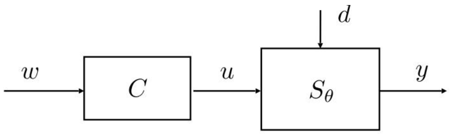
Control systems

Xin Wang

I. Control system



Notation:

- S_Θ: System to be controlled
- C: Controller
- Θ: System parameters
- y: Controlled variable i.e. output
- u: Control variable (accessible)
- d: Disturbance factors
- w: Reference variable i.e. point

A. Control system objective

- Act on u to maintain $y \approx w$ in the presence of uncertainty

$$d = \bar{d} + \Delta d$$
$$\Theta = \bar{\Theta} + \Delta\Theta$$

  - $\bar{d}$ and $\bar{\Theta}$ are known nominal values i.e. expected
  - Δd and $\Delta\Theta$ are uncertainties
- Uncertainty Δd may have a known upper bound

$$|\Delta d| < \bar{D}$$

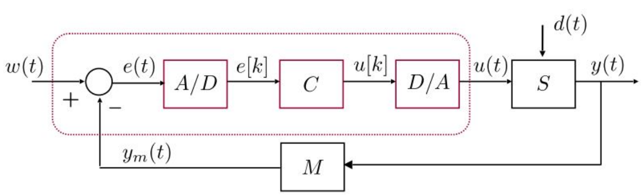
II. Controller

- Two kinds of controllers:
  - Analog: Receives analog inputs and outputs analog
  - Digital: Processes digital sampled variables in computing devices
- Conversion between two types requires: **ADC** and **DAC**
- Converters are synchronised via clock signal - period T_s
- Discrete-time variables can be expressed with time index

$$t_k = kT_s \Rightarrow k$$

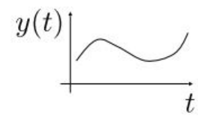
A. Digital control systems

- **Hybrid systems** - analog and digital variables

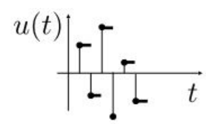


III. Conversions

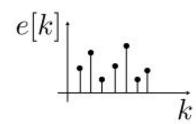
- Types of variables:
  - Continuous-time



  - Piece-wise constants: Constant value between two sampling times

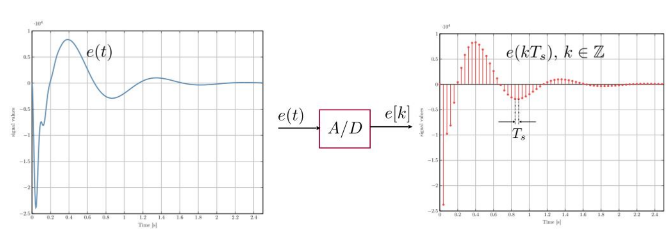


  - Discrete-time



  - Analog: Values change with continuity
  - Digital: Values are quantised

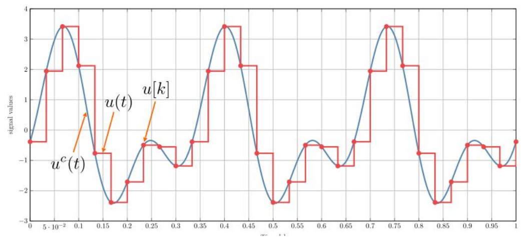
A. A/D conversion



- Conversion uses the **sampling mechanism**
  - **Sampling frequency**: $f_s = \frac{1}{T_s}$
- Implications:
  - Loss of information
  - Quantisation of noise and distortion

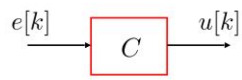
B. D/A conversion

- **Zero-order hold**: Stair-wise delayed approximation continuous-time function

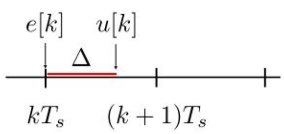


IV. Digital discrete-time controller and control systems

- Controller is **computational algorithm**



- **Temporisation** i.e. delay
- Controller computation time should satisfy: $\Delta < T_s$



A. Control System

- **Error variable**: $e(t) = w(t) - y(t)$
- General requirements:
  - **Static precision**: $y(t) \approx w(t)$ or $e(t) = 0$
  - **Dynamic precision**:
    - Quick enough response time
    - Dampens possible oscillations
    - Able to track varying variables $w(t)$
  - Insensitive to disturbances i.e. reject $d(t)$
  - Robust: Above condition hold for unknown system parameters