# EE2 Mathematics – Probability & Statistics

## Solution 2

1.

$$\begin{aligned} P(D) &= P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) \\ &= \frac{15}{150} \cdot \frac{1}{3} + \frac{5}{100} \cdot \frac{1}{3} + \frac{5}{50} \cdot \frac{1}{3} \\ &= \frac{1}{30} + \frac{1}{60} + \frac{1}{30} \\ &= \frac{5}{60} = \frac{1}{12} \end{aligned}$$

2. Let F denote failure and N denote a normal environment. Then:

$$P(N) = 0.99 \quad P(\overline{N}) = 0.01 \quad P(F|N) = 0.02 \quad P(F|\overline{N}) = 0.5$$

   (a) The probability of failure is

$$\begin{aligned} P(F) &= P(F|N)P(N) + P(F|\overline{N})P(\overline{N}) \\ &= 0.02 \cdot 0.99 + 0.5 \cdot 0.01 \\ &= 0.0248 \end{aligned}$$

   (b) We require $P(\overline{N}|F)$. Using Bayes' theorem we have:

$$\begin{aligned} P(\overline{N}|F) &= \frac{P(F|\overline{N})P(\overline{N})}{P(F)} \\ &= \frac{0.5 \cdot 0.01}{0.0248} \\ &\approx 0.2016 \end{aligned}$$

3. Let A and B denote the probability that sensors A and B fail, respectively. Given $P(A) = 0.04$, $P(B) = 0.12$, and $P(A \cap B) = 0$, we can construct the table:

| | $A$ | $\overline{A}$ | |
|---|---|---|---|
| $B$ | 0.00 | 0.12 | 0.12 |
| $\overline{B}$ | 0.04 | 0.84 | 0.88 |
| | 0.04 | 0.96 | 1.00 |

   (a) $P(\overline{A}|\overline{B}) = 0.84/0.88 \approx 0.955$

   (b) $P(\overline{B}|\overline{A}) = 0.84/0.96 = 0.875$

4. Again, we can construct a probability table:

| | $A$ | $\overline{A}$ | |
|---|---|---|---|
| $B$ | 0.77 | 0.03 | 0.80 |
| $\overline{B}$ | 0.08 | 0.12 | 0.20 |
| | 0.85 | 0.15 | 1.00 |

(a) $P(A \cap B) = 0.77$

(b) $P\left[(A \cap B) \cup (\overline{A} \cap \overline{B})\right] = 0.77 + 0.12 = 0.89$

(c) $P(B|\overline{A}) = \frac{P(B \cap \overline{A})}{P(\overline{A})} = 0.03/0.15 = 0.2$