The derivations of the upper bound for the paper:

MAC Protocol for Multi-channel Heterogeneous Networks Based on Deep Reinforcement Learning

Xiaowen Ye, Yiding Yu, Liqun Fu

I. Introduction

This is a supplementary document to the paper: MAC Protocol for Multi-channel Heterogeneous Networks Based on Deep Reinforcement Learning. In this document, we give the upper bound on the network throughput when the M-DLMA node coexists with the nodes using other protocols in the multi-channel heterogeneous network, then we use the upper bound as the benchmark for our paper. Well noted, if you read online and find the page cannot be loaded, please download it locally for reading.

We consider a multi-channel heterogeneous network (HetNet) consisting of multiple types radio networks and an access point (AP). These radio networks use different MAC protocols, such as TDMA, q-ALOHA, FW-ALOHA and M-DLMA. With the exception of the radio network using the M-DLMA protocol, each radio network has a dedicated channel for transmitting packet to AP. The goal of the M-DLMA node is to coexist in harmony with other nodes in multi-channel HetNets without knowing the MAC protocols of other nodes while efficiently using the spectrum.

In order to get the optimal network throughput in various scenarios, we use the **model-aware** node that knows the MAC mechanisms of coexisting nodes to replace the M-DLMA node. In particular, the model-aware node has different optimal transmission strategies for different scenarios.

A. Single-node Model-aware

1) Coexistence with TDMA networks: We consider the coexistence of one model-aware node and TDMA nodes. Suppose that there are Y time slots in a frame, TDMA nodes use X time slots within Y time slots. In order to maximize the network throughput, the model-aware node will take full advantage of the available time slots not used by TDMA nodes. As a result, the optimal network throughput is 1.

In particular, the optimal individual throughputs as the following TABLE I.

TABLE I
THE OPTIMAL INDIVIDUAL THROUGHPUTS FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND TDMA NODES.

Model-aware node	1-X/Y
TDMA node	X/Y

2) Coexistence with q-ALOHA networks: We consider the coexistence of one model-aware node and N q-ALOHA nodes in the network. The transmission probability of q-ALOHA node i $(i = 1, 2, \cdots, N)$ in each time slot is denoted by q_i. To derive the optima results achieved by the model-aware node, we assume that the transmission probability of the model-aware node is b. Then the total network throughput can be calculated as follows:

$$f(b) = b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\Big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\Big), \quad (1)$$

Thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\Big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\Big), \quad (2)$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \quad (3)$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}f(b)/\mathrm{d}b < 0$, if $b = 0$, $f(b)$ can get the maximum value; when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, if $b = 1$, $f(b)$ can get the maximum value. As a result, the optimal policy of the model-aware node is as follows: it does not transmit in each time slot when $\mathrm{d}f(b)/\mathrm{d}b < 0$; it transmits in each time slot when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, i.e.,

$$b^* = \begin{cases} 0, & \text{if } \mathrm{d}f(b)/\mathrm{d}b < 0, \\ 1, & \text{otherwise.} \end{cases} \quad (4)$$

Then the optimal network throughput is

$$\begin{cases} \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), & \text{if } \mathrm{d}f(b)/\mathrm{d}b < 0, \\ \prod_{i=1}^{N}(1-q_i), & \text{otherwise.} \end{cases} \quad (5)$$

3) Coexistence with FW-ALOHA networks: We consider the coexistence of one model-aware node and one FW-ALOHA node with the contention window size W. As shown in Fig. 1, we construct a Markov chain to analyze the optimal network throughput. When the model-aware node stays in the t-th time slot, we define state i $(i = 0, 1, 2, \cdots, W-1)$ to be the number of continuous idle time slots of FW-ALOHA node that has been observed by the model-aware node. The state transition probabilities are given in Fig. 1. We can get

Xiaowen Ye and Liqun Fu are with the School of Information, and the Key laboratory of Underwater Acoustic Communication and Marine Information Technology Ministry of Education, Xiamen University, Xiamen, China. Email:{xiaowen@stu.xmu.edu.cn, liqun@xmu.edu.cn}

Yiding Yu is with the Department of Information Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong Special Administrative Region, China. Email:yy016@ie.cuhk.edu.hk

the state stationary probability p_i of state i by applying the balance equations as follows:

$$\begin{cases} p_0 = \frac{1}{W}p_0 + \frac{1}{W-1}p_1 + \cdots + \frac{1}{2}p_{W-2} + P_{W-1} \\ p_1 = (1-\frac{1}{W})p_0 \\ p_2 = (1-\frac{1}{W-1})p_0 \\ \vdots \\ p_{W-2} = \frac{2}{3}p_{W-3} \\ p_{W-1} = \frac{1}{2}p_{W-2} \\ p_0 + p_1 + p_2 + \cdots + p_{W-1} = 1 \end{cases} \tag{6}$$

Thus, we can calculate the state stationary probability as

$$p_i = \frac{2(W-i)}{W(W+1)}, \tag{7}$$

When the model-aware node stays in state i and takes the action $a_i \in \{0, 1\}$ (i.e., if transmits, $a_i = 1$; otherwise, $a_i = 0$) at time slot t, the FW-ALOHA node may transmit with the probability of $1/(W-i)$ or not transmit with the probability of $\big(1 - i/(W-i)\big)$. Then, the network throughput can be calculated as follows:

$$\begin{aligned} F(a_i) &= \sum_{i=0}^{W-1} \big(p_i a_i (1 - \frac{1}{W-i})\big) + \sum_{i=0}^{W-1} \big(p_i (1-a_i)\frac{1}{W-i}\big) \\ &= \sum_{i=0}^{W-1} \big(\frac{2a_i(W-i-1)}{W(W+1)} + \frac{2(1-a_i)}{W(W+1)}\big) \\ &= 2\sum_{i=0}^{W-1} \frac{(W-2)a_i - ia_i + 1}{W(W+1)}. \end{aligned} \tag{8}$$

Thus, the goal of the model-aware node is

$$\begin{aligned} &\text{maximize } F(a_i) \\ &\text{subject to } 0 \le \sum_{i=0}^{W-1} a_i \le W, \\ &a_i = 0 \text{ or } 1. \end{aligned} \tag{9}$$

Without loss of generality, let $\sum_{i=0}^{W-1} a_i = j$, then $j \in \{0, 1, 2, \cdots, W\}$. Because $a_i = 0$ or 1, thus

$$\sum_{i=0}^{W-1} i a_i \ge 0 \times 1 + 1 \times 1 + \cdots + (j-1) \times 1 = \frac{j(j-1)}{2}. \tag{10}$$

Combining (8), (9) and (10), our objective becomes

$$\begin{aligned} &\text{maximize } F(j) = \frac{-j^2 + (2W-3)j + 2W}{W(W+1)} \\ &\text{subject to } j \in \{0, 1, 2, \cdots, W\}. \end{aligned} \tag{11}$$

Further, we turn (11) into a convex optimization problem, i.e.,

$$\begin{aligned} &\text{minimize } -F(j) = -\frac{-j^2 + (2W-3)j + 2W}{W(W+1)} \\ &\text{subject to } j \in \{0, 1, 2, \cdots, W\}, \end{aligned} \tag{12}$$

and thus,

$$-\frac{\mathrm{d}F(j)}{\mathrm{d}j} = \frac{2j - (2W-3)}{W(W+1)}, \tag{13}$$

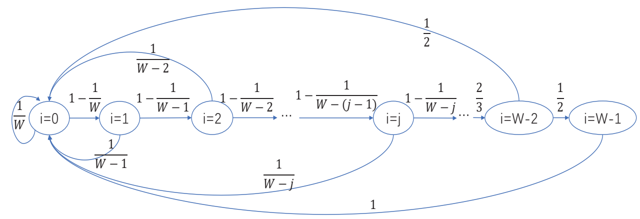

Fig. 1. Markov chain of the model-aware node when coexisting with the FW-ALOHA node

$$-\frac{\mathrm{d}^2F(j)}{\mathrm{d}j^2} = \frac{2}{W(W+1)} > 0, \tag{14}$$

indicating that $-F(j)$ is convex in $j \in \{0, 1, 2, \cdots, W\}$. When $-\mathrm{d}F(j)/\mathrm{d}j = 0$, i.e., $j = (W - 3/2)$, $-F(j)$ has the minimum value, conversely, $F(j)$ has the maximum value. Specially, j should be an integer, let $\lfloor x \rfloor$ denote the maximum integer below x, thus $j = \lfloor W - 3/2 \rfloor$, i.e., $j = W - 2$.

Therefore, the optimal network throughput for the coexistence of one model-aware node and one FW-ALOHA node is

$$\frac{W^2 - W + 2}{W(W+1)}. \tag{15}$$

4) Coexistence with TDMA networks and q-ALOHA networks: We consider the coexistence of one model-aware node, TDMA nodes and N q-ALOHA nodes. Let p denote the radio of the number of time slots where TDMA nodes transmit in a frame, and q_i denote transmission probability of the q-ALOHA node i ($i = 1, 2, \cdots, N$). In this coexistence scenario, in order to take full advantage of the underutilized spectrum, for the time slots where TDMA nodes do not transmit, the model-aware node will transmit on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node will try to coexist with q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

At time slot t, if the TDMA channel is used by TDMA nodes, in order to maximize the total network throughput, the model-aware will try to share the q-ALOHA channel with q-ALOHA nodes as in Section I-A2. We assume that the transmission probability of the model-aware node is b when it coexists with q-ALOHA nodes, then the network throughput at time slot t is

$$1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{16}$$

At time slot t, if TDMA channel is not used by TDMA nodes, the model-aware node will utilize the idle TDMA channel, thereby the q-ALOHA channel is only used by q-ALOHA nodes. Then the network throughput at time slot t is

$$1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{17}$$

Therefore, the average network throughput $f(b)$ is

$$f(b) = p\Big(1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big) + (1-p)\Big(1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big), \tag{18}$$

and thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = p\Big(\prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big), \tag{19}$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \tag{20}$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}F(b)/\mathrm{d}b < 0$, if $b = 0$, $F(b)$ has the maximum value:

$$1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{21}$$

When $\mathrm{d}F(b)/\mathrm{d}b \geq 0$, if $b = 1$, $F(b)$ has the maximum value:

$$p\big(1+\prod_{i=1}^{N}(1-q_i)\big) + (1-p)\Big(1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big), \tag{22}$$

For convenience of illustration, let $z = \mathrm{d}F(b)/\mathrm{d}b = p\Big(\prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$. For this coexistence scenario, the optimal access policy of the model-aware node is as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel. For the time slots where TDMA nodes transmit, if $z<0$, the model-aware node does not transmit on the q-ALOHA channel; if $z\geq 0$, the model-aware node transmits on the q-ALOHA channel.

Therefore, when the model-aware node coexists with a mix of TDMA nodes and q-ALOHA nodes, the optimal network throughput is summarized as the following TABLE II.

TABLE II
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES AND q-ALOHA NODES.

$z<0$
$1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$
$1+p\big(\prod_{i=1}^{N}(1-q_i)\big)+(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$

Further, the optimal individual throughputs of each network in this coexistence scenario are summarized as the following TABLE III.

5) Coexistence with TDMA networks and FW-ALOHA networks: We consider the coexistence of one model-aware node, TDMA nodes and one FW-ALOHA node. Let p denote the radio of the number of time slots where TDMA nodes transmit in a frame, and W denote the contention window size of the FW-ALOHA node. For this coexistence scenario, in order to efficiently use the underutilized spectrum, the optimal access policy of the model-aware node is as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node coexists with the FW-ALOHA node. Then the optimal network throughput can be calculated as follows:

TABLE III
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES AND q-ALOHA NODES.

$z<0$	
Model-aware	$1-p$
TDMA	p
q-ALOHA	$\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$	
Model-aware	$1-p+p\big(\prod_{i=1}^{N}(1-q_i)\big)$
TDMA	p
q-ALOHA	$(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$

At time slot t, if TDMA nodes transmit on the TDMA channel, the model-aware node will coexist with the FW-ALOHA node as in Section I-A3. Then the network throughput is

$$1+\frac{W^2-W-2}{W(W+1)}+\frac{4}{W(W+1)}. \tag{23}$$

At time slot t, if TDMA nodes do not transmit, the model-aware node will occupy the idle TDMA channel. Then the network throughput is

$$1+\frac{2}{W+1}, \tag{24}$$

where $2/(W+1)$ is the average transmission probability of the FW-ALOHA node in each time slot [1].

Therefore, when the model-aware node coexists with a mix of TDMA nodes and one FW-ALOHA node, the optimal network throughput is

$$1+p\frac{W^2-3W+2}{W(W+1)}+\frac{2}{W+1}. \tag{25}$$

Further, the optimal individual throughputs of each network in this coexistence scenario are summarized as the following TABLE IV.

TABLE IV
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES AND ONE FW-ALOHA NODE.

Model-aware	$(1-p)+p\frac{W^2-W-2}{W(W+1)}$
TDMA	p
FW-ALOHA	$(1-p)\frac{2}{W+1}+p\frac{4}{W(W+1)}$

6) Coexistence with TDMA networks, q-ALOHA networks and FW-ALOHA networks: We consider the coexistence of one model-aware node coexists and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node. Let p denote the ratio of the number of time slots where TDMA nodes transmit in a frame, q_i denote the transmission probability of q-ALOHA node i ($i = 0, 1, \cdots, N$), and W denote the contention window size of the FW-ALOHA node. For convenience of illustration, let $z = p\Big(\prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$ as in Section I-A4. The optimal access policy of the model-aware node in this coexistence scenario depends on q_i and W. In particular, we consider two cases as follows:

a). $z<0$: As analyzed in Section I-A4, when one model-aware node coexists with a mix of TDMA nodes and q-ALOHA nodes, if $z<0$, the model-aware node will never occupy the q-ALOHA channel. Therefore, when one model-aware node coexists with a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node, if $z<0$, the optimal access policy of the model-aware node is the same as in Section I-A5: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node will try to coexist harmoniously with the FW-ALOHA node. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$1 + \frac{p(W^2 - 3W + 2) + 2W}{W(W+1)} + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{26}$$

b). $z\geq 0$: When $z\geq 0$, the model-aware node has two optimal access policies. **The first optimal access policy** is the same as I-A6 a). Then the optimal network throughput as (26). **The second optimal access policy** is the same as in Section I-A4: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node will try to coexist harmoniously with the q-ALOHA nodes. As a result, the FW-ALOHA channel is only used by the FW-ALOHA node. Then the optimal network throughput can be calculated as follows:

$$1 + p\big(\prod_{i=1}^{N}(1-q_i)\big) + (1-p)\Big(\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big) + \frac{2}{W+1}. \tag{27}$$

Now, we compare (26) and (27) as follows:

$$\begin{aligned}(26) - (27) =& p\Big(\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big) - \\ & \big(\prod_{i=1}^{N}(1-q_i)\big) + \frac{W^2 - 3W + 2}{W(W+1)}\Big) \\ =& pg.\end{aligned} \tag{28}$$

Thus, when $g\geq 0$, the optimal network throughput as (26), i.e., the model-aware node will select the first optimal policy to access the multi-channel. When $g<0$, the optimal network throughput as (27), i.e., the model-aware node will select the second optimal policy to access the multi-channel.

Combining the analysis of *a)* and *b)*, we can summarize the optimal total network throughput for the coexistence of one model-aware node and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node as TABLE V.

TABLE V
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z<0$ or ($z\geq 0$ and $g\geq 0$)
$1 + \frac{p(W^2-3W+2)+2W}{W(W+1)} + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$ and $g<0$
$1 + p\big(\prod_{i=1}^{N}(1-q_i)\big) + (1-p)\Big(\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big) + \frac{2}{W+1}$

Further, TABLE VI summarizes the optimal individual throughputs of each network for this coexistence scenario.

TABLE VI
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z<0$ or ($z\geq 0$ and $g\geq 0$)	
Model-aware	$(1-p) + p\frac{W^2-W-2}{W(W+1)}$
TDMA	p
q-ALOHA	$\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
FW-ALOHA	$\frac{2}{W+1} - p\frac{2W-4}{W(W+1)}$
$z\geq 0$ and $g<0$	
Model-aware	$1 - p + p\big(\prod_{i=1}^{N}(1-q_i)\big)$
TDMA	p
q-ALOHA	$(1-p)\Big(\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$
FW-ALOHA	$\frac{2}{W+1}$

B. Multi-node model-aware

1) Coexistence with TDMA networks and q-ALOHA networks: We consider the coexistence of multiple model-aware nodes and a mix of TDMA nodes and q-ALOHA nodes. Let p denote the ratio of the number of time slots where TDMA nodes transmit in a frame, and q_i denote the transmission probability of q-ALOHA node i ($i = 0, 1, \cdots, N$). To derive the optimal results achieved by multiple model-aware nodes, we assume that multiple model-aware nodes are aware of each other and can cooperate to occupy the free time slots that are not used by other protocols-based nodes. For this coexistence scenario, in order to take full advantage of the underutilized channels, multiple model-aware nodes will be divided into two parts. One part shares TDMA channel with TDMA nodes, then the optimal TDMA channel throughput as analyzed in Section I-A1; the other part shares q-ALOHA channel with q-ALOHA nodes, then the optimal q-ALOHA channel throughput as analyzed in Section I-A2. Let b denote the total transmission probability of the part of multiple model-aware nodes coexisting with q-ALOHA nodes. Thus, the optimal network throughput can be calculated as follows:

$$f(b) = 1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{29}$$

Thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{30}$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \tag{31}$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}f(b)/\mathrm{d}b<0$, if $b=0$, $f(b)$ can get the maximum value; when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, if $b=1$, $f(b)$ can get the maximum value. For convenience of illustration, let $z = \mathrm{d}f(b)/\mathrm{d}b$. Thus, the optimal policy of multiple model-aware nodes is as follows: multiple model-aware nodes are divided into two parts. One part coexists with TDMA nodes, i.e., using the idle time slots of the TDMA channel to transmit packets. For the other part, if $z<0$, it does not transmit in each time slot; otherwise, it transmits on the q-ALOHA channel in each time slot.

Particularly, TABLE VII summarizes the optimal network throughput for this coexistence scenario.

TABLE VII
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF MULTIPLE MODEL-AWARE NODES AND A MIX OF TDMA NODES AND q-ALOHA NODES.

$z<0$
$1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$
$1+\prod_{i=1}^{N}(1-q_i)$

Further, the optimal individual throughputs of each network for this coexistence scenario are summarized in TABLE VIII.

TABLE VIII
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF MULTIPLE MODEL-AWARE NODES AND A MIX OF TDMA NODES AND q-ALOHA NODES.

$z<0$	
Model-aware	$(1-p)$
TDMA	p
q-ALOHA	$\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$	
Model-aware	$1-p+\prod_{i=1}^{N}(1-q_i)$
TDMA	p
q-ALOHA	0

2) Coexistence with TDMA networks and FW-ALOHA networks: We consider the coexistence of multiplt model-aware nodes and a mix of TDMA nodes and one FW-ALOHA node. Let p denote the ratio of the number of time slots where TDMA nodes transmit, and W denote the contention window size of the FW-ALOHA node. To derive the optimal results achieved by multiple model-aware nodes, we assume that multiple model-aware nodes are aware of each other and can cooperate to occupy the free time slots that are not used by other protocols-based nodes. For this coexistence scenario, the optimal policy of multiple model-aware nodes is as follows: in order to efficiently the underutilized spectrum, multiple model-aware nodes are divided into two parts. One part coexists with TDMA nodes as in Section I-A1, and the other part coexists with FW-ALOHA nodes as Section I-A3. Then the optimal network throughput can be calculated as follows:

$$1+\frac{W^2-W-2}{W(W+1)}+\frac{4}{W(W+1)}. \tag{32}$$

Further, the optimal individual throughputs of each network for this coexistence scenario are summarized in TABLE IX.

TABLE IX
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF MULTIPLE MODEL-AWARE NODES AND A MIX OF TDMA NODES AND FW-ALOHA NODES.

Model-aware	$1-p+\frac{W^2-W-2}{W(W+1)}$
TDMA	p
FW-ALOHA	$\frac{4}{W(W+1)}$

3) Coexistence with TDMA networks, q-ALOHA networks and FW-ALOHA networks: We consider the coexistence of L model-aware nodes and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node. Let p denote the ratio of the number of time slots where TDMA nodes transmit, q_i denote the transmission probability of q-ALOHA node i ($i=0,1,\cdots,N$), and W denote the contention window size of the FW-ALOHA node. To derive the optimal results achieved by multiple model-aware nodes, we assume that L model-aware nodes are aware of each other and can cooperate to occupy the free time slots that are not used by other protocols-based nodes. In this coexistence scenario, the optimal access policy is different when $L=2$ and when $L>2$. Therefore, we will discuss the two coexistence scenarios, respectively.

(1) $L=2$. When two model-aware nodes coexist with TDMA nodes, q-ALOHA nodes and one FW-ALOHA node, let $z = p\Big(\prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$ as in Section I-A4, then the optimal policy of the model-aware

nodes depends on z and W. In particular, we consider two cases as follows.

a). $z{<}0$: As analyzed in Section I-A4, when one model-aware node coexists with a mix of TDMA nodes and q-ALOHA nodes, if $z{<}0$, the model-aware node will never occupy the q-ALOHA channel. Therefore, when two model-aware node coexist with a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node, if $z{<}0$, in order to maximize the total network throughput, the optimal access policy of two model-aware nodes is as follows: One of two model-aware nodes will coexist with TDMA nodes as analyzed Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this model-aware node transmits on the TDMA channel; otherwise, it does not transmit. The other of two model-aware nodes will coexist harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$1+\sum_{i=1}^{N}\big[q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big]+\frac{W^2-W-2}{W(W+1)}+\frac{4}{W(W+1)}. \tag{33}$$

b). $z{\geq}0$: When $z{\geq}0$, in order to maximize the total network throughput, the model-aware node has two optimal access policies. **The first optimal access policy** is as follows: two model-aware nodes will be divided into two parts. For one of two model-aware nodes, for the time slots where TDMA nodes do not transmit, it transmits on the TDMA channel; for the time slots where TDMA nodes transmit, it coexists with q-ALOHA nodes, as in Section I-A4. For the other of two model-aware nodes, it coexists with the FW-ALOHA node, as in Section I-A3. **The second optimal access policy** is as follows: two model-aware nodes will be divided into two parts. For one of two model-aware nodes, for the time slots where TDMA nodes do not transmit, it transmits on the TDMA channel; for the time slots where TDMA nodes transmit, it coexists with FW-ALOHA nodes, as in Section I-A5. For the other of two model-aware nodes, it will transmit in the q-ALOHA channel in each time slot because of $z{\geq}0$, as in Section I-A2.

For the first optimal access policy, the optimal network throughput can be calculated as follows:

$$p\Big(1+\prod_{i=1}^{N}(1-q_i)\Big)+(1-p)\Big(1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)+\frac{W^2-W+2}{W(W+1)}. \tag{34}$$

For the second optimal access policy, the optimal network throughput can be calculated as follows:

$$p\Big(1+\frac{W^2-W+2}{W(W+1)}\Big)+(1-p)\Big(1+\frac{2}{W+1}\Big)+\prod_{i=1}^{N}(1-q_i), \tag{35}$$

where $2/(W+1)$ is the average transmission probability of the FW-ALOHA node [1].

Now, we compare (34) and (35) as follows:

$$\begin{aligned}(33)-(34)=&(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)-\\&\big(\prod_{i=1}^{N}(1-q_i)\big)+\frac{W^2-3W+2}{W(W+1)}\Big)\\=&(1-p)g.\end{aligned} \tag{36}$$

Thus, when $g{\geq}0$, the optimal network throughput as (34), i.e., the model-aware nodes will select the first optimal policy to access the multi-channel. When $g{<}0$, the optimal network throughput as (35), i.e., the model-aware nodes will select the second optimal policy to access the multi-channel.

Combining the analysis of *a)* and *b)*, we can summarize the optimal total network throughput for the coexistence of two model-aware nodes and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node as TABLE X.

TABLE X
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF TWO MODEL-AWARE NODES AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z{<}0$
$1+\sum_{i=1}^{N}\big[q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big]+\frac{W^2-W+2}{W(W+1)}$
$z{\geq}0$ and $g{<}0$
$1+p\frac{W^2-3W+2}{W(W+1)}+\frac{2}{W+1}+\prod_{i=1}^{N}(1-q_i)$
$z{\geq}0$ and $g{\geq}0$
$1+p\prod_{i=1}^{N}(1-q_i)+(1-p)\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+\frac{W^2-W+2}{W(W+1)}$

Further, TABLE XI summarizes the optimal individual throughputs of each network for this coexistence scenario.

(2) $L{>}2$. When more than two model-aware nodes coexist with TDMA nodes, q-ALOHA nodes and one FW-ALOHA node, let $z=p\Big(\prod_{i=1}^{N}(1-q_i)-\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$ as in Section I-A4, then the optimal policy of the model-aware nodes depends on z. In particular, we consider two cases as follows.

a). $z{<}0$: As analyzed in Section I-A4, when one model-aware node coexists with a mix of TDMA nodes and q-ALOHA nodes, if $z{<}0$, the model-aware node will never occupy the q-ALOHA channel. Therefore, when two model-aware node coexist with a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node, if $z{<}0$, in order to maximize the total network throughput, the optimal access policy of two model-aware nodes is as follows: these model-aware nodes should be divided into two part. One part coexists with TDMA nodes as analyzed in Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this model-aware node transmits on the TDMA channel; otherwise, it does not transmit. The other part coexists harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput as (33).

TABLE XI
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF TWO MODEL-AWARE NODES AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z<0$	
Model-aware	$(1-p)+\frac{W^2-W-2}{W(W+1)}$
TDMA	p
q-ALOHA	$\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)$
FW-ALOHA	$\frac{4}{W(W+1)}$
$z\geq 0$ and $g<0$	
Model-aware	$(1-p)+p\frac{W^2-W-2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)$
TDMA	p
q-ALOHA	0
FW-ALOHA	$\frac{2}{W+1}+p\frac{4-2W}{W(W+1)}$
$z\geq 0$ and $g\geq 0$	
Model-aware	$1-p+p\left(\prod_{i=1}^{N}(1-q_i)\right)+\frac{W^2-W-2}{W(W+1)}$
TDMA	p
q-ALOHA	$(1-p)\left(\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)\right)$
FW-ALOHA	$\frac{4}{W(W+1)}$

b). $z\geq 0$: In order to maximize the total network throughput, the optimal access policy of these model-aware nodes is as follows: these model-aware nodes should be divided into three part. The first part coexists with TDMA nodes as analyzed in Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this model-aware node transmits on the TDMA channel; otherwise, it does not transmit; the second part transmits on the q-ALOHA channel in each time slot because of $z\geq 0$, as analyzed in Section I-A2; the last part coexists harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. Then the optimal network throughput can be calculated as follows:

$$1+\prod_{i=1}^{N}(1-q_i)+\frac{W^2-W-2}{W(W+1)}+\frac{4}{W(W+1)}. \qquad (37)$$

Combining the analysis of *a)* and *b)*, we can summarize the optimal total network throughput for the coexistence of more than two model-aware nodes and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node as TABLE XII.

Further, TABLE XIII summarizes the optimal individual throughputs of each network for the coexistence of more than two model-aware nodes and a mix of TDMA nodes, q-ALOHA nodes and one FW-ALOHA node.

TABLE XII
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF MORE THAN TWO MODEL-AWARE NODES AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z<0$
$1+\sum_{i=1}^{N}\left[q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right]+\frac{W^2-W+2}{W(W+1)}$
$z\geq 0$
$1+\prod_{i=1}^{N}(1-q_i)+\frac{W^2-W+2}{W(W+1)}$

TABLE XIII
THE OPTIMAL INDIVIDUAL THROUGHPUTS OF EACH NETWORK FOR THE COEXISTENCE OF MORE THAN TWO MODEL-AWARE NODES AND A MIX OF TDMA NODES, q-ALOHA NODES AND ONE FW-ALOHA NODE.

$z<0$	
Model-aware	$(1-p)+\frac{W^2-W-2}{W(W+1)}$
TDMA	p
q-ALOHA	$\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)$
FW-ALOHA	$\frac{4}{W(W+1)}$
$z\geq 0$	
Model-aware	$(1-p)+\prod_{i=1}^{N}(1-q_i)+\frac{W^2-W-2}{W(W+1)}$
TDMA	p
q-ALOHA	0
FW-ALOHA	$\frac{4}{W(W+1)}$

REFERENCES

[1] G. Bianchi, L. Fratta, and M. Oliveri, "Performance evaluation and enhancement of the csma/ca mac protocol for 802.11 wireless lans," in *Proc. IEEE PIMRC*, Oct. 1996, pp. 392–396.