

1: Introduction

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I. WHAT IS A SIGNAL

- A function of one or more variables representing data
- Represented by a physical aspects e.g. voltage
- Measured by observing physical aspects

II. TYPES OF SIGNALS

A. Continuous-time and discrete-time

- Continuous:
 - Signal defined for all time t
 - E.g. sound
- Discrete:
 - Signal defined only at discrete instants of time nT_s
 - E.g. Digital music

B. Even and odd

- All signals composed of even and odd components

$$x(t) = x_e(t) + x_o(t)$$

- Even:

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

- Positive and negative are the same

$$x_e(t) = x_e(-t)$$

- Odd:

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- Positive and negative are different

$$x_o(t) = -x_o(-t)$$

C. Periodic and non-periodic

- Continuous-time periodic T_0 signal:

$$x(t) = x(t + T_0) \quad \forall t$$

where T_0 is the **fundamental period**

- Discrete-time periodic N_0 signal:

$$x[n] = x[n + N_0]$$

with **fundamental angular frequency** $\Omega_0 = \frac{2\pi}{N_0}$

D. Deterministic and Random signals

- **Deterministic signal**: Specified function of time i.e. no uncertainty about signal value at any observed instant of time
- **Random process**: Describes group of random signals
- **Random signal**: Cannot be determined before occurring
 - Different waveform and occurring probabilities
 - A particular occurrence of a random process

E. Energy and Power signal

- Described in terms of a 1Ω load
- **Instantaneous power**:

$$p(t) = \frac{v^2(t)}{R} = Ri^2(t) = x^2(t)$$

- **Total energy**:

$$E = \lim_{T \rightarrow \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

$$E = \sum_{n=-\infty}^{n=\infty} x^2[n]$$

- **Average power**:

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t) dt$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

- Signals either **energy signals** or **power signals**

- **Energy signal**: $0 < E < \infty$ and 0 power
- **Power signal**: $0 < P < \infty$ and infinite energy
- Signals can be neither power or energy signals

III. SPECIAL SIGNALS

- Unit impulse function:

- Discrete-time:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

- Continuous-time:

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

for $t = 0$

- Unit-step function:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

- Relationship:

$$\delta(t) = \frac{d}{dt} u(t) \text{ and } u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

IV. SIGNAL OPERATIONS

- Amplification:

$$y(t) = c * x(t)$$

$$y[n] = c * x[n]$$

- Addition:

$$y(t) = x_1(t) + x_2(t)$$

$$y[n] = x_1[n] + x_2[n]$$

- Multiplication or modulation:

$$y(t) = x_1(t) * x_2(t)$$

$$y[n] = x_1[n] * x_2[n]$$

- Differentiation:

$$y(t) = \frac{d}{dt}x(t)$$

- Integration:

$$y(t) = \int_{-\infty}^t x\tau d\tau$$

- Time scaling:

$$y(t) = x(at)$$

$$y[n] = x[kn]$$

- $a > 1$: Compression
- $0 < a < 1$: Expansion

- Time shifting:

- $t_0 > 0$: Shift to right i.e. delay

$$y(t) = x(t - t_0)$$

- $t_0 < 0$: Shift to left i.e. advance

$$y(t) = x(t + t_0)$$

- Time reflection:

$$y(t) = x(-t)$$

A. Precedence Rule

- Time-shift first **before** time-scaling