Analysis Module Documentation

Adam, Brandon Cann, Xin Wang [1]

May 12, 2020
v1.0

[1]document compiled by Xin Wang

Contents

1 Introduction

This document will research the theory of analysing a circuit and design the *Analyser* module. The *Analyser* module will create the matrices required to perform the nodal analysis and, using the matrices, perform a quick scan to ensure the circuit described is a practical circuit. As stated previous in the Netlist documentation, the netlist input format may be correct but the circuit is unrealistic e.g. a resistor is only connected on one end.

The initial stage is will be to implement a version of the software that supports basic components like resistors and voltage source. Support for capacitors and inductors, and non-linear components will be added at a later stage.

This document is primarily used for planning the most effective way to approach the problem as well as allowing fellow teammates to engage productively on a equal knowledge footing.

2 Modified Nodal Analysis

2.1 Node Voltage Method

Building from first principals, node voltage method is the foundational concept required to analyse a circuit but it is not easily converted into an algorithm. Modified Nodal Analysis seeks to establish an universal algorithm for solving a circuit.

- Establish a reference node (Ground)
- Name remaining nodes
- Name current through each voltage source
- Apply KCL at each node. **Currents out of node is taken to be positive.**
- Write equation and solve for unknowns

$$
\begin{aligned}
\frac{v_a}{R_1} - i_{v1} &= 0 \\
i_{v1} + \frac{v_b}{R_3} + \frac{v_b - v_c}{R_2} &= 0 \\
i_{v2} + \frac{v_c - v_b}{R_2} &= 0 \\
v_b - v_a &= V_1 \\
v_c &= V_2
\end{aligned}
\qquad
\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & -1 & 0 \\ 0 & \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} & 1 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\begin{bmatrix} v_a \\ v_b \\ v_c \\ i_{v1} \\ i_{v2} \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ V1 \\ V2 \end{bmatrix}
$$

Figure 1: Process of conversion of equations into a matrix [1]

2.2 Characteristics of the MNA matrix

All circuits when presented in matrix for, will always result in the form:

$$Ax = b$$

In the given example matrix[1]:

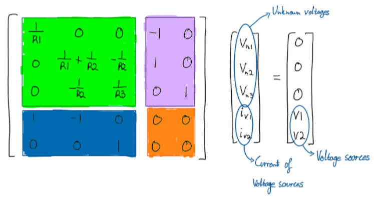

Figure 2: Characteristics of $Ax = b$

The following characteristics will always hold:

- Matrix A
  - The highlighted part is size $[N \times N]$ and contains values of passive elements
  - The main diagonal is positive values only. Non-diagonal values are either 0 or negative.
  - If an element is grounded, it will appear along diagonal.
- Matrix x: Matrix of unknown quantities:
  - General dimension: $[(N + M) \times 1]$
  - Top N elements are simply node voltages
  - Bottom M elements are the currents related to voltage sources[2]
- Matrix b: Matrix containing known quantities:
  - General dimension: $[(N + M) \times 1]$
  - Top N elements are either 0 or sum of independent current sources
  - Bottom M elements are independent voltage sources

Solution is found by:

$$x = A^{-1}b$$

[1] Red highlight is $[N \times N]$ and matrix is $[(N + M) \times (N + M)]$ where N is the nodes in the circuit and M is the number of independent sources.

[2] Dependent current and voltage sources have not been considered yet

2.3 Notations

The naming notation is the following:[1].

- Ground is labelled **Node 0**
- Each node is given a label starting from 1 to N
- Node voltage name: v_N
- Current through V sources: $i\_VoltageSourceName$
- Independent voltage sources: $vname$
- Independent voltage sources: $iname$

3 Algorithm design for MNA

Matrices need that need to be generated:

- A
- x
- b

3.1 A matrix

The A matrix has dimension $[(M+N) \times (M+N)]$ and is combined from four smaller matrices: A_a, A_b, A_c and A_d [1].

The respective matrices are defined as:

- A_a: $n \times n$ matrix - Passive element connections
- A_b: $n \times m$ matrix - Voltage source connections
- A_c: $m \times n$ matrix - Similar to matrix A_b
- A_d: $m \times m$ matrix - 0 if independent sources are considered

3.1.1 A_a matrix

- Size $N \times N$
- Each element in diagonal matrix is the conductance of each element connected to respective node
- Elements not on diagonal are negative conductances of the element connected to respective node.
- Non-grounded element will have one entry.
- Non-grounded element will have four entries.

For example:

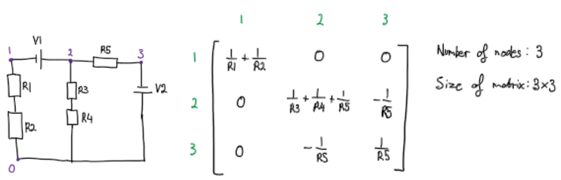

Figure 3: The relationship between diagonal and non-diagonal elements

3.1.2 A_b matrix

- Size $N \times M$
- Only 1, 0 and -1 entries
- Direction of voltage source matter. Negative terminal is -1 and positive terminal is 1.
- A grounded voltage source will have one entry.
- Non-grounded voltage source will have two entries.

The positive terminal of V1 in the above circuit is connected to node 2 as indicated in matrix element $[1, 2]$ where 1 is Vi and 2 is the node the terminal is connected to.

For example:

$$\begin{array}{c c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \end{array}$$

Figure 4: Matrix showing the direction of voltage sources

3.1.3 A_c matrix

- Size $M \times N$
- **Usually the transpose of A_b**
  - Not the case if dependent sources are involved.

3.1.4 A_d matrix

- Size $M \times M$
- Usually composed of 0
  - Not the case if dependent sources are involved.

3.2 x matrix

The x matrix has dimension $[N \times M]$ and is combined from two smaller matrices: x_a and x_b.

The respective matrices are defined as:

- x_a: $N \times 1$ - Unknown node voltages
- x_b: $M \times 1$ - Unknown currents through voltage sources

3.2.1 x_a matrix

The naming follows the notation set out earlier.

There is no entry for Ground (Node 0).

Figure 5: General notation of the x_a matrix

3.2.2 x_b matrix

Corresponds to the number of voltage sources. Example given in Figure 2.

3.3 b matrix

This matrix has dimension $[(N \times M)) \times 1]$ and contains the independent current and voltage sources.

The matrix is composed of two matrices: b_a and b_b:

- b_a: $N \times 1$ Either 0 or the sum of independent current sources.
- b_b: $M \times 1$ Values of independent voltage sources.

4 Implementation

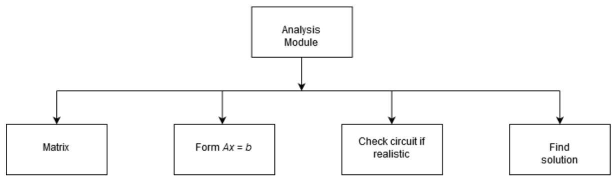

Figure 6: *Analysis module* breakdown

4.1 struct Matrix

A matrix is represented by a vector of vectors.

A breakdown of the Matrix data type and its methods.

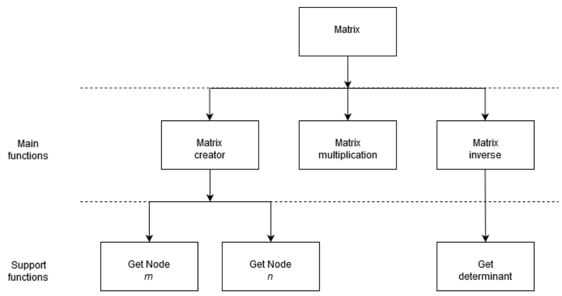

Figure 7: *Matrix submodule* breakdown

References

[1] Professor Erik Cheever. *Analysis of Circuits*. 2005 - 2019. DOI: https://lpsa.swarthmore.edu/Systems/Electrical/mna/MNA1.html.