Random Walk, Diffusion, and Cluster Growth

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Outline

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1D Random Walk

RMS distance of an ensemble of random walkers after n steps

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n}\Delta x \tag{1}$$

Diffusive motion:

$$< x^2(t) >= 2Dt \tag{2}$$

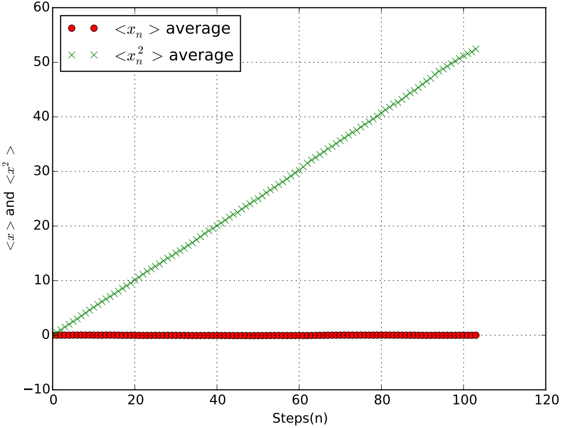
where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

2D Random Walk (1)

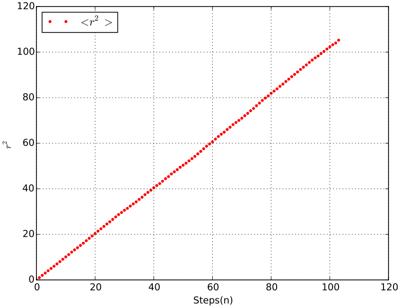
Diffusive motion:

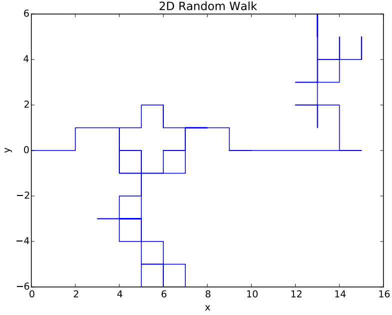
$$< r^2(t) >= 2Dt \tag{3}$$

Where $D = (\Delta x)^2/(4\Delta t)$.



2D Random Walk (2)



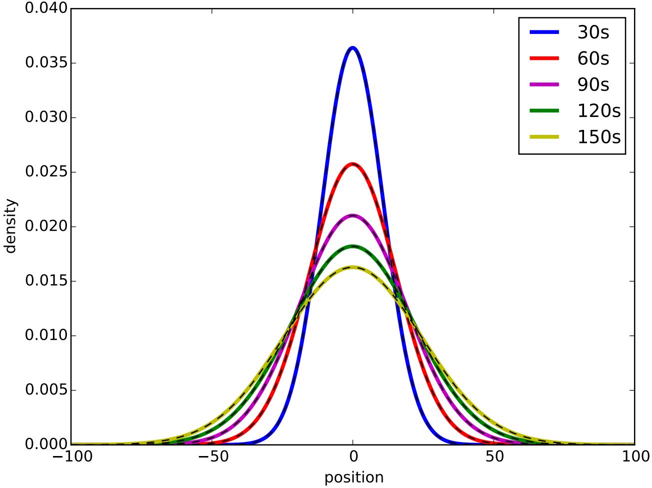


Diffusion

Background on diffusion
One-dimensional normal distribution

$$\rho(x,t) = \frac{1}{\sqrt{2\pi\,\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \tag{4}$$

Diffusion time dependence

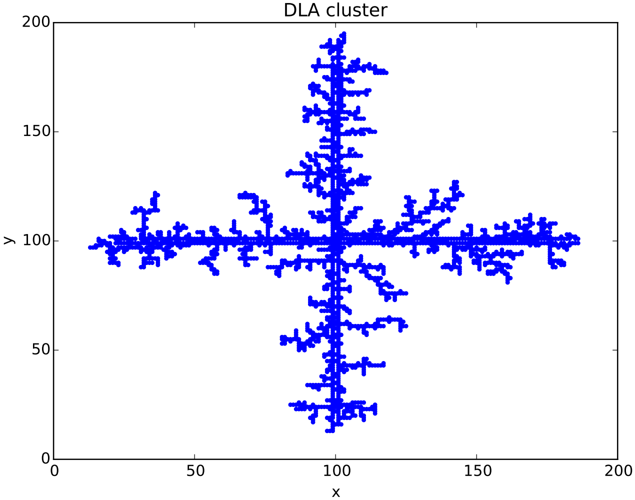


Crystal Growth with Diffusion Limited Aggregation (DLA)

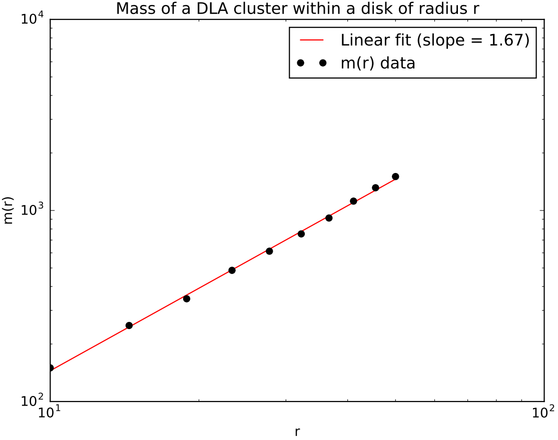
DLA Method:

1. Consider a lattice of points with a seed particle at the origin
2. Release a particle from a random location a distance R from the origin
3. Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
4. Repeat for a large number of particles, e.g. 10,000

Example DLA cluster



Fractal Dimension d_f



Average value over 10 runs: $d_f = 1.56(09)$
Expected value: $d_f = 1.65$