# Random Walk, Diffusion, and Cluster Growth

Connor Hann, Xiaomeng Jia, Peifan Liu and Xinyu Wu

Physics Department, Duke University

March 24, 2016

Duke
UNIVERSITY

# Outline

Duke

# 1D Random Walk

RMS distance of an ensemble of random walkers after $n$ steps

$$\sqrt{<x_n^2>} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}<\Delta x_i \Delta x_j>} = \sqrt{n}\Delta x \tag{1}$$

Diffusive motion:
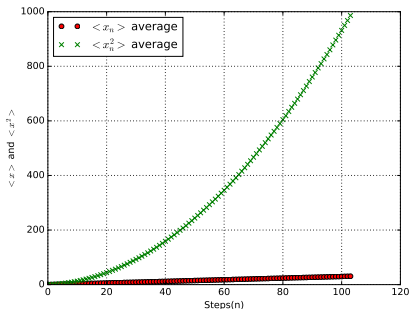
$$<x^2(t)> = 2Dt \tag{2}$$

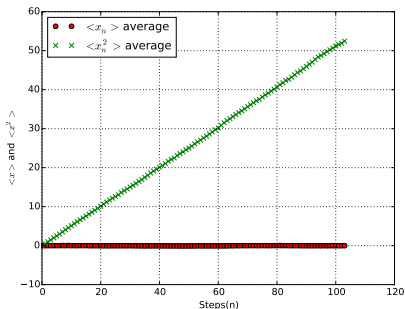where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.
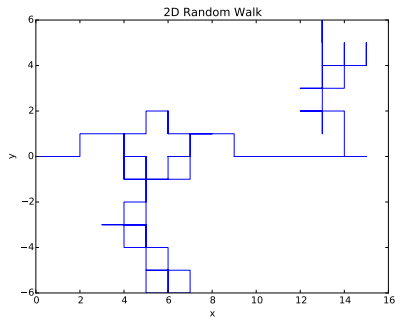
Duke

# 2D Random Walk (1)

Diffusive motion:

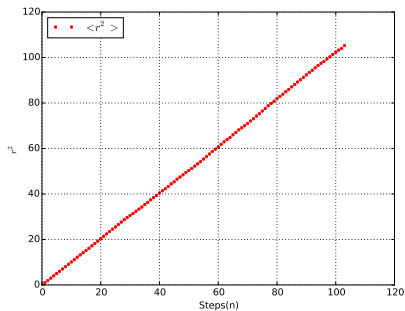$$< x^2(t) >=< y^2(t) >= \frac{< r^2(t) >}{2} = 2Dt \tag{3}$$

where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.



left: random walking; right: biased random walking (0.4 for up and right, 0.1 for down and left)

# 2D Random Walk (2)

# Diffusion

Diffusion equation

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho \tag{4}$$

discretize $t = k\Delta t, x = i\Delta x$,

$$\rho_{i,k+1} = \rho_{i,k} + D\frac{\Delta t}{\Delta x^2}(\rho_{i+1,k} + \rho_{i-1,k} - 2\rho_{i,k}) \tag{5}$$

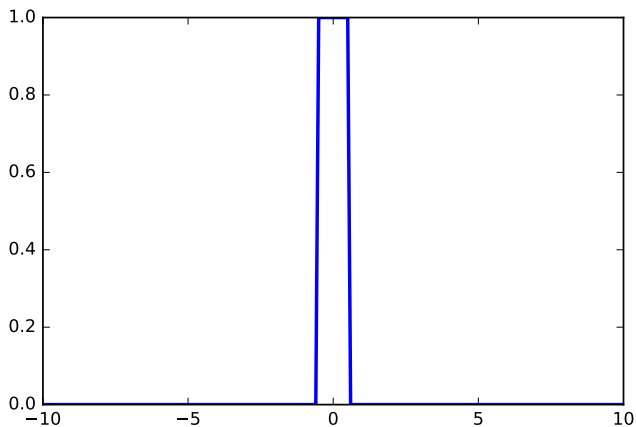with $\Delta t < \frac{\Delta x^2}{2D}$

Duke

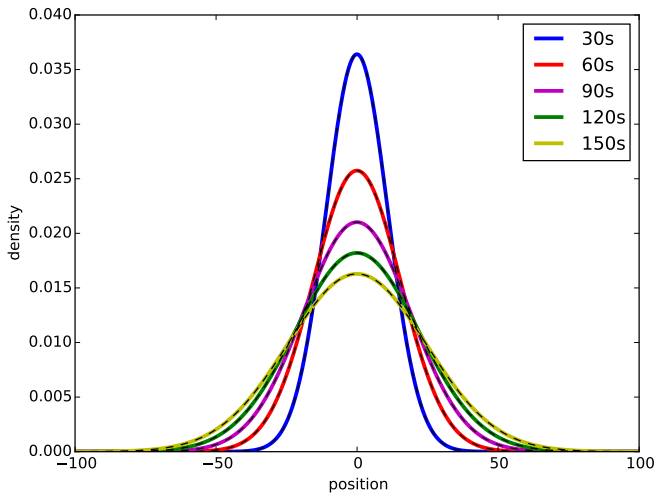# Normal Distribution

One-dimensional normal distribution

$$\rho(x, t) = \frac{1}{\sqrt{2\pi\,\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \tag{6}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(t)} x^2 exp(-\frac{x^2}{2\sigma(t)^2}) = \sigma(t)^2 \tag{7}$$

# Animation

# Diffusion time dependence



$$\langle x^2 \rangle = \sigma(t)^2 = 2Dt$$

# Cluster Growth with Diffusion Limited Aggregation (DLA)

DLA Algorithm:

1. Consider a lattice of points with a seed particle at the origin
2. Release a particle from a random location a distance $R$ from the origin
3. Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
4. If the particle hits the perimeter, add the particle to the cluster
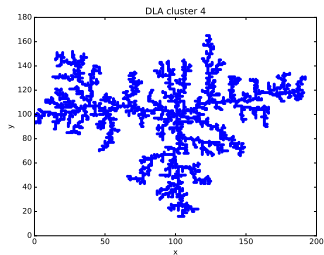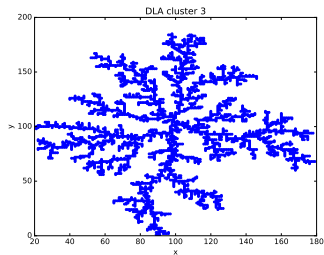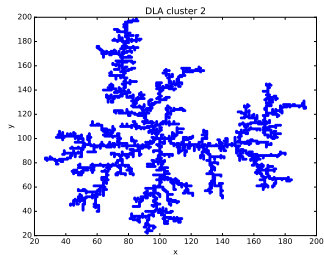5. Repeat until cluster reaches the desired size(Radius $= 100$)
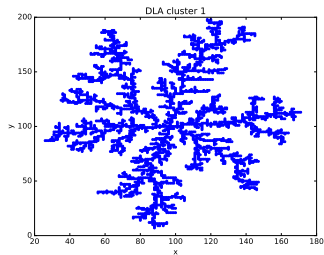
We used a 2D Array for the lattice, and 0 for unoccupied lattice point, 1 for occupied lattice point.
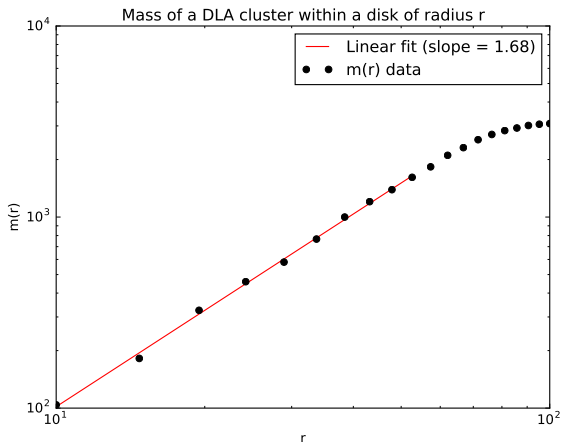Animation

Duke

# Example DLA cluster
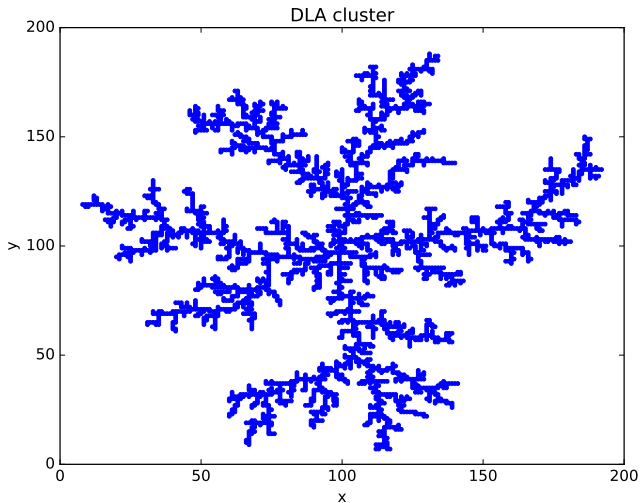
# Typical clusters

# Fractal Dimension $d_f$ (1)

$$m(r) \sim r^{d_f}$$
$$\log(m) = d_f \log(r)$$



Mass of a DLA cluster within a disk of radius r

# Fractal Dimension $d_f$ (2)

| Cluster | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---------|------|------|------|------|------|------|------|------|------|------|
| $d_f$ | 1.74 | 1.63 | 1.77 | 1.55 | 1.66 | 1.58 | 1.64 | 1.74 | 1.74 | 1.75 |

Average value over 10 runs: $d_f = 1.68(07)$
Expected value: $d_f = 1.65$

Duke
UNIVERSITY