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CS 760 Machine Learning Homework #4

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Problem1
Answer: By back propagation, I call the hidden layer z1 and z2 for simplicity:

$z_1 = 1*0.5+0*0.5-1*1 = -0.5$

$z_2 = 1*2-0*(-0.5)-1*1 = 1$

$y_{z1} = 1/(1+\exp(0.5)) = 0.3775$

$y_{z2} = 1/(1+\exp(-1)) = 0.7310$

$z_3 = 0.3775*2+0.7310*2-2 = 0.217$

$z_4 = 0.3775*1+0.7310*0.5 = 0.743$

$y_1 = 1/(1+\exp(-0.217)) = 0.5540$

$y_2 = 1/(1+\exp(-0.743)) = 0.6777$

$\delta_1 = 0.5540*(1-0.5540)*(0-0.5540) = -0.1369$

$\delta_2 = 0.6777*(1-0.6777)*(1-0.6777) = 0.0704$

$\delta_{z1} = 0.3775*(1-0.3775)*((-0.1369)*2+(0.0704)*1) = -0.0478$

$\delta_{z2} = 0.7310*(1-0.7310)*((-0.1369)*2+(0.0704)*0.5) = -0.0469$

$w_{10} = -2+0.1*(-0.1369)*1 = -2.0137$

$w_{11} = 2+0.1*(-0.1369)*0.3775 = 1.9948$

$w_{12} = 2+0.1*(-0.1369)*0.7310 = 1.9900$

$w_{20} = 0+0.1*(0.0704)*1 = 0.0070$

$w_{21} = 1+0.1*(0.0704)*0.3775 = 1.0027$

$w_{22} = 0.5+0.1*(0.0704)*0.7310 = 0.5051$

$w_{z10} = -1+0.1*(-0.0478)*1 = -1.0048$

$w_{z11} = 0.5+0.1*(-0.0478)*1 = 0.4952$

$w_{z12} = 0.5+0.1*(-0.0478)*0 = 0.5$

$w_{z20} = -1+0.1*(-0.0469)*1 = -1.0047$

$w_{z21} = 2+0.1*(-0.0469)*1 = 1.9953$

$w_{z22} = -0.5+0.1*(-0.0469)*0 = -0.5$

After one round calculation, the result is:

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$$z_1 = 1*0.4952 + 0*0.5 - 1*1.0048 = -0.5096$$
$$z_2 = 1*1.9953 + 0*(-0.5) - 1*1.0047 = 0.9906$$
$$y_{z1} = 1/(1+\exp(0.5096)) = 0.3753$$
$$y_{z2} = 1/(1+\exp(-0.9906)) = 0.7292$$
$$z_3 = 0.3753*1.9948 + 0.7292*1.99 - 1*2.0137 = 0.1861$$
$$z_4 = 0.3753*1.0027 + 0.7292*0.5051 + 1*(0.007) = 0.7516$$
$$y_1 = 1/(1+\exp(-0.1861)) = 0.5464$$
$$y_2 = 1/(1+\exp(-0.7516)) = 0.6795$$

We can clearly see that after one round of back-propagation, y1 gets more closer to 0 and y2 gets more closer to 1.

Problem2
Answer:
By using the sigmoid function, the input layer to hidden layer is “AND” relationship and from “hidden layer” to “output layer” is “OR” relationship.