

## EE2 Mathematics – Probability & Statistics

### Exercise 3

1. During the first Gulf War, the Pentagon claimed that the Patriot anti-aircraft system had “successfully engaged over 80 percent” of the SCUD missiles Iraq had launched at Saudi Arabia. In televised footage of Patriot-SCUD engagements, there were thirteen misses and one hit. Assuming that the engagements were independent and the probability of a successful engagement is  $p = 0.8$ , compute the probability of thirteen misses and comment on the Pentagon's claim.
2. In chess, a knight can move two squares in one direction and one square in a perpendicular direction. So, for example, a knight placed on the square marked ‘K1’ can move to any of the squares marked 1, while a knight placed on the square ‘K2’ can move to any of the squares marked 2.

	K2						
			2				
2		2					
				1		1	
			1				1
					K1		
			1				1
				1		1	

Find the expected value and the variance of the number of moves that can be made by a knight placed at random on an empty  $8 \times 8$  chessboard.

3. (a) For any random variable  $X$ , show that

$$\text{Var}(X) = \text{E}[X(X \pm 1)] - \text{E}(X)[\text{E}(X) \pm 1].$$

- (b) Use this formula to compute the variance of  $X \sim \text{Bin}(n, p)$ .

Hint: Start with  $\text{E}[X(X - 1)]$  and follow the approach we used to compute the expectation of  $X$ .

4. (a) Consider the infinite series

$$H(q) = \sum_{k=1}^{\infty} kq^{k-1} = 1 + 2q + 3q^2 + \dots$$

Find another infinite series  $G(q)$  such that  $H(q) = \frac{d}{dq}G(q)$ , and use this expression to evaluate  $H(q)$ .

- (b) Consider the infinite series

$$I(q) = \sum_{k=1}^{\infty} k(k-1)q^{k-2} = 2 + 6q + 12q^2 + \dots$$

Show that  $I(q) = \frac{d}{dq}H(q)$ , where  $H(q)$  is defined as in the previous part. Evaluate  $I(q)$ .

- (c) Using the results from the first two parts, find the expectation and variance of the geometric distribution.

Hint: For the variance, it is easier to start with  $E[X(X+1)]$ .