Need to work out $P^{\vec{m}}$ projection operator.

For example, if both chains are prepared in so $|\psi_0\rangle$, then take a 1-site measurement, results in $\frac{1}{\sqrt{2}}(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle)$ at kth site, it would be

$$|Bell^{\vec{m}}\rangle = \frac{1}{\sqrt{2}}(C_k^\dagger|vac\rangle_k \otimes C_k^\dagger|vac\rangle_k \otimes I + |vac\rangle_k \otimes |vac\rangle_k \otimes I)$$

which means kth sites are either both filled or both empty

and $P^{\vec{m}} = |Bell^{\vec{m}}\rangle\langle Bell^{\vec{m}}|$.

However, this product form is generally hard to evaluate when we have to change basis. Instead, we can consider the operator $C_i^\dagger C_i$ and $C_i C_i^\dagger$. By anticommunication relation,

$$\begin{cases} C_i C_i^\dagger C_i^\dagger |vac\rangle = 0 \\ C_i^\dagger C_i C_i^\dagger |vac\rangle = C_i^\dagger |vac\rangle = |1\rangle_i \\ C_i C_i^\dagger |vac\rangle = |vac\rangle = |0\rangle_i \\ C_i^\dagger C_i |vac\rangle = 0 \end{cases} \qquad (C_i^\dagger C_i^\dagger = 0 \ , \ C_i|vac\rangle = 0)$$

thus $C_i^\dagger C_i$ filters out $|1\rangle_i$ while $C_i C_i^\dagger$ filters out $|0\rangle_i$

Therefore we can write $P^{\vec{m}} = C_i C_i^\dagger \otimes C_i C_i^\dagger + C_i^\dagger C_i \otimes C_i^\dagger C_i$

What we need to evaluate is the partial trace of correlation matrix, which can be expressed as

$$C_{mn} = \sum_j \langle \psi | C_m^\dagger C_j \otimes C_j^\dagger C_n | \psi \rangle \Rightarrow$$ I want to get a closed formula for this, but it seems rather complicated

where $|\psi\rangle = \frac{P^{\vec{m}}}{\sqrt{p^{\vec{m}}}} |G\rangle \otimes |G\rangle$ and $|G\rangle$ is the ground state.

Take the easiest example for 2-site fermionic chain, which is maximally entangled on ground state

$$|G\rangle = \frac{1}{\sqrt{2}} (C_1^\dagger |vac\rangle + C_0^\dagger |vac\rangle)$$

If we evaluate EE for site 1 with $|G\rangle \otimes |G\rangle$, it is

$$C_{11} = \sum_{i=0}^{1} \langle G | \otimes \langle G | C_1^\dagger C_i \otimes C_i^\dagger C_1 | G \rangle \otimes |G\rangle = 1$$

which results in $S = 0$

After measurement, the ground state is projected into

$$|\psi\rangle = \frac{1}{\sqrt{2}} (C_1^\dagger |vac\rangle \otimes C_1^\dagger |vac\rangle + C_0^\dagger |vac\rangle \otimes C_0^\dagger |vac\rangle)$$

For this state, $\langle \psi | C_m^\dagger C_j \otimes C_j^\dagger C_n | \psi \rangle = \delta_{mn}$, so it's easy to evalute that $C_{11} = \sum_{i=0}^{1} \langle \psi | C_1^\dagger C_i \otimes C_i^\dagger C_1 | \psi \rangle = \frac{1}{2}$

which results in $S = \ln 2$. and this entanglement is purely induced by measurement $P^{\vec{m}}$