EE2 Mathematics – Probability & Statistics

Exercise 9

1. Consider two random variables X_1 and X_2 characterized by the joint probability density function

$$f_{X_1,X_2}(x_1,x_2) = e^{-(x_1+x_2)}, \qquad x_1 \geq 0, x_2 \geq 0.$$

   Make the change of variables $Y_1 = X_1$ and $Y_2 = X_1 + X_2$ and compute the joint probability density function $f_{Y_1,Y_2}(y_1,y_2)$ and the marginal $f_{Y_2}(y_2)$.

2. Consider two random variables X_1 and X_2 characterized by the joint probability density function

$$f_{X_1,X_2}(x_1,x_2) = 8x_1x_2, \qquad 0 < x_1 < x_2 < 1.$$

   Compute the probability density function $f_{Y_1}(y_1)$ of $Y_1 = X_1/X_2$.