Random Walk, Diffusion, and Cluster Growth

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March 24, 2016

Outline

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1D Random Walk

RMS distance of an ensemble of random walkers after n steps

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n}\Delta x \tag{1}$$

Diffusive motion:

$$< x^2(t) > = 2Dt \tag{2}$$

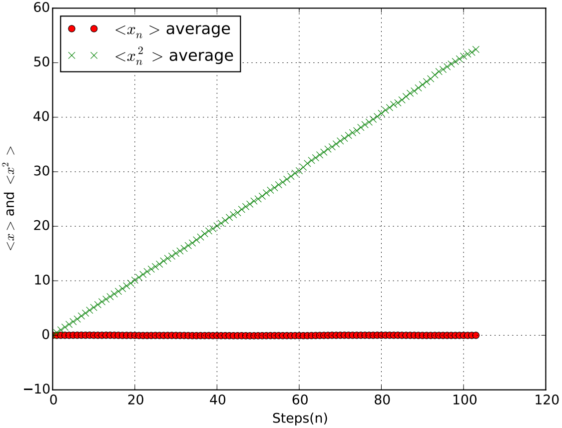
where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

2D Random Walk (1)

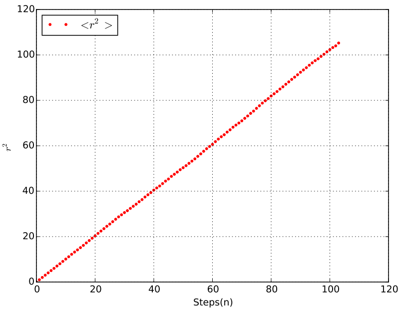
Diffusive motion:

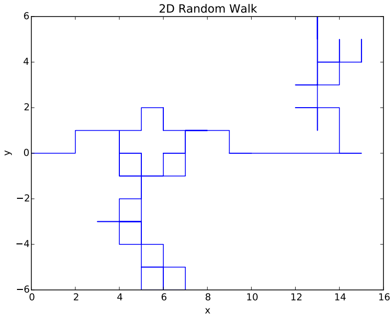
$$< r^2(t) >= 2Dt \tag{3}$$

Where $D = (\Delta x)^2/(4\Delta t)$.



2D Random Walk (2)





Diffusion

Diffusion equation

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho \tag{4}$$

discretize $t = k\Delta t, x = i\Delta x$,

$$\rho_{i,k+1} = \rho_{i,k} + D\frac{\Delta t}{\Delta x^2}(\rho_{i+1,k} + \rho_{i-1,k} - 2\rho_{i,k}) \tag{5}$$

with $\Delta t < \frac{\Delta x^2}{2D}$

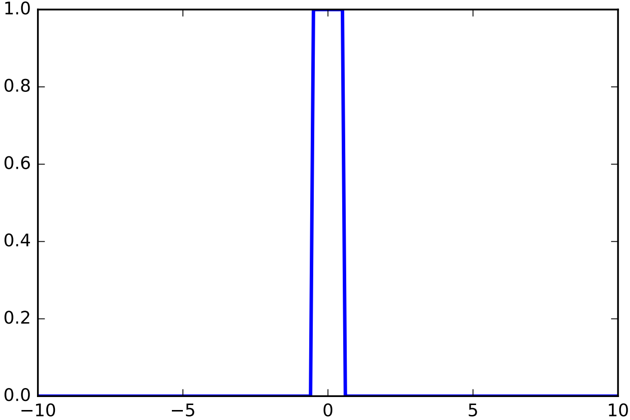
Normal Distribution

One-dimensional normal distribution

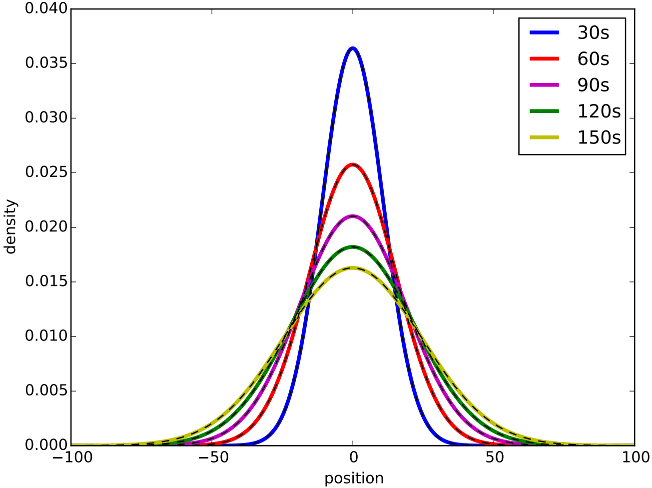
$$\rho(x,t) = \frac{1}{\sqrt{2\pi\,\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \tag{6}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(t)} x^2 exp(-\frac{x^2}{2\sigma(t)^2}) = \sigma(t)^2 \tag{7}$$

Animation



Diffusion time dependence



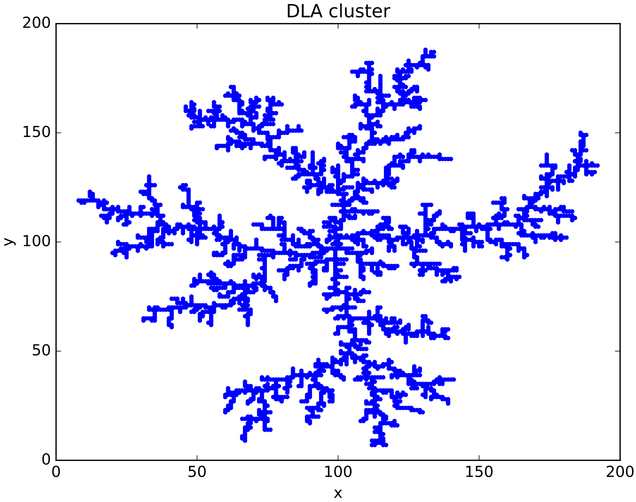
$$\langle x^2 \rangle = \sigma(t)^2 = 2Dt \tag{8}$$

Cluster Growth with Diffusion Limited Aggregation (DLA)

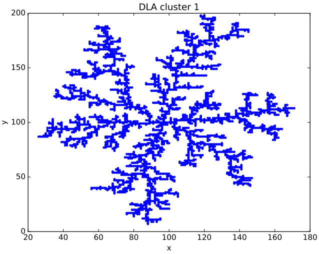
DLA Method:

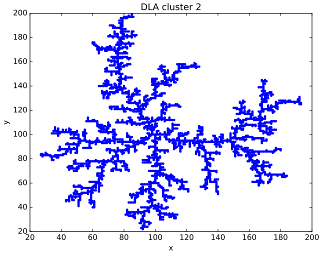
1. Consider a lattice of points with a seed particle at the origin
2. Release a particle from a random location a distance R from the origin
3. Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
4. Repeat until cluster reaches the desired size

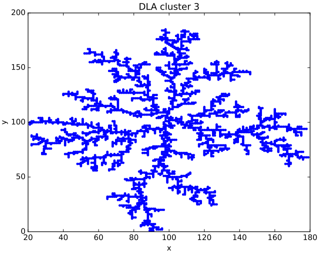
Example DLA cluster

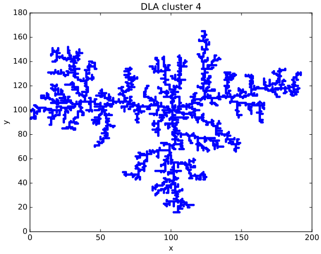


Typical clusters



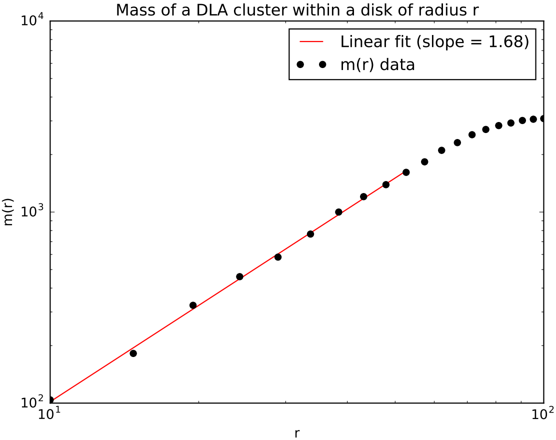






Fractal Dimension d_f (1)

$$m(r) \sim r^{d_f}$$
$$\log(m) = d_f \log(r)$$



Fractal Dimension d_f (2)

Cluster	1	2	3	4	5	6	7	8	9	10
d_f	1.74	1.63	1.77	1.55	1.66	1.58	1.64	1.74	1.74	1.75

Average value over 10 runs: $d_f = 1.68(07)$
Expected value: $d_f = 1.65$