Question 4.

(a) ① Logical model:

If we define 'goat in A' as 'A'; 'goat in B' as 'B'; 'goat in C' as 'C' we model the info as follows:

$$\begin{cases} A \vee B \vee C \\ A \Rightarrow \neg B \wedge \neg C \\ B \Rightarrow \neg A \wedge \neg C \\ C \Rightarrow \neg A \wedge \neg B \end{cases}$$

② probabilistic model.

with no prior knowledge, we have $P(A) = P(B) = P(C) = \frac{1}{3}$

(b) From logical model, we can not judge which action is a better choice.

(c) From probabilistic model, $P(A) = P(B) = P(C)$, so there is not an obvious choice of a best action.

(d) update logical model

If we choose location A, and CBMHBot tells there is not a goat in location B, so we have $\neg B$.

$\because B \Rightarrow \neg A \wedge \neg C$

$\therefore \neg B \Rightarrow A \vee C$

(e) Update probabilistic model.

⇩

$$P_{new}(A) = P(A|\neg B) = \frac{P(A, \neg B)}{P(\neg B)} = \frac{P(A, \neg B, C) + P(A, \neg B, \neg C)}{1 - P(B)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

(* ∵ A and C can not be true at the same time, so $P(A, \neg B, C) = 0$)

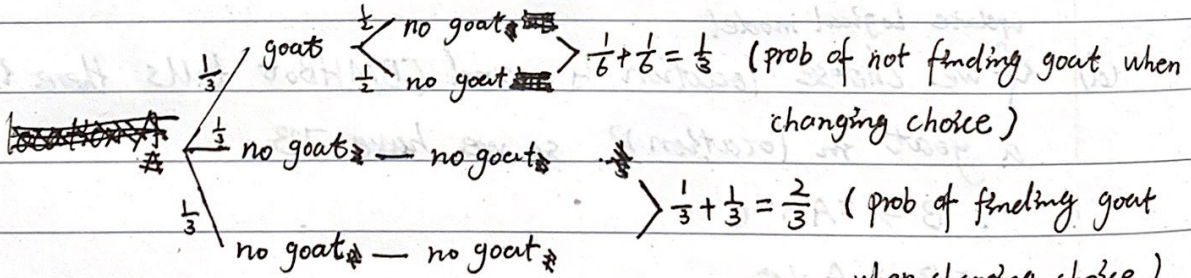
like above, $P_{new}(C) = P(C|\neg B) = \frac{1}{2}$

(f) under the info we infered from ~~logistic~~ logical model in (d), we have 'A∨C'. But we can still not tell which is a better choice between A and C

(g) Under the probability we computed from probabilistic model in (e), we have $P(A) = \frac{1}{2}$ and $P(C) = \frac{1}{2}$, still, we can not tell which is a better choice between A and C.

(h) like what we describe in (f), we can not decide whether to stick with location A or change.

(i)



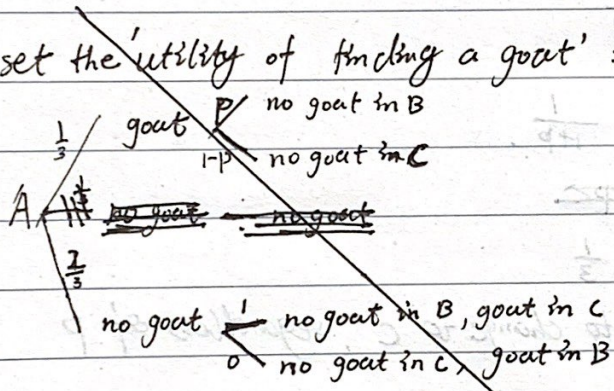
~~CBMHBot's choice~~

CD Bot's choice | CBMHBot's choice

As we can see from the graphs above, ~~if~~ ~~not~~ sticking to the initial choice leads to ~~a~~ a probability of $\frac{1}{3}$ of finding a goat. While changing the choice leads to a probality of ~~finding~~ ~~goat~~ $\frac{2}{3}$ of finding a goat.

(i) From ~~above~~ all the analysis before, ~~the~~ Probabilistic Goat Discovery Bot is more successful. When under the ~~help~~ info of CBMHBot, Probabilistic Goat Discovery Bot reach a probability of $\frac{2}{3}$ of finding a goat. While Logical Goat Discovery Bot only has a probability of $\frac{1}{2}$ of finding a goat.

Bonus: set the 'utility of finding a goat' = 1



utility of sticking = $\frac{1}{3} \times 1 + \frac{2}{3} \times 0 = \frac{1}{3}$

utility of changing to C = $\frac{1}{3}(P + (1-P)) \times 0 + \frac{2}{3} \times 1 \times 1 = \frac{2}{3}$

Rational choice is to change to C, regardless of P.

* redo Bonus in Next Page!

Bonus. let in_i represent goat in i location; O_i represent the place CBMHBot looks at; C_i represent the location we choose.

$\therefore P(in_3 \mid O_2, C_1)$ represent given that we choose A, CBMHBot looks at B, and the goat is in the C

$$P(in_3 \mid O_2, C_1) = \frac{P(O_2 \mid in_3, C_1) P(in_3 \mid C_1)}{P(O_2 \mid C_1)}$$

$$= \frac{P(O_2 \mid in_3, C_1) \cdot P(in_3 \mid C_1)}{\sum_{i=1}^{3} P(O_2 \mid in_i, C_1) P(in_i \mid C_1)}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 0 \times \frac{1}{3} + p \times \frac{1}{3}}$$

$$= \frac{1}{1+p}$$

So changing to C leads to $\frac{1}{1+p}$.

$\therefore p < 1 \quad \therefore \frac{1}{1+p} > \frac{1}{3}$

$\therefore$ rational choice is to change to C, regardless of p