EE2 Mathematics – Probability & Statistics

Exercise 5

1. The thickness of manufactured metal plates is normally distributed with mean $\mu = 20$mm and standard deviation $\sigma = 0.04$mm.

   (a) What proportion of the metal plates are thicker than 20.06mm?

   (b) Suppose that we want to set tolerance limits $(20 - k, 20 + k)$ and that metal plates whose thickness falls outside these limits will be scrapped. Find the value of k such that at most 2% scrap is produced.

   (c) Now suppose that the machine was incorrectly calibrated and that the true mean is in fact $\mu = 20.027$mm. If we are using the tolerance limits from part (b), what proportion of the metal plates will be scrapped?

   Pick your answers from:
   (i) 0.079 (ii) 0.035 (iii) 0.067 (iv) 0.048 (v) 0.093 (vi) 0.051 (vii) 0.084

2. The moment-generating function (MGF) of a random variable X is defined as

$$M_X(t) = \mathrm{E}(e^{tX}),$$

   wherever this expectation exists. For example, if $X \sim \text{Bernoulli}(p)$, its MGF is

$$\begin{aligned} M_X(t) = \mathrm{E}(e^{tX}) &= \sum_x e^{tx} P(X = x) \\ &= e^{t.0} P(X = 0) + e^{t.1} P(X = 1) = 1 - p + pe^t . \end{aligned}$$

   Find the MGF for the following distributions:

   (a) Poisson(θ)

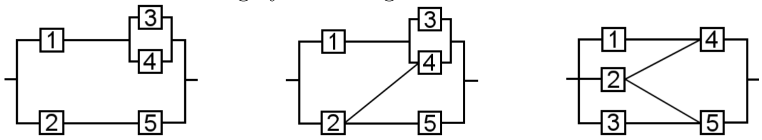
   (b) Geometric(θ)

   (c) $\mathrm{N}(0, \theta^2)$

   (d) Exp(θ)

   Pick your answers from:
   (i) $1 - \theta/t$ (ii) $\theta e^t/(1 - (1 - \theta)e^t)$ (iii) $e^{\theta^2 t^4}$ (iv) θe^{t-1}
   (v) $e^{\theta^2 t^2/2}$ (vi) $e^t/(1 - \theta + e^t)$ (vii) $\exp(\theta e^t - 1)$ (viii) $(1 - t/\theta)^{-1}$

3. Consider the following system diagrams:



Each of the three systems consists of five components with the same probability of failure p. Find the probability that each system functions.

Pick your answers from:

(i) $(1+p)(1-p)(1-p^2)$ (ii) $1-[1-(1-p)^2][1-(1-p)(1-p^2)]$

(iii) $1-[1-(1-p)^2][1-(1-p)^3]$ (iv) $(1-p)(1-p^2)+p\{1-[1-(1-p)^2]^2\}$

(v) $(1+p)(1-p)^2(1-p^2)$ (vi) $(1-p)(1-p)^2+p[1-(1-p)^2]^2$

4. * Suppose that X is a continuous random variable taking values between $-\infty$ and $+\infty$ with CDF $F_X(x)$. Sometimes we want to *fold* the distribution of X about the value $x=a$, that is we want the distribution function $F_Y(y)$ of the random variable Y obtained from X by taking $Y=X-a$ if $X>a$ and $Y=a-X$ if $X<a$. Find $F_Y(y)$ by working out directly $P(Y\le y)$. What is the density function of Y ? A particularly important application is the case when X has a $\mathrm{N}(\mu,\sigma^2)$ distribution, and we want to fold it about the value $a=\mu$. Apply your result to this case.