A supplementary for the paper: MAC Protocol for Multi-channel Heterogeneous Networks Based on Deep Reinforcement Learning

Xiaowen Ye[1], Yiding Yu[2], and Liqun Fu[1]
[1]School of Informatics, and the Key Laboratory of Underwater Acoustic Communication and Marine Information Technology of Ministry of Education, Xiamen University, Xiamen 361005, China
[2]Department of Information Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong Special Administrative Region, China.
Email: xiaowen@stu.xmu.edu.cn, yy016@ie.cuhk.edu.hk, liqun@xmu.edu.cn

I. Introduction

This is a supplementary document to the paper: MAC Protocol for Multi-channel Heterogeneous Networks Based on Deep Reinforcement Learning. In this document, we give the optimal throughputs on the network throughput when the MC-DLMA node coexists with the nodes using other protocols in various multi-channel heterogeneous wireless network (HetNet) scenarios, then we use the optimal throughputs as the upper bound benchmarks for our paper.

We consider a multi-channel heterogeneous wireless network (HetNet) consisting of multiple types radio networks and an access point (AP). These radio networks use different MAC protocols, such as TDMA, q-ALOHA, FW-ALOHA, and MC-DLMA. With the exception of the radio network using the MC-DLMA protocol, each radio network has a dedicated channel for transmitting packet to AP. The goal of the MC-DLMA protocol is to coexist in harmony with other nodes in multi-channel HetNets without knowing the MAC protocols of other nodes while efficiently using the spectrum.

In order to get the optimal network throughput in various scenarios, we use the **model-aware** node that knows the MAC mechanisms of coexisting nodes to replace the MC-DLMA node. In particular, the model-aware node has different optimal transmission strategies for different scenarios.

A. Single-node Model-aware

1) Coexistence with TDMA networks: We consider the coexistence of one model-aware node and multiple TDMA nodes. We assume that TDMA nodes use a total of X time slots out of Y time slots in a frame. In order to maximize the total network throughput, the model-aware node should take full advantage of the available time slots that are not used by TDMA nodes. As a result, the optimal network throughput is 1.

2) Coexistence with q-ALOHA networks: We consider the coexistence of one model-aware node and N q-ALOHA nodes. The transmission probability of q-ALOHA node i ($i = 1, 2, \cdots, N$) in each time slot is denoted by q_i. To derive the optima results achieved by the model-aware node, we assume that the transmission probability of the model-aware node in each time slot is b. Then the total network throughput can be calculated as follows:

$$f(b) \;=\; b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{1}$$

Thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{2}$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \tag{3}$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}f(b)/\mathrm{d}b < 0$, if $b = 0$, $f(b)$ can get the maximum value; when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, if $b = 1$, $f(b)$ can get the maximum value. As a result, the optimal policy of the model-aware node is as follows: it does not transmit in each time slot when $\mathrm{d}f(b)/\mathrm{d}b < 0$; it transmits in each time slot when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, i.e.,

$$b^* = \begin{cases} 0, & \text{if } \mathrm{d}f(b)/\mathrm{d}b < 0,\\ 1, & \text{otherwise.}\end{cases} \tag{4}$$

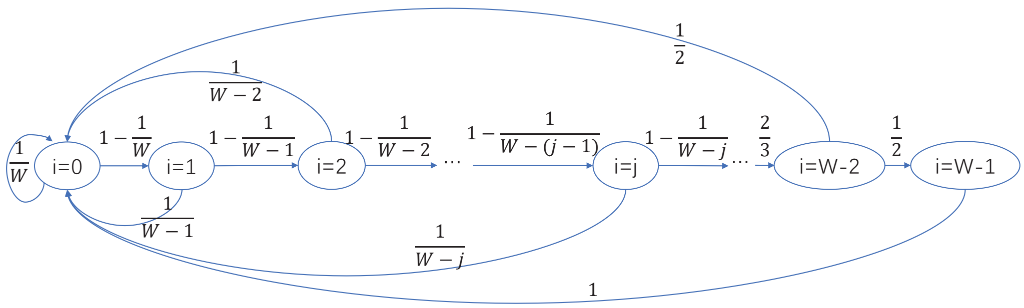

Fig. 1. Markov chain of the model-aware node when coexisting with the FW-ALOHA node

For convenience of illustration, let $z = \mathrm{d}f(b)/\mathrm{d}b = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$. Then the optimal network throughput is

$$\begin{cases} \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), & \text{if } z<0, \\ \prod_{i=1}^{N}(1-q_i), & \text{otherwise.} \end{cases} \tag{5}$$

3) Coexistence with FW-ALOHA networks: We consider the coexistence of one model-aware node and one FW-ALOHA node with the contention window size W. As shown in Fig. 1, we construct a Markov chain to analyze the optimal network throughput. When the model-aware node stays in the time slot t, we define state i ($i = 0, 1, 2, \cdots, W-1$) of the Markov chain to be the number of continuous idle time slots of FW-ALOHA node that has been observed by the model-aware node. The state transition probabilities are given in Fig. 1. We can get the state stationary probability p_i of state i by applying the balance equations as follows:

$$\begin{cases} p_0 = \frac{1}{W}p_0 + \frac{1}{W-1}p_1 + \cdots + \frac{1}{2}p_{W-2} + P_{W-1}, \\ p_1 = (1-\frac{1}{W})p_0, \\ p_2 = (1-\frac{1}{W-1})p_0, \\ \vdots \\ p_{W-2} = \frac{2}{3}p_{W-3}, \\ p_{W-1} = \frac{1}{2}p_{W-2}, \\ p_0 + p_1 + p_2 + \cdots + p_{W-1} = 1. \end{cases} \tag{6}$$

Thus, we can calculate the state stationary probability p_i as

$$p_i = \frac{2(W-i)}{W(W+1)}. \tag{7}$$

When the model-aware node stays in state i and takes the action $a_i \in \{0,\,1\}$ (i.e., if it transmits, $a_i = 1$; otherwise, $a_i = 0$) at time slot t, the FW-ALOHA node may transmit with the probability of $1/(W-i)$ or not transmit with the probability of $\big(1 - 1/(W-i)\big)$. Then, the total network throughput can be calculated as follows:

$$\begin{aligned} F(a_i) &= \sum_{i=0}^{W-1}\big(p_i a_i (1-\frac{1}{W-i})\big) + \sum_{i=0}^{W-1}\big(p_i(1-a_i)\frac{1}{W-i}\big) \\ &= \sum_{i=0}^{W-1}\big(\frac{2a_i(W-i-1)}{W(W+1)} + \frac{2(1-a_i)}{W(W+1)}\big) \\ &= 2\sum_{i=0}^{W-1}\frac{(W-2)a_i - ia_i + 1}{W(W+1)}. \end{aligned} \tag{8}$$

Thus, the goal of the model-aware node is to

$$\begin{aligned} &\text{maximize } F(a_i) \\ &\text{subject to } 0 \leq \sum_{i=0}^{W-1} a_i \leq W, \\ &a_i = 0 \text{ or } 1. \end{aligned} \tag{9}$$

Without loss of generality, let $\sum_{i=0}^{W-1} a_i = j$, then $j\in\{0,\, 1,\, 2,\, \cdots,\, W\}$. Because $a_i = 0$ or 1, thus

$$\sum_{i=0}^{W-1} i a_i \geq 0 \times 1 + 1 \times 1 + \cdots + (j-1) \times 1 = \frac{j(j-1)}{2}. \tag{10}$$

Combining (8), (9) and (10), our objective becomes

$$\begin{aligned} &\text{maximize } F(j) = \frac{-j^2 + (2W-3)j + 2W}{W(W+1)} \\ &\text{subject to } j\in\{0,\, 1,\, 2,\, \cdots,\, W\}. \end{aligned} \tag{11}$$

Further, we turn (11) into a convex optimization problem, i.e.,

$$\begin{aligned} &\text{minimize } -F(j) = -\frac{-j^2 + (2W-3)j + 2W}{W(W+1)} \\ &\text{subject to } j\in\{0,\, 1,\, 2,\, \cdots,\, W\}, \end{aligned} \tag{12}$$

and thus,

$$-\frac{\mathrm{d}F(j)}{\mathrm{d}j} = \frac{2j - (2W-3)}{W(W+1)}, \tag{13}$$

$$-\frac{\mathrm{d}^2 F(j)}{\mathrm{d}j^2} = \frac{2}{W(W+1)} > 0, \tag{14}$$

indicating that $-F(j)$ is convex in $j\in\{0,\, 1,\, 2,\, \cdots,\, W\}$. When $-\mathrm{d}F(j)/\mathrm{d}j = 0$, i.e., $j = (W - 3/2)$, $-F(j)$ has the minimum value. Conversely, $F(j)$ has the maximum value in $j = (W - 3/2)$. Specially, j should be an integer, let $\lfloor x \rfloor$ denote the maximum integer below x, thus $j = \lfloor W - 3/2 \rfloor$, i.e., $j = W - 2$. Therefore, for this coexistence scenario, the optimal access policy of the model-aware node can be summarized as follows: after observing a transmission of the FW-ALOHA node, the model-aware node transmits in all subsequent time slots except after observing consecutive $W-2$ idle time slots of the FW-ALOHA node, in which case the model-aware node refrains from transmission in the next two time slots. Furthermore, by substituting $j = W - 2$ into (11), we can get the optimal network throughput for the coexistence of one model-aware node and one FW-ALOHA node, i.e.,

$$\frac{W^2 - W + 2}{W(W+1)}. \tag{15}$$

4) Coexistence with TDMA networks and q-ALOHA networks: We consider the coexistence of one model-aware node, multiple TDMA nodes, and N q-ALOHA nodes. Let p denote the ratio of the number of time slots used by multiple TDMA nodes to the total number of time slots in a frame, and let q_i denote the transmission probability of q-ALOHA node i $(i = 1, 2, \cdots, N)$.

At time slot t, if the TDMA channel is used by TDMA nodes, in order to maximize the total network throughput, the model-aware node will try to share the q-ALOHA channel with N q-ALOHA nodes as in Section I-A2. We assume that the transmission probability of the model-aware node in each time slot is b when it coexists with q-ALOHA nodes, then the network throughput at time slot t can be calculated as

$$1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\Big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\Big). \tag{16}$$

At time slot t, if TDMA channel is not used by TDMA nodes, the model-aware node will utilize the idle TDMA channel; while the q-ALOHA channel is only used by q-ALOHA nodes. Then the network throughput at time slot t can be calculated as

$$1 + \sum_{i=1}^{N}\Big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\Big). \tag{17}$$

Therefore, the average network throughput $f(b)$ is

$$f(b) = p\Big(1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big) + (1-p)\Big(1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big), \tag{18}$$

and thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = p\Big(\prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big), \tag{19}$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \tag{20}$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}F(b)/\mathrm{d}b < 0$, if $b = 0$, $F(b)$ has the maximum value, i.e.,

$$1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{21}$$

When $\mathrm{d}F(b)/\mathrm{d}b \geq 0$, if $b = 1$, $F(b)$ has the maximum value, i.e.,

$$p\big(1 + \prod_{i=1}^{N}(1-q_i)\big) + (1-p)\Big(1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big). \tag{22}$$

For convenience of illustration, let $z = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$. For this coexistence scenario, the optimal access policy of the model-aware node can be summarized as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel. For the time slots where TDMA nodes transmit, if $z<0$, the model-aware node does not transmit on the q-ALOHA channel; if $z\geq 0$, the model-aware node transmits on the q-ALOHA channel. Furthermore, we summarize the optimal network throughput for the coexistence of one model-aware node, multiple TDMA nodes, and N q-ALOHA nodes as in TABLE I.

TABLE I

THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE, MULTIPLE TDMA NODES, AND N q-ALOHA NODES.

$z<0$	$1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$	$1 + p\big(\prod_{i=1}^{N}(1-q_i)\big) + (1-p)\Big(\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)$

5) Coexistence with TDMA networks and FW-ALOHA networks: We consider the coexistence of one model-aware node, multiple TDMA nodes, and one FW-ALOHA node. Let p denote the ratio of the number of time slots used by TDMA nodes to the total number of time slots in a frame, and let W denote the contention window size of the FW-ALOHA node.

At time slot t, if TDMA nodes transmit on the TDMA channel, the model-aware node will coexist with the FW-ALOHA node as in Section I-A3. Then the network throughput can be calculated as follows:

$$1 + \frac{W^2 - W + 2}{W(W+1)}. \tag{23}$$

At time slot t, if TDMA nodes do not transmit, in order to efficiently use the underutilized spectrum, the model-aware node will occupy the idle TDMA channel. Then the network throughput can be calculated as follows:

$$1 + \frac{2}{W+1}, \tag{24}$$

where $2/(W+1)$ is the average transmission probability of the FW-ALOHA node in each time slot [1].

Therefore, when the model-aware node coexists with multiple TDMA nodes and one FW-ALOHA node, the optimal network throughput can be calculated as follows:

$$p\big(1 + \frac{W^2 - W + 2}{W(W+1)}\big) + (1-p)\big(1 + \frac{2}{W+1}\big) = p\frac{W^2 - 3W + 2}{W(W+1)} + \frac{W+3}{W+1}. \tag{25}$$

In summary, for this coexistence scenario, the optimal access policy of the model-aware node is as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node coexists with the FW-ALOHA node.

6) Coexistence with q-ALOHA networks and FW-ALOHA networks: We consider the coexistence of one model-aware node, N q-ALOHA nodes, and one FW-ALOHA node. Let q_i denote the transmission probability of q-ALOHA node i ($i=1,2,\cdots,N$), and let W denote the contention window size of the FW-ALOHA node. For convenience of illustration, let $z=\prod_{i=1}^{N}(1-q_i)-\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)$ as in Section I-A2. The optimal access policy of the model-aware node in this coexistence scenario depends on q_i and W. In particular, we consider two cases in the following.

Case a). $z<0$: As analyzed in Section I-A2, when one model-aware node coexists with N q-ALOHA nodes, if $z<0$, the model-aware node will never occupy the q-ALOHA channel. Therefore, when one model-aware coexists with N q-ALOHA nodes and one FW-ALOHA node, if $z<0$, the optimal access policy of the model-aware node is as follows: the model-aware node always tries to coexist harmoniously with the FW-ALOHA node as in Section I-A3. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$\frac{W^2-W+2}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big). \tag{26}$$

Case b). $z\geq 0$: When $z\geq 0$, the model-aware node has two alternative optimal access policies.

* **The first optimal access policy** is as follows: for the model-aware node to coexist with N q-ALOHA nodes and the FW-ALOHA node in different time slots. Specifically, as analyzed in Section I-A3, the optimal access policy for the coexistence of one model-aware node and one FW-ALOHA node is: after observing a transmission of the FW-ALOHA node, the model-aware node transmits in all subsequent time slots except after observing consecutive $W-2$ idle time slots of the FW-ALOHA node, in which case the model-aware node refrains from transmission in the next two time slots. Furthermore, according to Section I-A2, when $z\geq 0$, the optimal access policy for the coexistence of one model-aware node and q-ALOHA nodes is: the model-aware node transmits on the q-ALOHA channel in each time slot. Therefore, for this coexistence scenario, the optimal policy of the model-aware node can be summarized as follows: after observing a transmission of the FW-ALOHA node, the model-aware node transmits in all subsequent time slots on the FW-ALOHA channel except after observing consecutive $W-2$ idle time slots of the FW-ALOHA node; for the last two time slots out of W time slots, the model-aware node should transmit on the q-ALOHA channel. Similar to (8), the optimal network throughput for this coexistence scenario can be calculated as follows:

$$\sum_{i=0}^{W-3}\big(p_i(1-\frac{1}{W-i})\big)+\prod_{i=1}^{N}(1-q_i)\sum_{i=W-2}^{W-1}p_i+\sum_{i=W-2}^{W-1}(p_i\frac{1}{W-i})+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\sum_{i=0}^{W-3}p_i, \tag{27}$$

where p_i is the state stationary probability in Fig. 1, which is given in (7), i.e., $p_i=2(W-i)/(W(W+1))$. The first item in (27) is the throughput achieved by the model-aware node on the FW-ALOHA channel, the second item in (27) is the throughput achieved by the model-aware node on the q-ALOHA channel, the third item in (27) is the throughput achieved by the FW-ALOHA node, and the last item in (27) is the throughput achieved by q-ALOHA nodes.
By combining (7) and (27), we can get the optimal network throughput as follows:

$$\frac{W^2-W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)\frac{6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\frac{(W+3)(W-2)}{W(W+1)}. \tag{28}$$

* **The second optimal access policy** is an "extreme" access policy compared with the first optimal access policy. Specifically, when the probability of "at least one q-ALOHA node is transmitting" in the network is very small, in order to maximize the total network throughput, the model-aware node will transmit on the q-ALOHA channel in each time slot; while the FW-ALOHA channel is only used by the FW-ALOHA node. Then the optimal network throughput can be calculated as follows:

$$\prod_{i=1}^{N}(1-q_i)+\frac{2}{W+1}, \tag{29}$$

where $2/(W+1)$ is the average transmission probability of the FW-ALOHA node in each time slot [1].

Now, we compare (28) and (29) as follows:

$$\begin{aligned}(28)-(29)=&\Big(\frac{W^2-W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)\frac{6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\frac{(W+3)(W-2)}{W(W+1)}\Big)-\Big(\prod_{i=1}^{N}(1-q_i)+\frac{2}{W+1}\Big)\\ =&\frac{W^2-3W+2}{W(W+1)}-\prod_{i=1}^{N}(1-q_i)\frac{W^2+W-6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\frac{(W+3)(W-2)}{W(W+1)}\\ =&\frac{W-2}{W(W+1)}\Big((W-1)-(W+3)\prod_{i=1}^{N}(1-q_i)+(W+3)\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)\\ =&\frac{W-2}{W(W+1)}\big((W-1)-(W+3)z\big).\end{aligned}\tag{30}$$

As a result, we can get that if $\big((W-1)/(W+3)\big)>z$, the network throughput achieved by the model-aware node based on the first optimal access policy is greater than that based on the second optimal access policy; otherwise, the network throughput achieved by the model-aware node based on the second optimal access policy is greater than that based on the first optimal access policy.

Overall, the optimal network throughput for the coexistence of one model-aware node, N q-ALOHA nodes, and one FW-ALOHA node can be summarized as in TABLE II.

TABLE II
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE, N q-ALOHA NODES, AND ONE FW-ALOHA NODE.

$z<0$	$\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+\frac{W^2-W+2}{W(W+1)}$
$0\leq z<\frac{W-1}{W+3}$	$\frac{W^2-W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)\frac{6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\frac{(W+3)(W-2)}{W(W+1)}$
$\frac{W-1}{W+3}\leq z$	$\prod_{i=1}^{N}(1-q_i)+\frac{2}{W+1}$

7) Coexistence with TDMA networks, q-ALOHA networks, and FW-ALOHA networks: We consider the coexistence of one model-aware node, multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node. Let p denote the ratio of the number of time slots used by TDMA nodes to the total number of time slots in a frame, q_i denote the transmission probability of q-ALOHA node i ($i=1,\cdots,N$), and W denote the contention window size of the FW-ALOHA node. For convenience of illustration, let $z=\prod_{i=1}^{N}(1-q_i)-\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$ as in Section I-A4. The optimal access policy of the model-aware node in this coexistence scenario depends on p, q_i, and W. In particular, we consider two cases in the following.

Case a). $z<0$: As analyzed in Section I-A4, when one model-aware node coexists with multiple TDMA nodes and N q-ALOHA nodes, if $z<0$, the model-aware node will never occupy the q-ALOHA channel. Therefore, when one model-aware node coexists with multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node, if $z<0$, the optimal access policy of the model-aware node is the same as in Section I-A5: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node will try to coexist harmoniously with the FW-ALOHA node. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$p\frac{(W^2-3W+2)}{W(W+1)}+\frac{W+3}{W+2}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big).\tag{31}$$

Case b). $z\geq0$: Similar to the analysis of *Case b)* in the Section I-A6, when $z\geq0$, there are two alternative optimal policies for the model-aware node.

* **The first optimal access policy** is as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node coexists

with q-ALOHA nodes and the FW-ALOHA node in different time slots as in Section I-A6 *Case b)* **"The first optimal access policy"**. Then the optimal network throughput can be calculated as follows:

$$
\begin{aligned}
&(1-p)\Big(1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+\frac{2}{W+1}\Big)+\\
&p\Big(1+\frac{W^2-W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)\frac{6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\frac{(W+3)(W-2)}{W(W+1)}\Big)\\
=&\frac{W+3}{W+1}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+p\frac{W^2-3W+2+6\prod_{i=1}^{N}(1-q_i)-6\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)}{W(W+1)}. \qquad (32)
\end{aligned}
$$

* **The second optimal access policy** is an "extreme" access policy compared with the first optimal access policy. Specifically, when the probability of "at least one q-ALOHA node is transmitting" in the network is very small, in order to maximize the total network throughput, the optimal policy of the model-aware node is as follows: for the time slots where TDMA nodes do not transmit, the model-aware node transmits on the TDMA channel; for the time slots where TDMA nodes transmit, the model-aware node will transmit on the q-ALOHA channel in each time slot. As a result, the FW-ALOHA channel is only used by the FW-ALOHA node. Then the optimal network throughput can be calculated as follows:

$$
\begin{aligned}
&1+p\big(\prod_{i=1}^{N}(1-q_i)\big)+(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)+\frac{2}{W+1}\\
=&p\big(\prod_{i=1}^{N}(1-q_i)\big)+(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)+\frac{W+3}{W+1}, \qquad (33)
\end{aligned}
$$

where $2/(W+1)$ is the average transmission probability of the FW-ALOHA node in each time slot [1].

Now, we compare (32) and (33) as follows:

$$
\begin{aligned}
&(32)-(33)\\
=&p\frac{W-2}{W(W+1)}\Bigg(\Big(\big(\prod_{i=1}^{N}(1-q_i)\big)-\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)(W+3)-(W-1)\Bigg)\\
=&p\frac{W-2}{W(W+1)}\big((W-1)-z(W+3)\big). \qquad (34)
\end{aligned}
$$

Thus, if $z<\big((W-1)/(W+3)\big)$, the network throughput achieved using the first optimal access policy is higher than that using the second optimal access policy; otherwise, the network throughput achieved using the second optimal access policy is higher than that using the first optimal access policy.

Overall, we summarize the optimal network throughput for the coexistence of one model-aware node, multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node as in TABLE III.

TABLE III
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF ONE MODEL-AWARE NODE AND A MIX OF TDMA NODES, q-ALOHA NODES, AND ONE FW-ALOHA NODE.

$z<0$	$1+\frac{p(W^2-3W+2)+2W}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$0\leq z<\frac{W-1}{W+3}$	$\frac{W+3}{W+1}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+p\frac{W^2-3W+2+6\prod_{i=1}^{N}(1-q_i)-6\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)}{W(W+1)}$
$\frac{W-1}{W+3}\leq z$	$p\big(\prod_{i=1}^{N}(1-q_i)\big)+(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)+\frac{W+3}{W+1}$

B. Multi-node model-aware

1) Coexistence with TDMA networks and q-ALOHA networks: We consider the coexistence of multiple model-aware nodes, multiple TDMA nodes, and N q-ALOHA nodes. Let p denote the ratio of the number of time slots used by TDMA nodes to the total number of time slots in a frame, and q_i denote the transmission probability of q-ALOHA node i $(i=1,\cdots,N)$. To derive the optimal results achieved by multiple model-aware nodes, we assume that multiple model-aware nodes are aware of each other and can cooperate to occupy the available time slots that are not used by other protocols-based nodes. For this coexistence scenario, in order to take full advantage of the underutilized channels, the optimal access policy of multiple model-aware nodes is to be divided into two parts. One part shares the TDMA channel with TDMA nodes, thus the optimal throughput on the TDMA channel as analyzed in Section I-A1; the other part shares the q-ALOHA channel with q-ALOHA nodes, thus the optimal throughput on the q-ALOHA channel as analyzed in Section I-A2. Let b denote the total transmission probability of the part of multiple model-aware nodes coexisting with q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$f(b) = p + (1-p) + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big) = 1 + b\prod_{i=1}^{N}(1-q_i) + (1-b)\sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{35}$$

Thus

$$\frac{\mathrm{d}f(b)}{\mathrm{d}b} = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big), \tag{36}$$

$$\frac{\mathrm{d}^2 f(b)}{\mathrm{d}b^2} = 0, \tag{37}$$

indicating that $f(b)$ is convex in b. When $\mathrm{d}f(b)/\mathrm{d}b < 0$, if $b = 0$, $f(b)$ can get the maximum value; when $\mathrm{d}f(b)/\mathrm{d}b \geq 0$, if $b = 1$, $f(b)$ can get the maximum value. For convenience of illustration, let $z = \mathrm{d}f(b)/\mathrm{d}b = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$. Then the optimal access policy of multiple model-aware nodes is as follows: multiple model-aware nodes are divided into two parts. One part coexists with TDMA nodes, i.e., using the idle time slots of the TDMA channel to transmit packets. For the other part, if $z<0$, it does not transmit in each time slot; otherwise, it transmits on the q-ALOHA channel in each time slot. Furthermore, TABLE IV summarizes the optimal network throughput for this coexistence scenario.

TABLE IV
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF MULTIPLE MODEL-AWARE NODES, MULTIPLE TDMA NODES, AND N q-ALOHA NODES.

$z<0$	$1 + \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$
$z\geq 0$	$1 + \prod_{i=1}^{N}(1-q_i)$

2) Coexistence with TDMA networks and FW-ALOHA networks: We consider the coexistence of multiple model-aware nodes, multiple TDMA nodes, and one FW-ALOHA node. Let p denote the ratio of the number of time slots to the total number of time slots in a frame, and W denote the contention window size of the FW-ALOHA node. To derive the optimal results achieved by multiple model-aware nodes, we assume that multiple model-aware nodes are aware of each other and can cooperate to occupy the available time slots that are not used by other protocols-based nodes. For this coexistence scenario, the optimal policy of multiple model-aware nodes is as follows: in order to efficiently the underutilized spectrum, multiple model-aware nodes are divided into two parts. One part coexists with TDMA nodes as in Section I-A1, and the other part coexists with one FW-ALOHA node as Section I-A3. Then the optimal network throughput can be calculated as follows:

$$1 + \frac{W^2 - W + 2}{W(W+1)} = \frac{2W^2 + 2}{W(W+1)}. \tag{38}$$

3) Coexistence with q-ALOHA networks and FW-ALOHA networks: We consider the coexistence of multiple model-aware nodes, N q-ALOHA nodes, and one FW-ALOHA node. Let q_i denote the transmission probability of q-ALOHA node i ($i = 1, \cdots, N$) and W denote the contention window size of the FW-ALOHA node. For convenience of illustration, let $z = \mathrm{d}f(b)/\mathrm{d}b = \prod_{i=1}^{N}(1-q_i) - \sum_{i=1}^{N}\big(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)$ as in Section I-A2. To derive the optimal results achieved by multiple model-aware nodes, we assume that multiple model-aware nodes are aware of each other and can cooperate to occupy the available time slots that are not used by other protocols-based nodes. For this coexistence scenario, in order to take full advantage of the underutilized channels, the optimal access policy of multiple model-aware nodes is to be divided into two parts. One part coexists with q-ALOHA nodes, i.e., if $z<0$, it does not transmit in each time slot; otherwise, it transmits on the q-ALOHA channel in each time slot. For the other part, it coexists with the FW-ALOHA node as analyzed in Section I-A6. Then the optimal network throughput for this coexistence scenario can be summarized as in TABLE V.

TABLE V
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF MULTIPLE MODEL-AWARE NODES, N q-ALOHA NODES, AND ONE FW-ALOHA NODE.

$z<0$	$\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)+\frac{W^2-W+2}{W(W+1)}$
$z\geq 0$	$\prod_{i=1}^{N}(1-q_i)++\frac{W^2-W+2}{W(W+1)}$

4) Coexistence with TDMA networks, q-ALOHA networks, and FW-ALOHA networks: We consider the coexistence of L model-aware nodes, multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node. Let p denote the ratio of the number of time slots used by TDMA nodes to the total number of time slots in a frame, q_i denote the transmission probability of q-ALOHA node i ($i=0,1,\cdots,N$), and W denote the contention window size of the FW-ALOHA node. To derive the optimal results achieved by multiple model-aware nodes, we assume that L model-aware nodes are aware of each other and can cooperate to occupy the free time slots that are not used by other protocols-based nodes. For convenience of illustration, let $z=\prod_{i=1}^{N}(1-q_i)-\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)$ as in Section I-A4. In this coexistence scenario, the optimal access policy is different when $L=2$ and when $L>2$. Therefore, we will discuss these two coexistence scenarios, respectively.

(1) $L=2$. When two model-aware nodes coexist with TDMA nodes, q-ALOHA nodes, and the FW-ALOHA node, the optimal policy of the model-aware nodes depends on z and W. In particular, we consider two cases in the following.

Case a). $z<0$: As analyzed in Section I-A4, when one model-aware node coexists with multiple TDMA nodes and N q-ALOHA nodes, if $z<0$, in order to maximize the total network throughput, the model-aware node will never occupy the q-ALOHA channel. Therefore, when two model-aware nodes coexist with multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node, if $z<0$, the optimal access policy of two model-aware nodes is as follows: one of two model-aware nodes will coexist with TDMA nodes as analyzed Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this model-aware node transmits on the TDMA channel; otherwise, it does not transmit. The other of two model-aware nodes will coexist harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput can be calculated as follows:

$$1+\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)+\frac{W^2-W+2}{W(W+1)}=\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)+\frac{2W^2+2}{W(W+1)}. \tag{39}$$

Case b). $z\geq 0$: When $z\geq 0$, in order to maximize the total network throughput, the model-aware node has three access policies to choose from.

* **The first access policy** is as follows: two model-aware nodes will be divided into two parts. For one of two model-aware nodes, for the time slots where TDMA nodes do not transmit, it transmits on the TDMA channel; for the time slots where TDMA nodes transmit, it coexists with q-ALOHA nodes, as in Section I-A4. For the other of two model-aware nodes, it coexists with the FW-ALOHA node, as in Section I-A3. Then the optimal network throughput can be calculated as follows:

$$\begin{aligned}&p\big(1+\prod_{i=1}^{N}(1-q_i)\big)+(1-p)\Big(1+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\Big)+\frac{W^2-W+2}{W(W+1)}\\ =&p\prod_{i=1}^{N}(1-q_i)+(1-p)\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)+\frac{2W^2+2}{W(W+1)}.\end{aligned} \tag{40}$$

* **The second access policy** is as follows: two model-aware nodes will be divided into two parts. For one of two model-aware nodes, for the time slots where TDMA nodes do not transmit, it transmits on the TDMA channel; for the time slots where TDMA nodes transmit, it coexists with FW-ALOHA nodes, as in Section I-A5. For the other of two model-aware nodes, it will transmit in the q-ALOHA channel in each time slot because of $z\geq 0$, as in Section I-A2. Then the optimal network throughput can be calculated as follows:

$$\begin{aligned}&p\Big(1+\frac{W^2-W+2}{W(W+1)}\Big)+(1-p)\Big(1+\frac{2}{W+1}\Big)+\prod_{i=1}^{N}(1-q_i)\\ =&\frac{W+3}{W+1}+p\frac{W^2-3W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i).\end{aligned} \tag{41}$$

* **The third access policy** is as follows: two model-aware nodes will be divided into two parts. For one of two model-aware nodes, it coexists with TDMA nodes on the TDMA channel, as in Section I-A1. For the other of two model-aware nodes,

it coexists with q-ALOHA nodes and the FW-ALOHA node in different time slots, as in Section I-A6 *Case b)* "**The first optimal access policy**". Then the optimal network throughput can be calculated as follows:

$$\begin{cases} 1+\prod_{i=1}^{N}(1-q_i)+\dfrac{2}{W+1}, & \text{if } z{\geq}\frac{W-1}{W+3}, \\ 1+\dfrac{W^2-W+2}{W(W+1)}+\prod_{i=1}^{N}(1-q_i)\dfrac{6}{W(W+1)}+\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)\dfrac{(W+3)(W-2)}{W(W+1)}, & \text{if } z{<}\frac{W-1}{W+3}. \end{cases} \tag{42}$$

First, we compare (40) and (41) as follows:

$$\begin{aligned}(40)-(41)=&(1-p)\Big(\sum_{i=1}^{N}\big(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\big)-\big(\prod_{i=1}^{N}(1-q_i)\big)+\frac{W^2-3W+2}{W(W+1)}\Big)\\ =&(1-p)\big(\frac{W^2-3W+2}{W(W+1)}-z\big)\end{aligned} \tag{43}$$

Thus, when $\big((W^2-3W+2)/(W(W+1))\big){>}z$, the network throughput achieved using the first optimal access policy is greater than that using the second optimal access policy; otherwise, the network throughput achieved using the second optimal access policy is greater than that using the first optimal access policy.

Further, we compare (41) and (42) as follows:

$$(41)-(42)=\begin{cases} p\dfrac{W^2-3W+2}{W(W+1)}{>}0, & \text{if } z{\geq}\frac{W-1}{W+3}, \\ \dfrac{W-2}{W(W+1)}\big((p-1)(W-1)+(W+3)z\big), & \text{if } z{<}\frac{W-1}{W+3}. \end{cases} \tag{44}$$

According to (44), we can get that when $\big((W-1)/(W+3)\big){\leq}z$, the network throughput achieved using the second optimal access policy is greater than the network throughput achieved using the third optimal access policy. Furthermore, according to (43), when $\big((W^2-3W+2)/(W(W+1))\big){\leq}z$, the network throughput achieved using the second optimal access policy is greater than the network throughput achieved using the first optimal access policy. Since

$$\frac{W-1}{W+3}{>}\frac{W^2-3W+2}{W(W+1)}, \tag{45}$$

we can get that when $z{\geq}\big((W-1)/(W+3)\big)$, the second access policy is optimal and the corresponding optimal network throughput is given in (41).

Now, we discuss the case when $\big((W-1)/(W+3)\big){>}z{\geq}0$ in the following.

Compare (40) and (42) as follows:

$$(40)-(42)=z\big(p-\frac{6}{W(W+1)}\big). \tag{46}$$

We can get that when $p{\geq}\big(6/W(W{+}1)$, the network throughput achieved using the first optimal access policy is greater than that using the third optimal access policy; otherwise, the network throughput achieved using the third optimal access policy is greater than that using the first optimal access policy. According to (43), we can get that when $\big((W^2-3W+2)/(W(W+1))\big){\geq}z$, the network throughput achieved using the first optimal access policy is greater than that using the second optimal access policy; otherwise, the network throughput achieved using the second optimal access policy is greater than that using the first optimal access policy. According to (44), we can get that when $z{>}\big((W-1)(1-p)/(W+3)\big)$, the network throughput achieved using the second optimal access policy is greater than that using the third optimal access policy; otherwise, the network throughput achieved using the third optimal access policy is greater than that using the second optimal access policy. In summary, when $\big((W-1)/(W+3)\big){>}z{\geq}0$, we can get

$$\begin{cases} \text{The first policy} > \text{the second policy}, & \text{if } \frac{W^2-3W+2}{W(W+1)}{>}z; \\ \text{The first policy} > \text{the third policy}, & \text{if } p{>}\frac{6}{W(W+1)}; \\ \text{The second policy} > \text{the third policy}, & \text{if } z{>}\frac{(W-1)(1-p)}{W+3}. \end{cases} \tag{47}$$

Furthermore, when $p{>}\big(6/(W(W+1))\big)$, we can get

$$\frac{(W-1)(1-p)}{W+3}{<}\frac{(W-2)(W-1)}{W(W+1)}{<}\frac{W-1}{W+3}. \tag{48}$$

Similarly, when $p \leq \big(6/(W(W+1))\big)$, we can get

$$\frac{(W-2)(W-1)}{W(W+1)} < \frac{(W-1)(1-p)}{W+3} < \frac{W-1}{W+3}. \tag{49}$$

Thus, we can get

$$\text{if } p > \frac{6}{W(W+1)} \begin{cases} \text{The first policy is optimal,} & \text{if } \frac{W^2-3W+2}{W(W+1)} > z \geq \frac{(W-1)(1-p)}{W+3}; \\ \text{The second policy is optimal,} & \text{if } \frac{W-1}{W+3} \geq z \geq \frac{W^2-3W+2}{W(W+1)}; \\ \text{The first policy is optimal,} & \text{if } \frac{(W-1)(1-p)}{W+3} > z \geq 0. \end{cases} \tag{50}$$

$$\text{if } p \leq \frac{6}{W(W+1)} \begin{cases} \text{The third policy is optimal,} & \text{if } \frac{(W-1)(1-p)}{W+3} > z \geq \frac{W^2-3W+2}{W(W+1)}; \\ \text{The second policy is optimal,} & \text{if } \frac{W-1}{W+3} \geq z \geq \frac{(W-1)(1-p)}{W+3}; \\ \text{The third policy is optimal,} & \text{if } \frac{W^2-3W+2}{W(W+1)} > z \geq 0. \end{cases} \tag{51}$$

Overall, we summarize the optimal network throughput for the coexistence of two model-aware nodes, multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node as TABLE VI.

TABLE VI
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF TWO MODEL-AWARE NODES, MULTIPLE TDMA NODES, N q-ALOHA NODES, AND ONE FW-ALOHA NODE.

$z<0$	$\sum_{i=1}^{N}\left(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\right) + \frac{2W^2+2}{W(W+1)}$
If $p > \frac{6}{W(W+1)}$	
$\frac{W^2-3W+2}{W(W+1)} > z \geq 0$	$p\prod_{i=1}^{N}(1-q_i) + (1-p)\sum_{i=1}^{N}\left(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\right) + \frac{2W^2+2}{W(W+1)}$
$z \geq \frac{W^2-3W+2}{W(W+1)}$	$\frac{W+3}{W+1} + p\frac{W^2-3W+2}{W(W+1)} + \prod_{i=1}^{N}(1-q_i)$
If $p \leq \frac{6}{W(W+1)}$	
$\frac{(W-1)(1-p)}{W(W+1)} > z \geq 0$	$\frac{2W^2+2}{W(W+1)} + \prod_{i=1}^{N}(1-q_i)\frac{6}{W(W+1)} + \sum_{i=1}^{N}\left(q_i \prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)\frac{(W+3)(W-2)}{W(W+1)}$
$z \geq \frac{(W-1)(1-p)}{W(W+1)}$	$\frac{W+3}{W+1} + p\frac{W^2-3W+2}{W(W+1)} + \prod_{i=1}^{N}(1-q_i)$

(2) $L>2$. When more than two model-aware nodes coexist with multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node, the optimal policy of the model-aware nodes depends on z. In particular, we consider two cases in the following.

Case a). $z<0$: As analyzed in Section I-A4, when one model-aware node coexists with TDMA nodes and q-ALOHA nodes, if $z<0$, in order to maximize the total network throughput, the model-aware node will never occupy the q-ALOHA channel. Therefore, when L model-aware nodes coexist with TDMA nodes, q-ALOHA nodes, and the FW-ALOHA node, if $z<0$, the optimal access policy of two model-aware nodes is as follows: these model-aware nodes should be divided into two parts. One part coexists with TDMA nodes as analyzed in Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this part transmits on the TDMA channel; otherwise, it does not transmit. The other part coexists harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. As a result, the q-ALOHA channel is only used by q-ALOHA nodes. Then the optimal network throughput as (39).

Case b). $z \geq 0$: In order to maximize the total network throughput, the optimal access policy of these model-aware nodes is as follows: these model-aware nodes should be divided into three parts. The first part coexists with TDMA nodes as analyzed in Section I-A1, i.e., for the time slots where TDMA nodes do not transmit, this part transmits on the TDMA channel; otherwise, it does not transmit. The second part transmits on the q-ALOHA channel in each time slot because of $z \geq 0$, as analyzed in Section I-A2. The last part coexists harmoniously with the FW-ALOHA node, as analyzed in Section I-A3. Then the optimal network throughput can be calculated as follows:

$$1 + \prod_{i=1}^{N}(1-q_i) + \frac{W^2-W+2}{W(W+1)} = \prod_{i=1}^{N}(1-q_i) + \frac{2W^2+2}{W(W+1)}. \tag{52}$$

In summary, we give the optimal total network throughput for the coexistence of more than two model-aware nodes, multiple TDMA nodes, N q-ALOHA nodes, and one FW-ALOHA node as in TABLE VII.

TABLE VII
THE OPTIMAL NETWORK THROUGHPUT FOR THE COEXISTENCE OF MORE THAN TWO MODEL-AWARE NODES, MULTIPLE TDMA NODES, N q-ALOHA NODES, AND ONE FW-ALOHA NODE.

$z<0$	$\sum_{i=1}^{N}\left(q_i\prod_{j=1_{j\neq i}}^{N}(1-q_j)\right)+\frac{2W^2+W}{W(W+1)}$
$z\geq 0$	$\prod_{i=1}^{N}(1-q_i)+\frac{2W^2+2}{W(W+1)}$

REFERENCES

[1] G. Bianchi, L. Fratta, and M. Oliveri, "Performance evaluation and enhancement of the csma/ca mac protocol for 802.11 wireless lans," in *Proc. IEEE PIMRC*, Oct. 1996, pp. 392–396.