

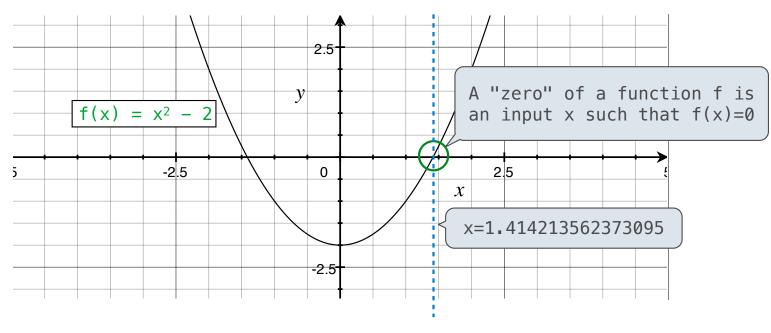


Example: Newton's Method (OPTIONAL)

(Once upon a time, this example was tested on midterms, but now it's not.)

Newton's Method Background

Quickly finds accurate approximations to zeroes of differentiable (smooth) functions



Application: a method for computing square roots, cube roots, etc.

The positive zero of $f(x) = x^2 - a$ is \sqrt{a} . (We're solving the equation $x^2 = a$.)

Newton's Method

Given a function f and initial guess x,

Repeatedly improve x:

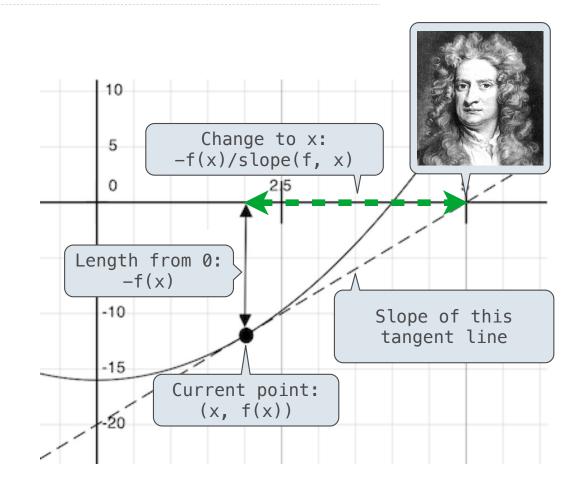
Compute the value of f at the guess: f(x)

Compute the slope of f at the guess: slope(f, x)

Update guess x to be:

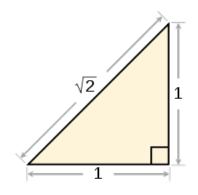
$$x - \frac{f(x)}{slope(f, x)}$$

Finish when f(x) = 0 (or close enough)



Using Newton's Method

How to find the square root of 2?



```
>>> f = lambda x: x*x - 2
>>> find_zero(f)
1.4142135623730951 Applies Newton's method
```

Fall 2012 Midterm 1 Question 4(a): Inverse

```
Implement inverse, which takes a one-argument
numerical function and returns its inverse.
def find_zero(f, x=1):
    """Return a zero of the function f."""
def sqrt(a):
    """Return the square root of a."""
    def f(x):
        return x*x - a
    return find zero(f)
def inverse(f):
    """Return the inverse function of f.
   >>> sqrt = inverse(lambda x: x * x)
   >>> sqrt(16)
    4.0
    return lambda y: find_zero(lambda x: f(x)-y)
```

The inverse of some function F is a function of argument X that returns you the Y, such that when you apply F to Y you recover the X.

Twenty-One Environment Diagram

Twenty-One Rules

Two players alternate turns, on which they can add 1, 2, or 3 to the current total

The total starts at 0

The game end whenever the total is 21 or more

The last player to add to the total loses

Some states are good; some are bad

$$21+ \leftarrow 20 \leftarrow 18 \leftarrow 16 \leftarrow 14 \leftarrow 12 \dots$$

$$17 \qquad 13 \qquad 13 \qquad \dots$$

(Demo)