1: Introduction

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I. What is a signal

- A function of one or more variables representing data
- Represented by a physical aspects e.g. voltage
- Measured by observing physical aspects

II. Types of signals

A. Continuous-time and discrete-time

- Continuous:
  - Signal defined for all time t
  - E.g. sound
- Discrete:
  - Signal defined only at discrete instants of time nT_s
  - E.g. Digital music

B. Even and odd

- All signals composed of even and odd components

$$x(t) = x_e(t) + x_o(t)$$

- Even:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

  - Positive and negative are the same

$$x_e(t) = x_e(-t)$$

- Odd:

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

  - Positive and negative are different

$$x_o(t) = -x_o(-t)$$

C. Periodic and non-periodic

- Continuous-time periodic T_0 signal:

$$x(t) = x(t + T_0)\ \forall t$$

  where T_0 is the **fundamental period**
- Discrete-time periodic N_0 signal:

$$x[n] = x[n + N_0]$$

  with **fundamental angular frequency** $\Omega_0 = \frac{2\pi}{N_0}$

D. Deterministic and Random signals

- **Deterministic signal**: Specified function of time i.e. no uncertainty about signal value at any observed instant of time
- **Random process**: Describes group of random signals
- **Random signal**: Cannot be determined before occurring
  - Different waveform and occurring probabilities
  - A particular occurrence of a random process

E. Energy and Power signal

- Described in terms of a 1Ω load
- **Instantaneous power**:

$$p(t) = \frac{v^2(t)}{R} = Ri^2(t) = x^2(t)$$

- **Total energy**:

$$E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

$$E = \sum_{n=-\infty}^{n=\infty} x^2[n]$$

- **Average power**:

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

$$P = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

- Signals either **energy signals** or **power signals**
  - **Energy signal**: $0 < E < \infty$ and 0 power
  - **Power signal**: $0 < P < \infty$ and infinite energy
  - Signals can be neither power or energy signals

III. Special signals

- Unit impulse function:
  - Discrete-time:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

  - Continuous-time:

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

  for $t = 0$
- Unit-step function:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u(t) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

- Relationship:

$$\delta(t) = \frac{d}{dt}u(t) \text{ and } u(t) = \int_{-\infty}^{t} \delta(\tau)$$

IV. Signal operations

- Amplification:

$$y(t) = c * x(t)$$
$$y[n] = c * x[n]$$

- Addition:

$$y(t) = x_1(t) + x_2(t)$$
$$y[n] = x_1[n] + x_2[n]$$

- Multiplication or modulation:

$$y(t) = x_1(t) * x_2(t)$$
$$y[n] = x_1[n] * x_2[n]$$

- Differentiation:

$$y(t) = \frac{d}{dt}x(t)$$

- Integration:

$$y(t) = \int_{-\infty}^{t} x\tau d\tau$$

- Time scaling:

$$y(t) = x(at)$$
$$y[n] = x[kn]$$

  - $a > 1$: Compression
  - $0 < a < 1$: Expansion
- Time shifting:
  - $t_0 > 0$: Shift to right i.e. delay

$$y(t) = x(t - t_0)$$

  - $t_0 < 0$: Shift to left i.e. advance

$$y(t) = x(t + t_0)$$

- Time reflection:

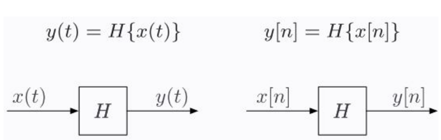
$$y(t) = x(-t)$$

A. *Precedence Rule*

- Time-shift first **before** time-scaling

V. Systems

- **System**: Operations applied on the input signal
- System operations can be represented mathematically



- Two important types of operations H:
  - Linear or non-linear function
  - Time-invariant or time-varying function

A. *Properties of a LTI system*

- Linearity: Satisfies **superposition** and **homogeneity**

$$x(t) = \sum_{i=1}^{N} a_i x_i(t)$$
$$y(t) = \sum_{i=1}^{N} a_i y_i(t)$$

  - Superposition:
    - Input: $x_1(t)$ and $x_2(t)$
    - Output: $y_1(t)$ and $y_2(t)$

$$x_1(t) + x_2(t) = y_1(t) + y_2(t)$$

  - Homogeneity:
    - Input: $x_1(t)$
    - Output: $y_1(t)$

$$x_1(t) = y_1(t)$$
$$a * x_1(t) = a * y_1(t)$$

- Time-invariance: H does not change over time
  - Time-shift in input equals time-shift of output
- Memory and memoryless systems:
  - **Memoryless**: Function depends on current input
  - **Memory**: Function depends on past and future inputs
- Stability: Stable if output bounded if input bounded
- **BIBO**: Bounded-input, bounded-output

$$|y(t)| \leq M_y < \infty \text{ given } |x(t)| \leq M_x < \infty$$

  where M_x and M_y are finite positive bounds
- Most linear system applications require BIBO stability
- Causality: Output depends on **past** and **present** inputs

$$y[n] = 0.25(x[n] + x[n-1] + x[n-2] + x[n-3])$$

- **Non-causal**: Present, future and past input required

$$y[n] = 0.25(x[n+2] + x[n+1] + x[n] + x[n-1])$$

- **Anticausal**: Only future signal required

$$y[n] = 0.33(x[n+3] + x[n+2] + x[n+1])$$