EE2 Mathematics – Probability & Statistics

Solution 1

1. There is more than one way to write some of these:
   (a) $A \cap \overline{B} \cap \overline{C}$
   (b) $A \cap B \cap \overline{C}$
   (c) $A \cap B \cap C$
   (d) $A \cup B \cup C$
   (e) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$
   (f) $(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)$
   (g) $(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$
   (h) $\overline{A} \cap \overline{B} \cap \overline{C}$
   (i) $\overline{A \cap B \cap C}$

2. The main tools are the commutative, distributive and associative laws.
   (a) $(A \cup B) \cap (A \cup \overline{B}) = A \cup (B \cap \overline{B}) = A \cup \emptyset = A$
   (b) $(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap \Omega = A$
   (c) $(A \cup B) \cap (B \cup C) = (A \cap C) \cup B,$
   (d) $(A \cup B) \cap (\overline{A} \cup B) \cap (A \cap \overline{B}) = ((A \cap \overline{A}) \cup B) \cap (A \cap \overline{B}) = B \cap A \cap \overline{B} = \emptyset$
   (e) $(A \cup B) \cap (\overline{A} \cup B) \cap (A \cup \overline{B}) = ((A \cap \overline{A}) \cup B) \cap (A \cup \overline{B}) = B \cap (A \cup \overline{B}) = (B \cap A) \cup (B \cap \overline{B}) = A \cap B.$

3. (a) $\Omega = \{(i, j) \, : \, i, j = 1, 2, \ldots, 6\}.$
   (b) We have:
   $B = \{(i, j) \, : \, i = 1, 2, 3 \text{ and } j = 1, 2, \ldots, 6\}.$
   $C = \{(i, j) \, : \, i + j = 6 \text{ and } i, j = 1, 2, \ldots, 6\}.$
   $P(\{(i, j)\}) = \frac{1}{36}$ for $i, j = 1, 2, \ldots, 6.$
   $P(B) = \frac{18}{36} = \frac{1}{2}$, $\quad P(C) = \frac{5}{36}.$
   $B \cap C = \{(1, 5), (2, 4), (3, 3)\}.$
   $P(B \cap C) = \frac{3}{36} = \frac{1}{12}$

4. (a) $P(A) - P(A \cap B) = 0.1.$
   (b) $P(A) + P(B) - P(A \cap B) = 0.7 + 0.8 - 0.6 = 0.9.$
   (c) $P(A \cap B) - P(A \cap B \cap C) = 0.6 - 0.5 = 0.1.$
   (d) $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) = 1.0.$$