Random Walk, Diffusion and Cluster Growth

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Abstract

In this article,

INTRODUCTION

1. 2D RANDOM WALK

A random walk is a mathematical formalization of a path that consists of a succession of random steps. For one dimensional case, a random walker can move one step in $\pm x$ directions with equal probability at each time step. If we make consider the statistical properties of an ensemble of random walkers, the expected value of the position $< x >$ is zero, since the expectation of each step is zero. However, the root-mean-squared distance(RMS) after n steps is:

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n}\Delta x \tag{1}$$

Here Δx is the step unit corresponding to a time unit Δt and a constant velocity v:

$$\Delta x = v \cdot \Delta t \tag{2}$$

Combined with

$$n = \frac{t}{\Delta t} \tag{3}$$

We can show that the motion is diffusive:

$$< x^2(t) >= 2Dt \tag{4}$$

where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

If we generalize the deduction to the 2D case, where the random walkers can move one step in four diections $(\pm x, \pm y)$ with equal probability, we can find that the motion is again diffusive, by evaluating the mean square distance:

$$< r^2(t) >= 2Dt \tag{5}$$

Where $D = (\Delta x)^2/(4\Delta t)$.

This argument can be verified numerically using Python. We write a program to simulate a random walker in 2 dimensions, taking steps of unit length in $\pm x$ or $\pm y$ direction on a discrete square lattice. For up to $n = 100$, by averaging over 10^4 different walks for each $n > 3$, the expected value of the position $< x >$ is zero at all time, and the mean square value $< x^2 >$ has a linear relation with n(remember $n = t/\Delta t$ is proportional to t), as can be seen in Fig.1. A typical 2D random walking pattern is show in Fig.2. Since we choose $\Delta t = \Delta x = 1$ here, the diffusion constant is determined using the slope of $< r^2(t) >$ and n(see Fig.3):$D = 1/4$.

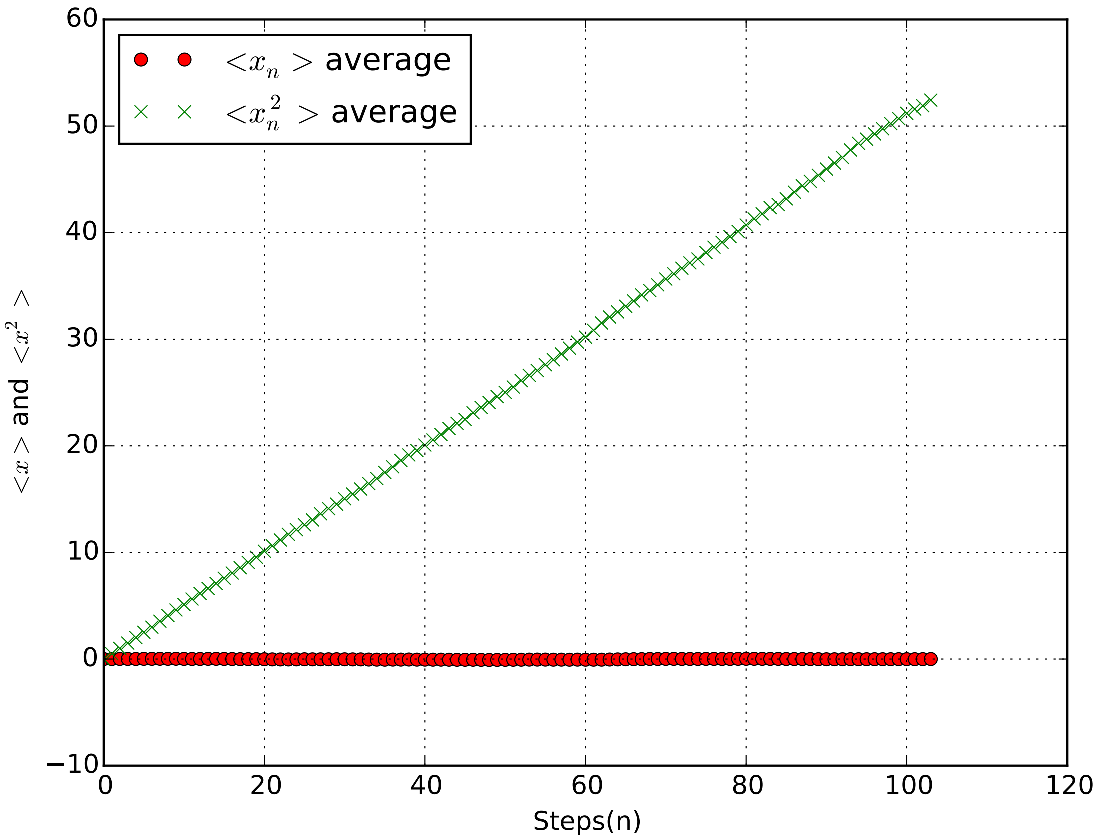

FIG. 1: $< x(t) >$ and $< x^2(t) >$ in 1D random walking, averaging over a 10000 random walker ensemble.

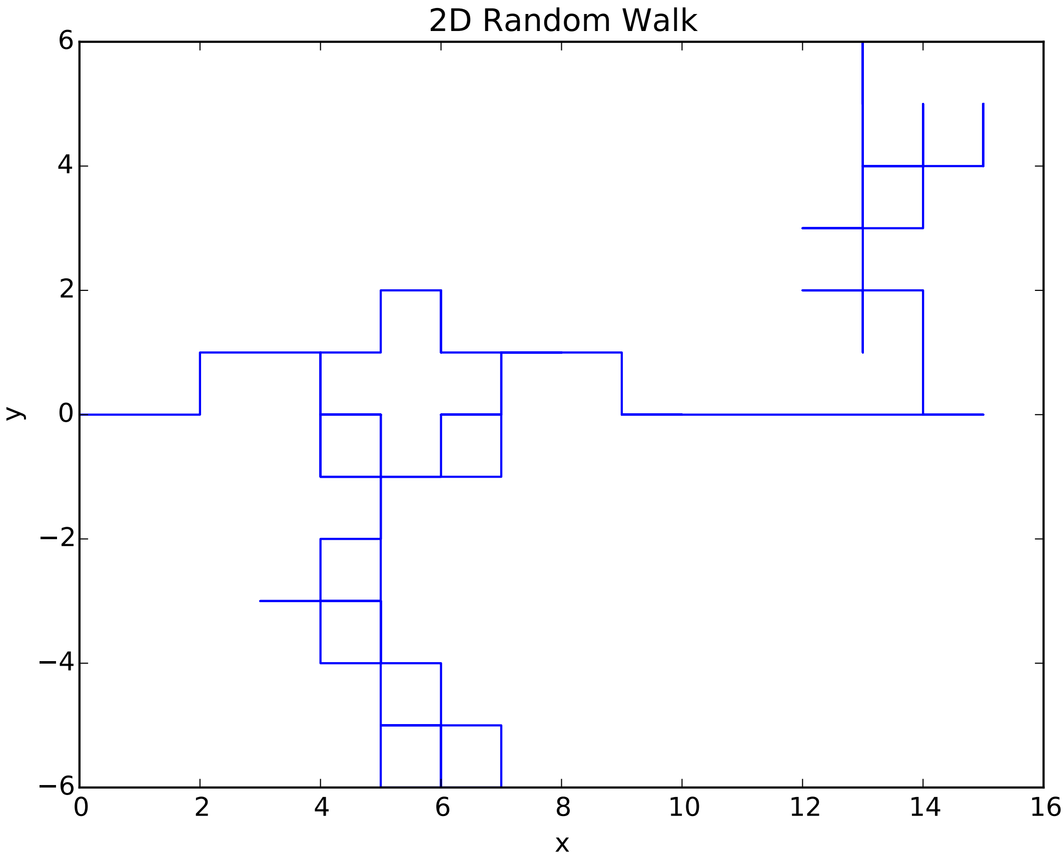

FIG. 2: A tipical 2D random walking pattern, starting from (0,0).

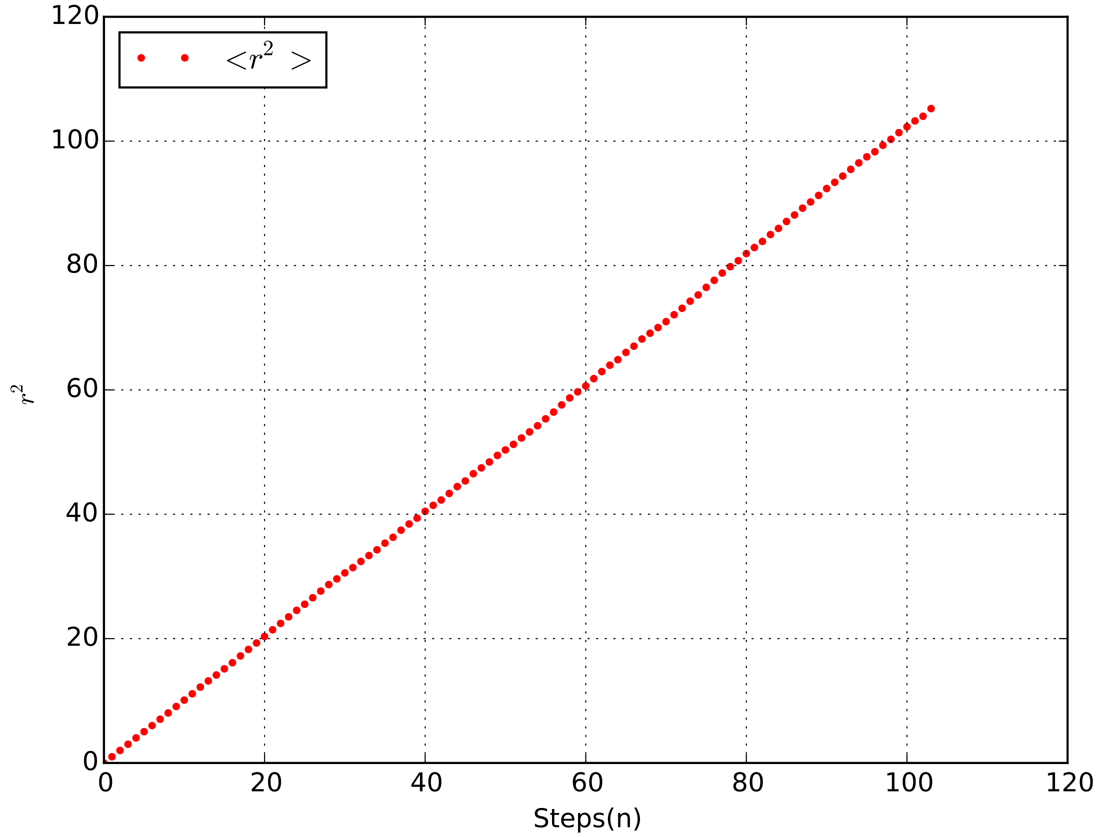

FIG. 3: $< r^2(t) >$ in 2D random walking, averaging over a 10000 random walker ensemble.

2. DIFFUSION EQUATION

3. CLUSTER GROWTH WITH THE DLA MODEL