60006 - Tutorial 2

Edge Detection, Hough Transform

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Question 1

The horizontally and vertically flipped kernels are defined as:

$$h_{x_{flipped}} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad h_{y_{flipped}} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Question 1.1: If we use zero padding at the boundary pixels, what is gradient magnitude of the second row after being convolved by 3x3 Prewitt filter kernels?

Convolution on the second row:

$$g_x = f * h_{x_{flipped}}$$

$$\begin{bmatrix} 12 & 15 & 33 & 51 & 90 & 75 & 21 & -150 \end{bmatrix}$$

$$g_y = f * h_{y_{flipped}}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Gradient magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$

$$\begin{bmatrix} 12 & 15 & 33 & 51 & 90 & 75 & 21 & 150 \end{bmatrix}$$

Question 1.2: If we use replicate padding (padding using the same value as the boundary pixel), what is the gradient magnitude of the second row?

Convolution on the second row:

$$g_x = f * h_{x_{flipped}}$$

$$\begin{bmatrix} 3 & 15 & 33 & 51 & 90 & 75 & 21 & 6 \end{bmatrix}$$

$$g_y = f * h_{y_{flipped}}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Gradient magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$

$$\begin{bmatrix} 3 & 15 & 33 & 51 & 90 & 75 & 21 & 6 \end{bmatrix}$$

Question 1.2: If we use replicate padding and the 3x3 Sobel filter, what is the result?

The Sobel filters kernels are defined as:

$$h_x = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad h_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

The horizontally and vertically flipped kernels are defined as:

$$h_{x_{flipped}} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad h_{y_{flipped}} = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Convolution on the second row:

$$g_x = f * h_{x_{flipped}}$$
$$\begin{bmatrix} 4 & 20 & 44 & 68 & 120 & 100 & 28 & 8 \end{bmatrix}$$
$$g_y = f * h_{y_{flipped}}$$
$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Gradient magnitude:

$$g = \sqrt{g_x^2 + g_y^2}$$
$$\begin{bmatrix} 4 & 20 & 44 & 68 & 120 & 100 & 28 & 8 \end{bmatrix}$$

Question 2

Calculate the gradient direction for the pixel in the centre using the Prewitt filters.

$$g_x = f * h_x = 10$$
$$g_y = f * h_y = -10$$

Gradient direction:

$$\arctan(\frac{g_y}{g_x}) = -45\,\text{deg}$$

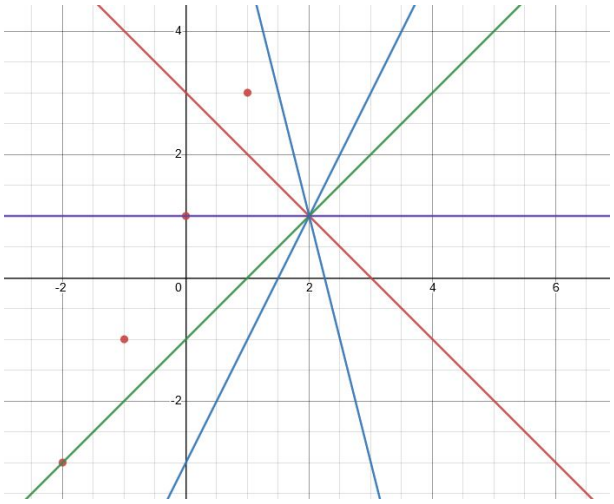
Question 3

The following points appear in an edge map: $(-2,-3), (-1,-1), (0,1), (1,3), (4,9)$. We would like to use the Hough transform to estimate a line model from the points.

- Find the line model parameters for these points. You can use the figure below (abscissa: m, ordinate: b) to do this.

$$(-2,-3) \rightarrow b = -3 + 2x$$
$$(-1,-1) \rightarrow b = -1 + 1x$$
$$(0,1) \rightarrow b = 1$$
$$(1,3) \rightarrow b = 3 - x$$
$$(4,9) \rightarrow b = 9 - 4x$$

- Please sketch the corresponding lines in the parameter space using the slope intercept form for the line model $y = mx + b$, i.e. $b = y - mx$.



The lines sintersect at $(2, 1)$. So the fitted line model is $y = 2x + 1$.

Question 4

Expand the loss function:

$$E(\beta) = Y^T - 2X^T\beta^T Y + X^T\beta^T X\beta$$

Differentiating:

$$\frac{\partial E}{\partial \beta} = -2X^T Y + 2X^T X\beta$$

To get the optimum, let $\frac{\partial E}{\partial \beta} = 0$:

$$-2X^T Y + 2X^T X\beta = 0$$
$$\beta = \frac{X^T Y}{X^T X}$$