EE2 Mathematics – Probability & Statistics

Solution 6

1. (a) Let T be the component lifetime, $T \sim \text{Exp}(1/2000)$. The probability is

$$P(T > 1000) = 1 - F_T(1000) = e^{-1000/2000} = e^{-0.5} \approx 0.607$$

(b) Let T_1, T_2 be the lifetimes of the two components, and let X be the time to failure of the device. The probability is

$$\begin{aligned} P(X > 1000) &= P(T_1 > 1000, T_2 > 1000) \\ &= P(T_1 > 1000)P(T_2 > 1000) = e^{-0.5}e^{-0.5} \approx 0.368 \end{aligned}$$

(c) Let Y be the time to failure of this device. The probability is

$$\begin{aligned} P(Y > 1000) &= 1 - P(Y < 1000) = 1 - P(T_1 < 1000, T_2 < 1000) \\ &= 1 - P(T_1 < 1000)P(T_2 < 1000) \\ &= 1 - (1 - e^{-0.5})(1 - e^{-0.5}) \approx 0.845 \end{aligned}$$

2. Define X, Y as in the previous answers. We have:

$$\begin{aligned} R_X(t) &= P(X > t) = (e^{-t/2000})^2 = e^{-t/1000} \\ H_X(t) &= -\log R_X(t) = t/1000 \\ h_X(t) &= H_X'(t) = 1/1000 \end{aligned}$$

$$\begin{aligned} R_Y(t) &= P(Y > t) = 1 - (1 - e^{-t/2000})^2 = e^{-t/2000}(2 - e^{-t/2000}) \\ H_Y(t) &= -\log R_Y(t) = t/2000 - \log(2 - e^{-t/2000}) \\ h_Y(t) &= H_Y'(t) = 1/2000 - \frac{e^{-t/2000}/2000}{2 - e^{-t/2000}} = \frac{2 - e^{t/2000}}{1000(1 - e^{t/2000})} \end{aligned}$$

3. Using the standard formulae:

$$\begin{aligned} H_T(t) &= \int_0^t z_T(u)\,du = \int_0^t \frac{\alpha}{u+1}du = [\alpha \log(u+1)]_0^t = \alpha \log(t+1) \\ R_T(t) &= \exp\{-H_T(t)\} = \exp\{-\alpha \log(t+1)\} = (t+1)^{-\alpha} \\ F_T(t) &= 1 - R_T(t) = 1 - (t+1)^{-\alpha} \\ f_T(t) &= F_T'(t) = \alpha(t+1)^{-(\alpha+1)} \end{aligned}$$

for $t > 0$ and zero otherwise.