Random Walk, Diffusion and Cluster Growth

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Abstract

In this article,

INTRODUCTION

1. 2D RANDOM WALK

A random walk is a mathematical formalization of a path that consists of a succession of random steps. For one dimensional case, a random walker can move one step in $\pm x$ directions with equal probability at each time step. If we make consider the statistical properties of an ensemble of random walkers, the expected value of the position $< x >$ is zero, since the expectation of each step is zero. However, the root-mean-squared distance(RMS) after n steps is:

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n}\Delta x \tag{1}$$

Here Δx is the step unit corresponding to a time unit Δt and a constant velocity v:

$$\Delta x = v \cdot \Delta t \tag{2}$$

Combined with

$$n = \frac{t}{\Delta t} \tag{3}$$

We can show that the motion is diffusive:

$$< x^2(t) >= 2Dt \tag{4}$$

where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

If we generalize the deduction to the 2D case, where the random walkers can move one step in four diections $(\pm x, \pm y)$ with equal probability, we can find that the motion is again diffusive, by evaluating the mean square distance:

$$< r^2(t) >= 2Dt \tag{5}$$

Where $D = (\Delta x)^2/(4\Delta t)$.

2. DIFFUSION EQUATION

3. CLUSTER GROWTH WITH THE DLA MODEL