Random Walk, Diffusion, and Cluster Growth

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Outline

1. 2D Random Walk
   - One-dimensional random walks
   - Two-dimensional random walks
2. Diffusion
   - Mathematical background
   - Numerical results
3. Cluster Growth
   - Background
   - Fractal Dimension

1D Random Walk

RMS distance of an ensemble of random walkers after n steps

$$\sqrt{< x_n^2 >} = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} < \Delta x_i \Delta x_j >} = \sqrt{n} \Delta x \tag{1}$$

Diffusive motion:

$$< x^2(t) >= 2Dt \tag{2}$$

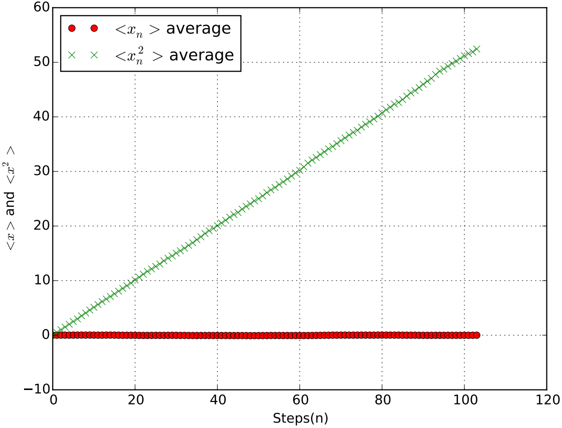
where $D = v \cdot \Delta x/2 = (\Delta x)^2/(2\Delta t)$ is the diffusion constant.

2D Random Walk (1)

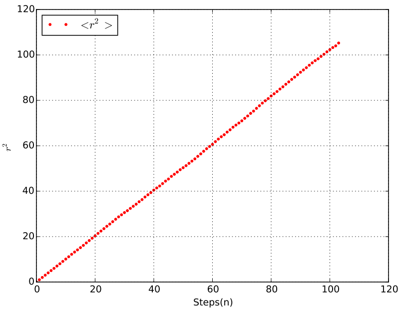
Diffusive motion:

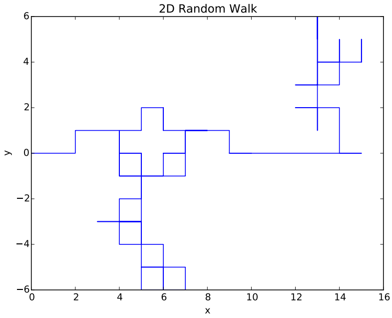
$$< r^2(t) >= 2Dt \tag{3}$$

Where $D = (\Delta x)^2/(4\Delta t)$.



2D Random Walk (2)





Diffusion

Diffusion equation

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho \tag{4}$$

discretize $t = k\Delta t, x = i\Delta x$,

$$\rho_{i,k+1} = \rho_{i,k} + D\frac{\Delta t}{\Delta x^2}(\rho_{i+1,k} + \rho_{i-1,k} - 2\rho_{i,k}) \tag{5}$$

with $\Delta t < \frac{\Delta x^2}{2D}$

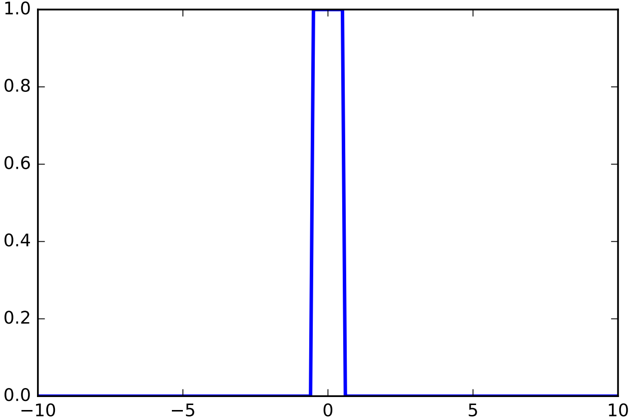
Normal Distribution

One-dimensional normal distribution

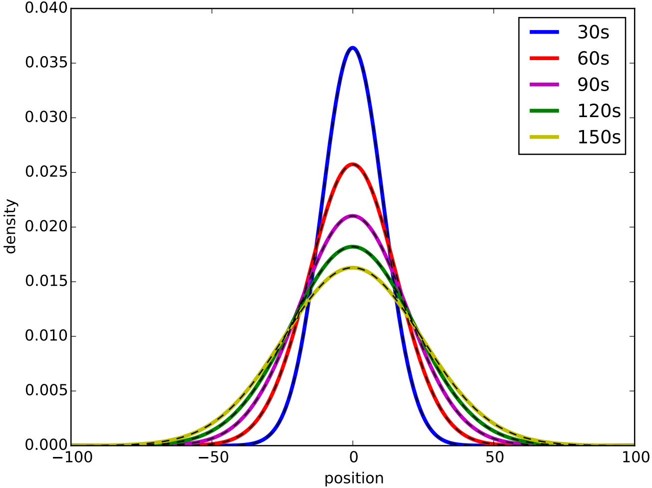
$$\rho(x, t) = \frac{1}{\sqrt{2\pi\,\sigma(t)^2}} \exp\left(-\frac{x^2}{2\sigma(t)^2}\right) \tag{6}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(t)} x^2 exp(-\frac{x^2}{2\sigma(t)^2}) = \sigma(t)^2 \tag{7}$$

Animation



Diffusion time dependence



$$\langle x^2 \rangle = \sigma(t)^2 = 2Dt$$

Cluster Growth with Diffusion Limited Aggregation (DLA)

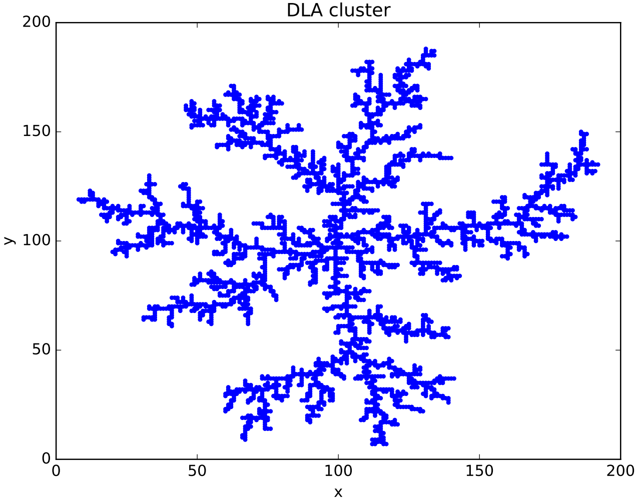
DLA Algorithm:

1. Consider a lattice of points with a seed particle at the origin
2. Release a particle from a random location a distance R from the origin
3. Let the particle perform a random walk until it hits the perimeter of the cluster or strays too far
4. If the particle hits the perimeter, add the particle to the cluster
5. Repeat until cluster reaches the desired size(Radius $= 100$)

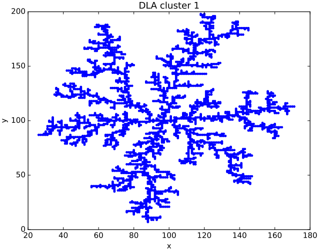
We used a 2D Array for the lattice, and 0 for unoccupied lattice point, 1 for occupied lattice point.

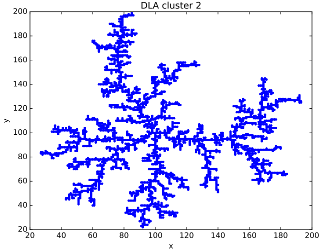
Animation

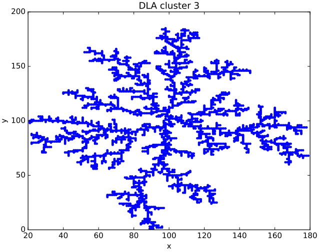
Example DLA cluster

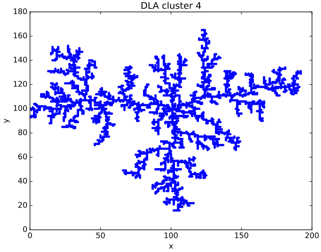


Typical clusters



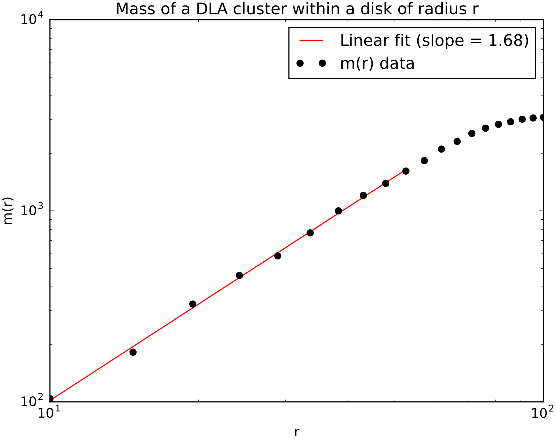






Fractal Dimension d_f (1)

$$m(r) \sim r^{d_f}$$
$$\log(m) = d_f \log(r)$$



Fractal Dimension d_f (2)

Cluster	1	2	3	4	5	6	7	8	9	10
d_f	1.74	1.63	1.77	1.55	1.66	1.58	1.64	1.74	1.74	1.75

Average value over 10 runs: $d_f = 1.68(07)$
Expected value: $d_f = 1.65$