EE2 Mathematics – Probability & Statistics

Exercise 8

1. Write down the likelihood and find the maximum-likelihood estimator of the unknown parameter given a random sample $X_1, \ldots, X_n$ from the following distributions:

   (a) Poisson(λ).

   (b) Bin(m, p), where m is known.

   (c) Geo(p).

   (d) Gamma(α, β), where α is known.

   Which of these estimators are unbiased?

   Note: if a random variable X follows a Gamma(α, β) distribution, its PDF is $f_X(x) = \beta^\alpha x^{\alpha-1} e^{-\beta x}/\Gamma(\alpha)$, $x > 0$.

2. Suppose that $X_1, \ldots, X_n$ is a random sample from the exponential distribution with parameter λ. Define $Y = \min_{1 \le i \le n} X_i$, the smallest observation in this sample.

   (a) Show that $F_Y(y) = 1 - (1 - F_{X_1}(y))^n$. What is the distribution of Y?

   (b) Construct an unbiased estimator of $\mu = 1/\lambda$ based only on Y.

   (c) Find the maximum-likelihood estimator of μ and show that it is unbiased.

   (d) Which of the two estimators is better?