

# Random Walk, Diffusion, and Cluster Growth

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# Outline

## 1 2D Random Walk

- One-dimensional random walks
- Two-dimensional random walks

## 2 Diffusion

- Mathematical background
- Numerical results

## 3 Crystal Growth

- Background
- Fractal Dimension

# 1D Random Walk

RMS distance of an ensemble of random walkers after  $n$  steps

$$\sqrt{\langle x_n^2 \rangle} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n \langle \Delta x_i \Delta x_j \rangle} = \sqrt{n} \Delta x \quad (1)$$

Diffusive motion:

$$\langle x^2(t) \rangle = 2Dt \quad (2)$$

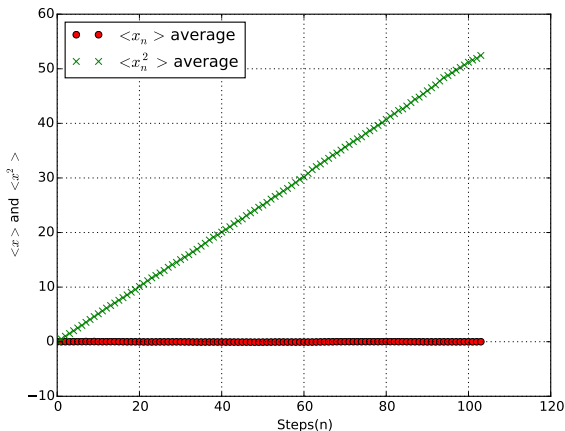
where  $D = v \cdot \Delta x / 2 = (\Delta x)^2 / (2\Delta t)$  is the diffusion constant.

## 2D Random Walk (1)

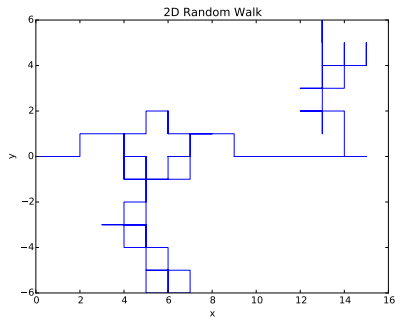
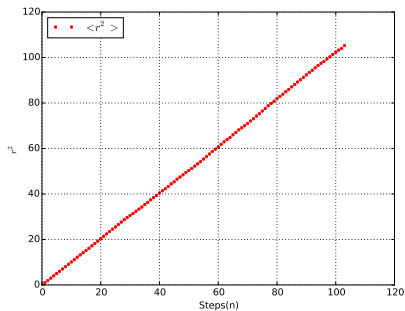
Diffusive motion:

$$\langle r^2(t) \rangle = 2Dt \quad (3)$$

Where  $D = (\Delta x)^2 / (4\Delta t)$ .



## 2D Random Walk (2)



# Diffusion

Diffusion equation

$$\frac{\partial \rho}{\partial t} = D \nabla^2 \rho \quad (4)$$

discretize  $t = k\Delta t, x = i\Delta x,$

$$\rho_{i,k+1} = \rho_{i,k} + D \frac{\Delta t}{\Delta x^2} (\rho_{i+1,k} + \rho_{i-1,k} - 2\rho_{i,k}) \quad (5)$$

with  $\Delta t < \frac{\Delta x^2}{2D}$

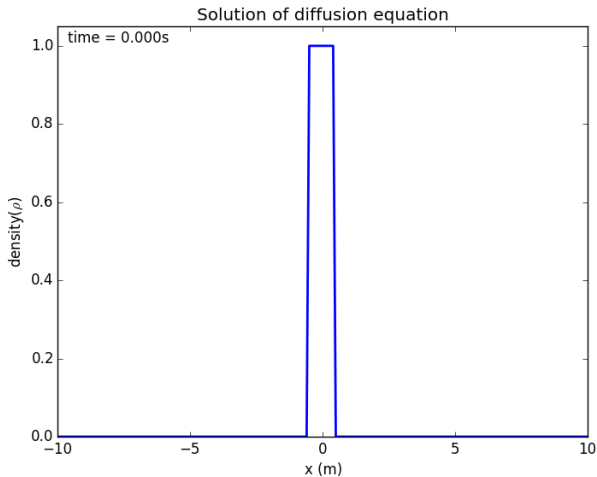
# Normal Distribution

One-dimensional normal distribution

$$\rho(x, t) = \frac{1}{\sqrt{2\pi \sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \quad (6)$$

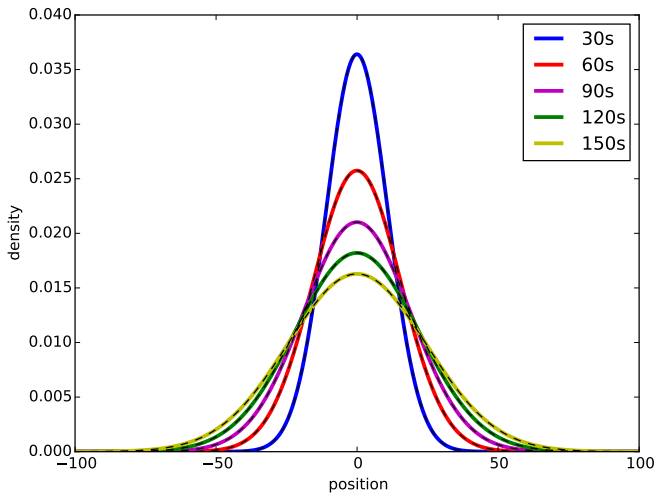
$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi \sigma(t)}} x^2 \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) = \sigma(t)^2 \quad (7)$$

# Animation





# Diffusion time dependence



$$\langle x^2 \rangle = \sigma(t)^2 = 2Dt$$

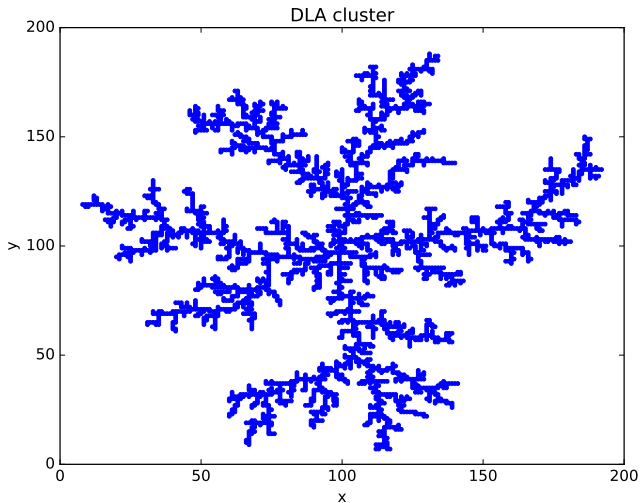
# Crystal Growth with Diffusion Limited Aggregation (DLA)

## DLA Algorithm:

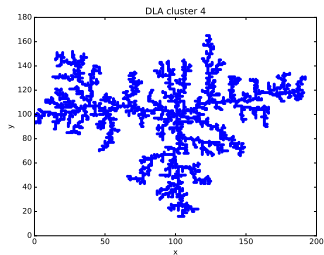
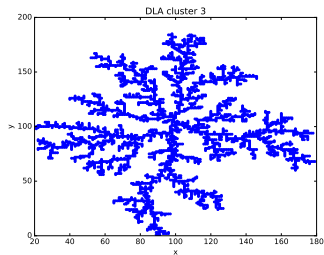
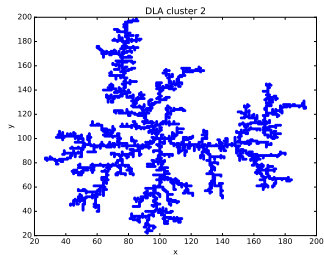
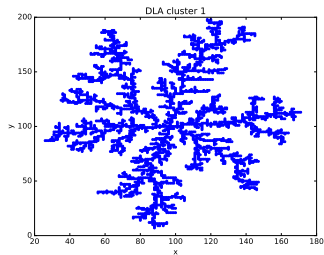
- 1 Consider a lattice of points with a seed particle at the origin
- 2 Release a particle from a random location a distance  $R$  from the origin
- 3 Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
- 4 Repeat until the cluster reaches the edge of the circle(Radius=100)

## Animation

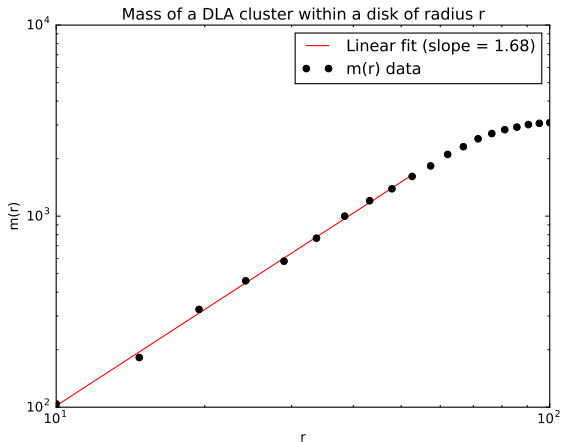
# Example DLA cluster



# Typical clusters



# Fractal Dimension $d_f$ (1)



## Fractal Dimension $d_f$ (2)

Cluster	1	2	3	4	5	6	7	8	9	10
$d_f$	1.74	1.63	1.77	1.55	1.66	1.58	1.64	1.74	1.74	1.75

Average value over 10 runs:  $d_f = 1.68(07)$

Expected value:  $d_f = 1.65$