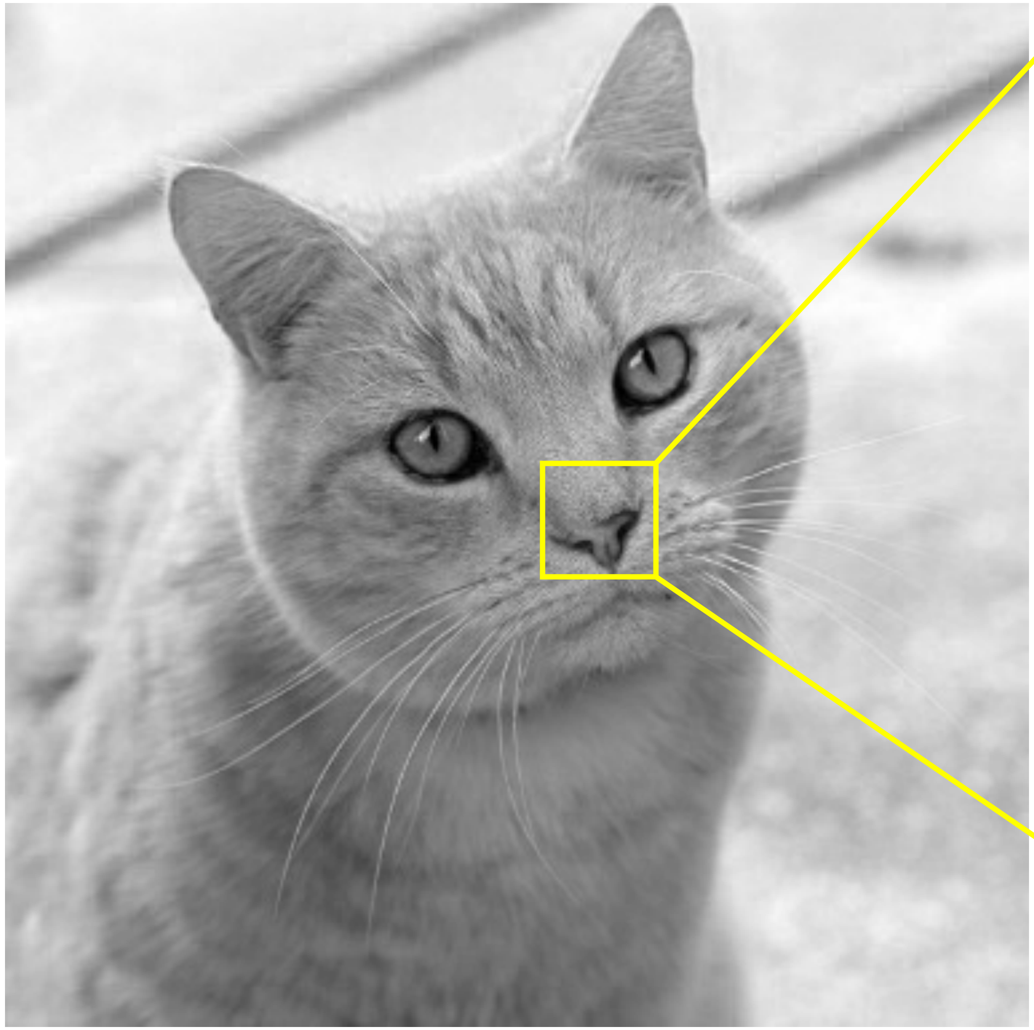


Colour channels



Red Green Blue

Grayscale image



199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

Image filtering

- For most of the time, we will use grayscale images as examples for image filtering.
- For colour images, we can perform filtering for each of it colour channel, e.g. RGB channels or HSV channels.

Example of image filtering

Input

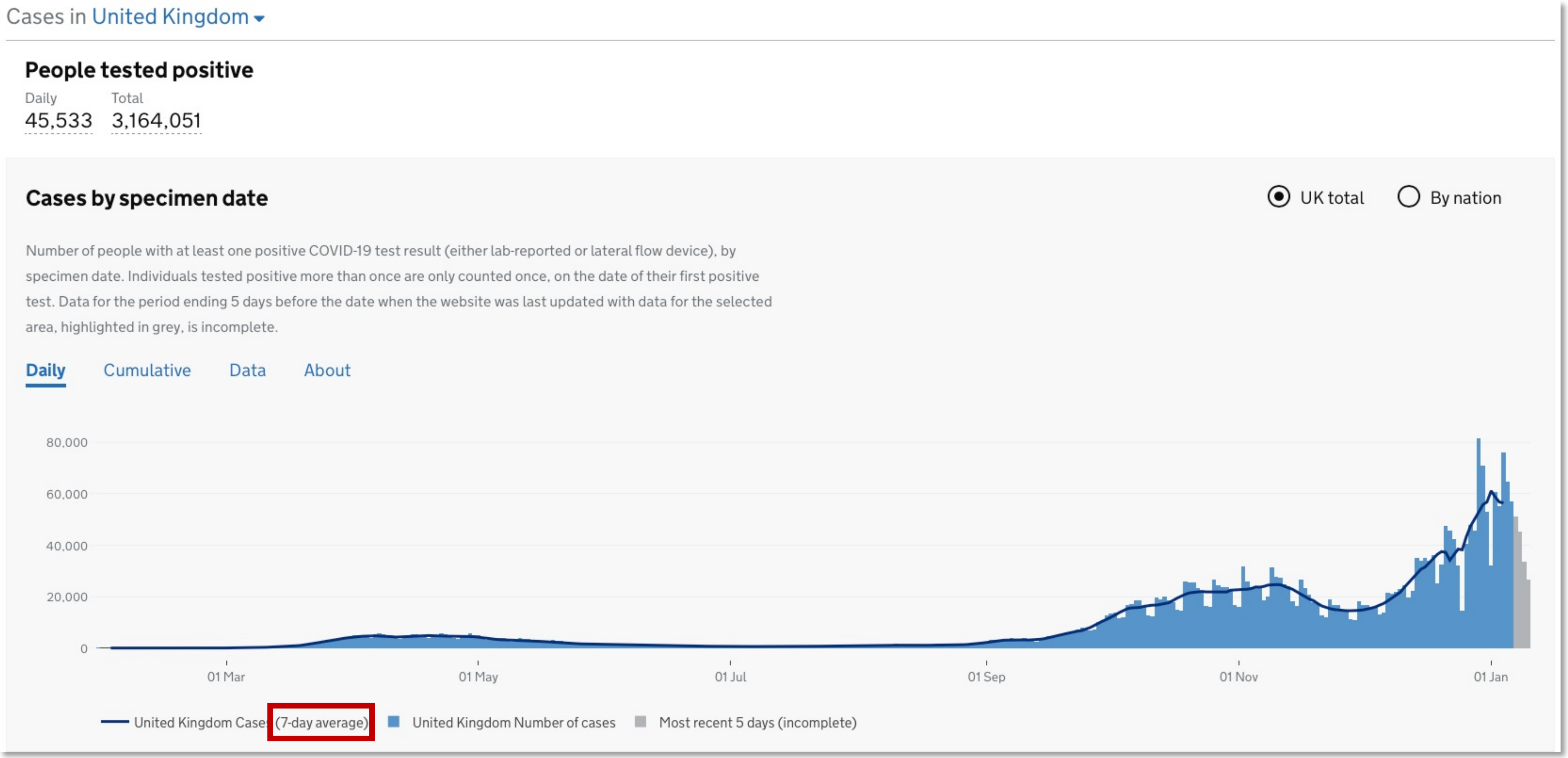
Output

Moving average filter

- We start from moving average filter, which is commonly used for both 1D signal (time series) and 2D signal (images) processing.
- It moves a window across the signal and calculates the average value within the window.

Moving average (MA) for stock market analysis.
https://stockcharts.com

Moving average filter



https://coronavirus.data.gov.uk/details/cases

Moving average (3x3 window)

Let us try this filter kernel.

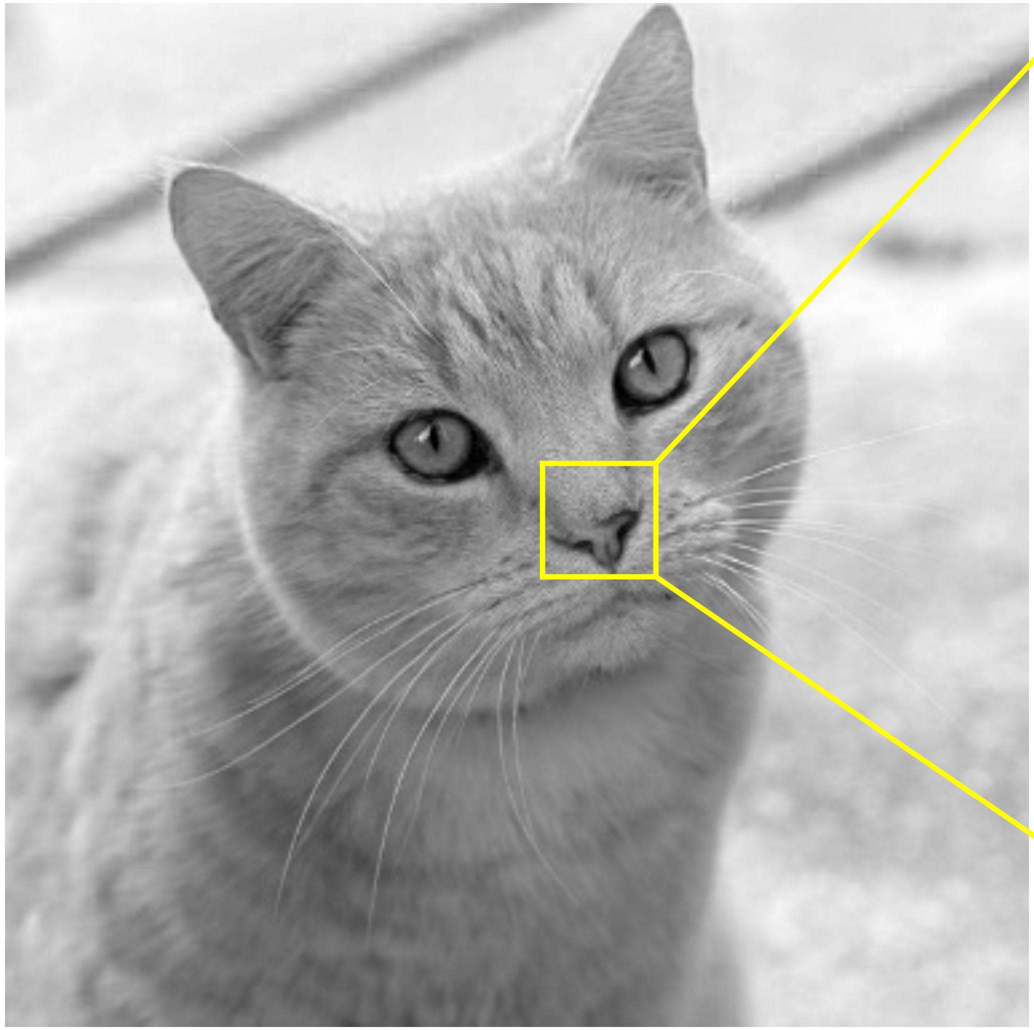
1	1	1
1	1	1
1	1	1

÷ 9

Input

Output

Grayscale image



199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

1/9 199	1/9 192	1/9 158	111	110	123	130	130
1/9 189	1/9 149	1/9 108	111	113	120	126	125
1/9 139	1/9 109	1/9 98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

	147						

199	1/9	1/9	1/9	110	123	130	130
189	1/9	1/9	1/9	113	120	126	125
130	1/9	1/9	1/9	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

	147	126					

199	192	1/9	1/9	1/9	123	130	130
189	149	1/9	1/9	1/9	120	126	125
130	100	1/9	1/9	1/9	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

	147	126	114				

199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	1/9	1/9	1/9
77	80	72	71	60	1/9	1/9	1/9
68	67	63	58	53	1/9	1/9	1/9

	147	126	114	114	118	122	
	117	108	107	111	113	113	
	99	99	102	106	107	105	
	91	94	93	93	94	94	
	85	86	81	78	78	79	
	76	74	68	62	62	64	

Padding

- The output image is smaller than the input image.
- How to deal with the boundary pixels?
- Padding
  - By constant value (e.g. 0 or the boundary pixel value)
  - By mirroring values
  - …

Padding

0 1/9	0 1/9	0 1/9						
0 1/9	190 1/9	192 1/9	158	111	110	123	130	130
0 1/9	189 1/9	14[illegible] 1/9	108	111	113	120	126	125
	130	100	98	108	113	113	114	120
	85	100	96	104	108	107	101	94
	85	95	98	96	100	103	100	96
	79	94	87	77	69	70	87	84
	77	80	72	71	60	52	59	64
	68	67	63	58	53	51	54	52

81							
	147	126	114	114	118	122	
	117	108	107	111	113	113	
	99	99	102	106	107	105	
	91	94	93	93	94	94	
	85	86	81	78	78	79	
	76	74	68	62	62	64	

199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

81	111	92	79	76	80	84	57
107	147	126	114	114	118	122	83
84	117	108	107	111	113	113	76
66	99	99	102	106	107	105	69
60	91	94	93	93	94	94	62
57	85	86	81	78	78	79	54
52	76	74	68	62	62	64	44
32	47	46	42	38	37	37	25

Moving average (3x3 window)

Let us try this filter kernel.

1	1	1
1	1	1
1	1	1

÷ 9

Input

Output

Moving average (7x7 window)

kernel

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

÷ 49

Input

Output

Computational complexity

- Image size: $N \times N$
- Kernel size: $K \times K$
- What is the computational complexity?
  - At each pixel, K^2 multiplications, then K^2-1 summations
  - Do this for N^2 pixels
  - That is N^2K^2 multiplications and $N^2(K^2-1)$ summations
  - The complexity is $O(N^2K^2)$
- Can we accelerate this?

Separable filter

- If a big filter can be separated as the consecutive operation of two small filters, then we can first filter the input image with small filter 1, then with small filter 2.

$$\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \end{bmatrix} * \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Average in a 2D window

Average across row

Average across column

1/3	1/3	1/3	158	111	110	123	130	130
	189	149	108	111	113	120	126	125
	130	100	98	108	113	113	114	120
	85	100	96	104	108	107	101	94
	85	95	98	96	100	103	100	96
	79	94	87	77	69	70	87	84
	77	80	72	71	60	52	59	64
	68	67	63	58	53	51	54	52

130							

199	192	158	111	110	123	130	130	
189	149	108	111	113	120	126	125	
130	100	98	108	113	113	114	120	
85	100	96	104	108	107	101	94	
85	95	98	96	100	103	100	96	
79	94	87	77	69	70	87	84	
77	80	72	71	60	52	59	64	
68	67	63	58	53	51	1/3	1/3	1/3

130	183	154	126	115	121	128	87
113	149	123	111	115	120	124	84
77	109	102	106	111	113	116	78
62	94	100	103	106	105	101	65
60	93	96	98	100	101	100	65
58	87	86	78	72	75	80	57
52	76	74	68	61	57	58	41
45	66	63	58	54	53	52	35

1/3							
1/3	183	154	126	115	121	128	87
1/3	149	123	111	115	120	124	84
77	109	102	106	111	113	116	78
62	94	100	103	106	105	101	65
60	93	96	98	100	101	100	65
58	87	86	78	72	75	80	57
52	76	74	68	61	57	58	41
45	66	63	58	54	53	52	35

81	111	92	79	76	80	84	57
107	147	126	114	114	118	122	83
84	117	108	107	111	113	113	76
66	99	99	102	106	107	105	69
60	91	94	93	93	94	94	62
57	85	86	81	78	78	79	54
52	76	74	68	62	62	64	44
32	47	46	42	38	37	37	25

The result is exactly the same.

Moving average (separable filter)

1/3	1/3	1/3

$*$

1/3
1/3
1/3

Input

Output

Computational complexity for separable filtering

- Image size: $N \times N$
- Two kernels: first one $1 \times K$, second one $K \times 1$
- What is the computational complexity?
  - At each pixel, K multiplications, then $K - 1$ summations
  - Do this for N^2 pixels
  - Do this twice for two kernels
  - That is $2N^2K$ multiplications and $2N^2(K - 1)$ summations
  - The complexity is $O(N^2K)$, versus the original complexity $O(N^2K^2)$
- It makes a difference if K is large.

What does a moving average filter do?

- It removes high-frequency signal (noise or sharpness).
- Result in a smooth but blurry image.

Moving average (colour image)

Input

Output

Types of image filters

- Identity filter
- Low-pass or smoothing filters
  - Moving average filter
  - Gaussian filter
- High-pass or sharpening filters
- Denoising filters
  - Median filter
- …

Identity filter

	1	

Filter kernel

Identity filter

Input

Output

Gaussian filter

- Kernel: 2D Gaussian distribution

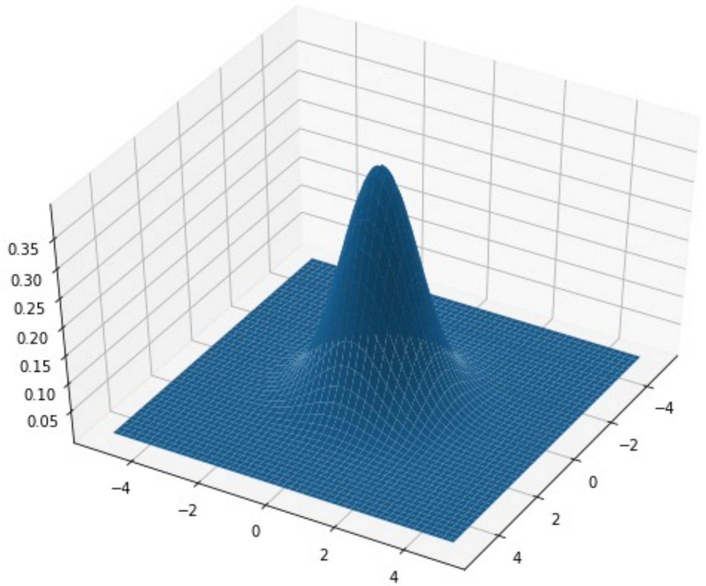
$$h(i,j) = \frac{1}{2\pi\sigma^2} e^{-\frac{i^2+j^2}{2\sigma^2}}$$

  - Its support is infinite, but we may ignore small values outside $[-k\sigma, k\sigma]$, e.g. $k = 3 \; or \; 4$.

- The 2D Gaussian filter is a separable filter, equivalent to two 1D Gaussian filters with the same σ, one along x-axis and the other along y-axis.

$$h(i,j) = h_x(i) \ * h_y(j)$$

$$h_x(i) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{i^2}{2\sigma^2}}$$



Gaussian distribution

0.003	0.013	0.022	0.013	0.003
0.013	0.060	0.098	0.060	0.013
0.022	0.098	0.162	0.098	0.022
0.013	0.060	0.098	0.060	0.013
0.003	0.013	0.022	0.013	0.003

Gaussian filter

Gaussian filter

Input

Output

Gaussian filter

Input

Output

Low-pass vs high-pass filters

- The moving average filter and Gaussian filter smooth or blur the image, keeping the low-frequency signals.
- They are called low-pass filters or smoothing filters.

- There are some filters that sharpen the image and highlight the high-frequency signals.
- They are called high-pass filters or sharpening filters.

High-pass filter

Design 1

$$\underset{\text{Identity}}{\begin{bmatrix} & & \\ & 1 & \\ & & \end{bmatrix}} + \underset{\text{High-frequency}}{\left(\begin{bmatrix} & & \\ & 1 & \\ & & \end{bmatrix} - \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix} \right)} = \begin{bmatrix} -1/9 & -1/9 & -1/9 \\ -1/9 & 1.9 & -1/9 \\ -1/9 & -1/9 & -1/9 \end{bmatrix}$$

Design 2

$$\underset{\text{Identity}}{\begin{bmatrix} & & \\ & 1 & \\ & & \end{bmatrix}} + \underset{\text{High-frequency}}{\begin{bmatrix} -1/8 & -1/8 & -1/8 \\ -1/8 & 1 & -1/8 \\ -1/8 & -1/8 & -1/8 \end{bmatrix}} = \begin{bmatrix} -1/8 & -1/8 & -1/8 \\ -1/8 & 2 & -1/8 \\ -1/8 & -1/8 & -1/8 \end{bmatrix}$$

Design …

High-pass filter

Input

Output

High-pass filter

Input

Output

Median filter

- A non-linear filter
- Often used for denoising
- How to perform median filtering?
  - Move the sliding window
  - Replace the centre pixel using the median value in the window

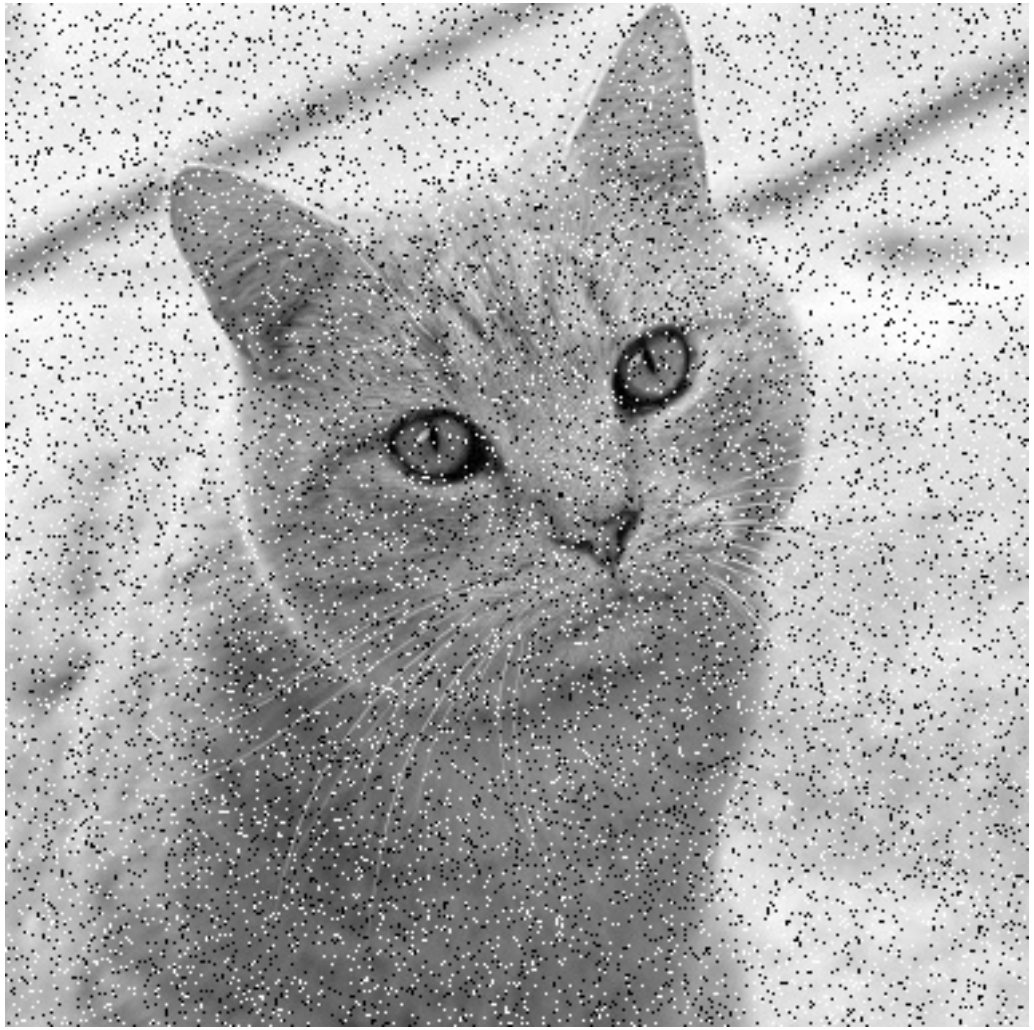
199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

	149						

199	192	158	111	110	123	130	130
189	149	108	111	113	120	126	125
130	100	98	108	113	113	114	120
85	100	96	104	108	107	101	94
85	95	98	96	100	103	100	96
79	94	87	77	69	70	87	84
77	80	72	71	60	52	59	64
68	67	63	58	53	51	54	52

192	189	149	111	111	123	126	130
189	149	111	111	113	114	123	125
130	100	104	108	111	113	114	120
95	98	98	100	107	107	103	100
85	94	96	96	100	100	96	94
80	85	87	77	71	70	84	84
77	77	72	69	60	59	59	64
68	68	67	60	53	53	52	54

Median filter



Corrupted by salt and pepper noise

Denoised image

Median filter

Corrupted by salt and pepper noise

Denoised image

Denoising filter

- Apart from median filter, there are other more complex denoising filters.
  - Non-local means
  - Block-matching and 3D filtering (BM3D)
  - ...

Buades et al. A non-local algorithm for image denoising. CVPR 2005.
Dabov et al. Image denoising by sparse 3-D transform-domain collaborative filtering. IEEE TIP 2007.

Who use these filters?

- Photographers and designers
- Denoising used by cameras
- Youtubers or maybe the person you talk to on Zoom

Photoshop

Smartphone camera

Zoom touch up filter

Image filtering

- Image representation
- Image filters and applications
  - Smoothing
  - Sharpening
  - Denoising

References

- Section 3.2: Linear filtering; Section 3.3.1: Non-linear filtering. Richard Szeliski, Computer Vision: Algorithms and Applications (http://szeliski.org/Book).