Assignment 5

Algorithm Design and Analysis

January 4, 2019

Notice:

1. Please submit a digital version to UCAS website http://sep.ucas.ac.cn/ before 11 pm. January 17.
2. Please choose **at least 3** problems from Problem 1-6, and finish 7-8 on OJ.
3. When you're asked to give an algorithm, you should do at least the following things:
   - Describe the basic idea of your algorithm in natural language **AND** pseudo-code;
   - Prove the correctness of your algorithm.
   - Analyse the complexity of your algorithm.

1 Load balance

You have some different computers and jobs. For each job, it can only be done on one of two specified computers. The load of a computer is the number of jobs which have been done on the computer. Give the number of jobs and two computer ID for each job. You task is to minimize the max load.

(hint: binary search) $O(n^3 \log n)$

2 Matrix

For a matrix filled with 0 and 1, you know the sum of every row and column. You are asked to give such a matrix which satisfys the conditions.

3 Problem Reduction

There is a matrix with numbers which means the cost when you walk through this point. you are asked to walk through the matrix from the top left point to the right bottom point and then return to the top left point with the minimal cost. Note that when you walk from the top to the bottom you can just walk to the right or bottom point and when you return, you can just walk to the top or left point. And each point CAN NOT be walked through more than once.

4 Network Cost

For a network, there is one source and one sink. Every edge is directed and has two value c and a. c means the maximum flow of the adge. a is a coefficient number which means that if the flow of the edge is x, the cost is ax^2.

Design an algorithm to get the Minimum Cost Maximum Flow.

5 Choose Numbers

Given a matrix $M = (M_{ij})^{n\times m}$ where $M_{ij} > 0$, for every two neighbor elements, **at least** one of them should be chosen. You are asked minimize the sum of chosen elements that meets the conditions.
(hint: neighbor means share common edge. That is, the neighbors of $M_{i,j}$ are $M_{i-1,j}, M_{i,j-1}, M_{i,j+1}, M_{i+1,j}$).

6 Maximum Weight Subgraph

Given an undirected graph with n weighted vertices and m weighted edges(weights are all positive). A subgraph of a graph is some set of the graph vertices and some set of the graph edges. The set of edges must meet the condition: both ends of each edge from the set must belong to the chosen set of vertices.
The weight of a subgraph is the sum of the weights of its edges, minus the sum of the weights of its vertices.
You need to find the maximum weight of subgraph of given graph.

7 Basic Maximum Flow Algorithm

Given a directed graph with N vertices and M edges. The i-th edge is from vertex U_i to vertex V_i with capacity C_i. Find the maximum flow from source S to sink T.

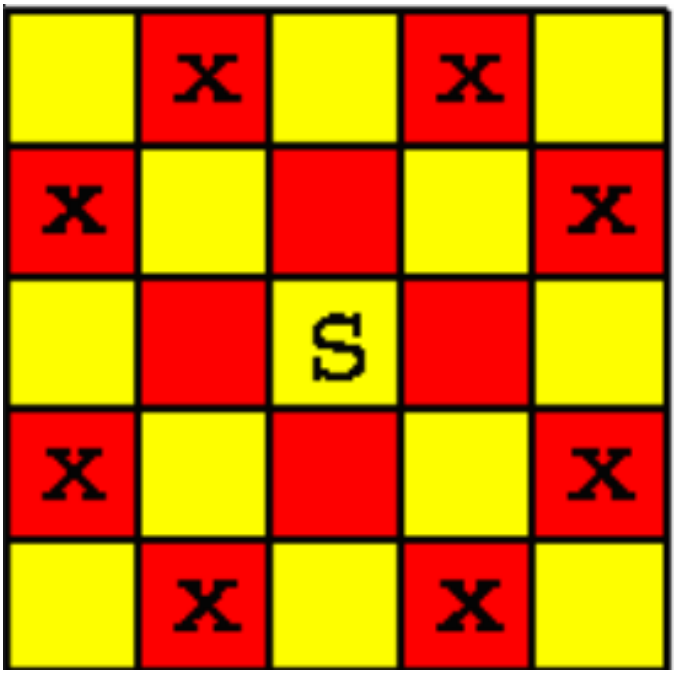
INPUT:
Input is given from Standard Input in the following format:
N M S T
U_1 V_1 C_1
U_2 V_2 C_2
..
U_N V_N C_N

OUTPUT:
Print maximum flow from source S to sink T.

8 Maximum Knights

Given a $N * N$ chessboard. There are M obstacles in the chessboard and the position of i-th obstacle is (X_i, Y_i). You are asked to find the maximum number of knights which can be placed in the chessboard at the same time, satisfied that,
1. No two knights can attack each other.
2. Knights can't be placed in obstacle.
3. There can be at most one knight in a grid.

(A Knight in chess can attack 8 positions, as shown in following figure)



INPUT:
Input is given from Standard Input in the following format:
N M
X_1 Y_1
X_2 Y_2
..
X_M Y_M

OUTPUT:
Print the maximum number of knights.