# Random Walk, Diffusion, and Cluster Growth

Xinyu Wu, Peifan Liu, Connor Hann, and Xiaomeng Jia

Physics Department, Duke University

March 24, 2016

# Outline

1. 2D Random Walk
   - One-dimensional random walks
   - Two-dimensional random walks
2. Diffusion
   - Mathematical background
   - Numerical results
3. Crystal Growth
   - Background
   - Fractal Dimension

# 1D Random Walk

RMS distance of an ensemble of random walkers after $n$ steps

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n}\Delta x \tag{1}$$

Diffusive motion:

$$< x^2(t) >= 2Dt \tag{2}$$

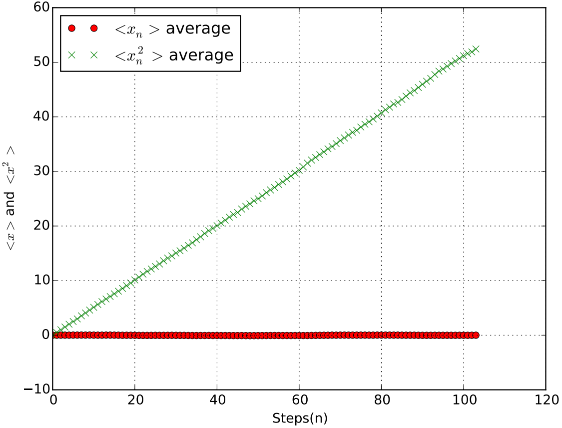
where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

# 2D Random Walk (1)

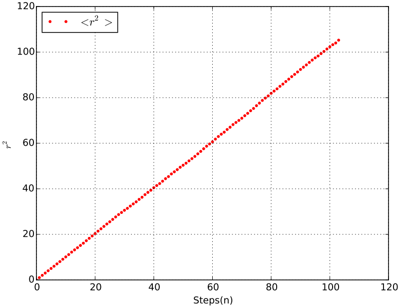
Diffusive motion:

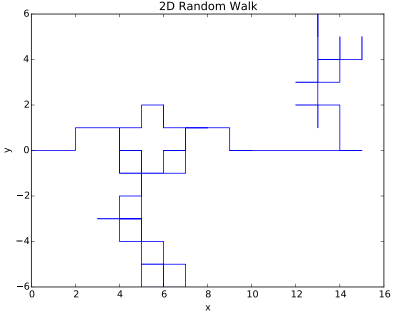
$$< r^2(t) >= 2Dt \tag{3}$$

Where $D = (\Delta x)^2/(4\Delta t)$.



# 2D Random Walk (2)



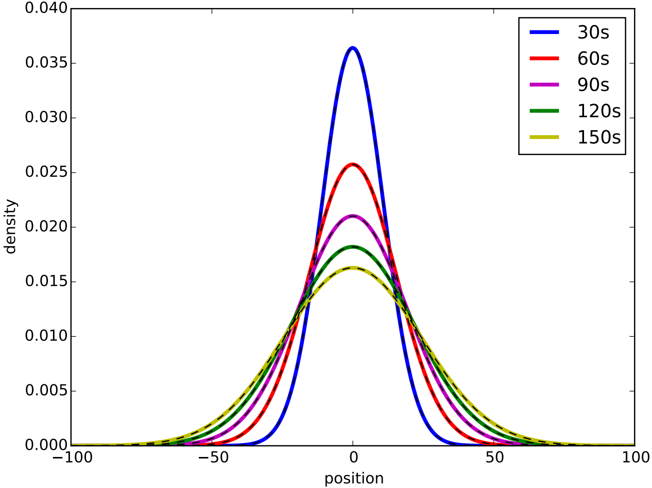
# Diffusion

Background on diffusion

One-dimensional normal distribution

$$\rho(x, t) = \frac{1}{\sqrt{2\pi\,\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \tag{4}$$

# Diffusion time dependence

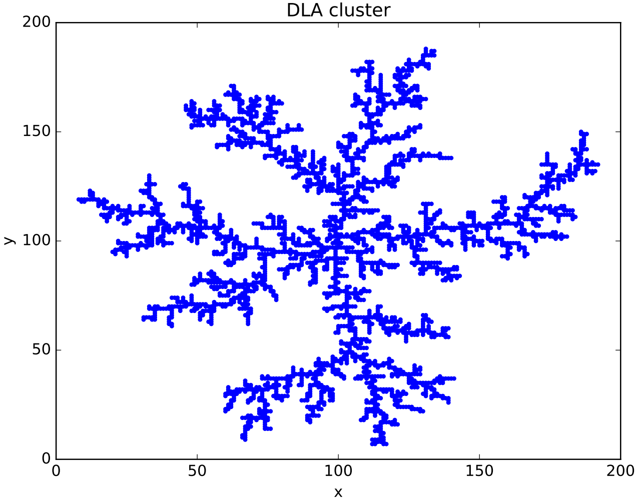


# Crystal Growth with Diffusion Limited Aggregation (DLA)

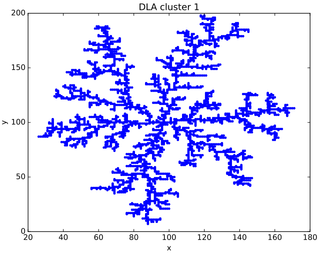
DLA Method:

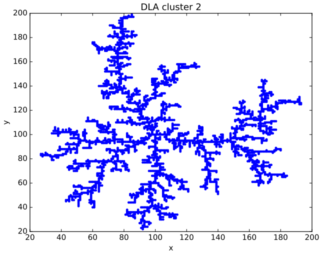
1. Consider a lattice of points with a seed particle at the origin
2. Release a particle from a random location a distance $R$ from the origin
3. Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
4. Repeat for a large number of particles, e.g. 10,000

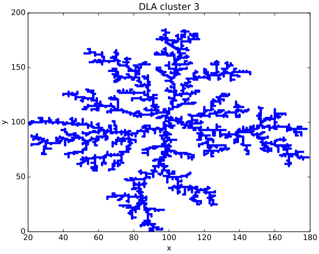
# Example DLA cluster

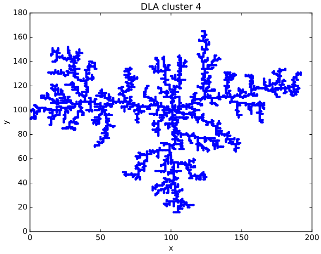


# Typical clusters

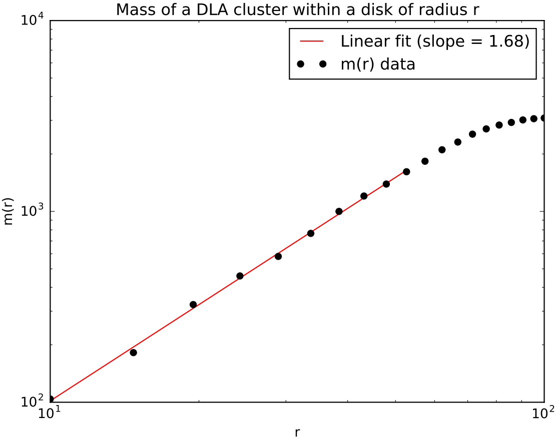


# Fractal Dimension $d_f$ (1)



# Fractal Dimension $d_f$ (2)

| Cluster | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $d_f$ | 1.74 | 1.63 | 1.77 | 1.55 | 1.66 | 1.58 | 1.64 | 1.74 | 1.74 | 1.75 |

Average value over 10 runs: $d_f = 1.68(07)$
Expected value: $d_f = 1.65$