Hw 7

1. Suppose that we have a neuron which, in a given time period, will fire with probability 0.1, yielding a Bernoulli distribution for the neuron's firing (denoted by the random variable $R = 0$ or 1) with $P(R = 1) = 0.1$.

(a) Compute the entropy $H(R)$ of this distribution (calculated in bits, i.e., using the base 2 logarithm)? *(1 point)*

$$H(R) = -P(R=1)\log_2 P(R=1) - P(R=0)\log_2 P(R=0)$$

$$= -0.1\log_2 0.1 - 0.9\log_2 0.9 = 0.47$$

(b) Now lets add a stimulus to the picture. Suppose that we think this neuron's activity is related to a light flashing in the eye. Let us say that the light is flashing in a given time period with probability 0.10. Call this stimulus random variable S. If there is a flash, the neuron will fire with probability 1/2. If there is no flash, the neuron will fire with probability 1/18. Call the random variable describing whether the neuron fires or not R. Compute the mutual information $I(S : R)$? *(2 points)*

S : light $\quad P(S=T) = 0.1 \quad P(S=F) = 0.9$

R : neuron $\quad P(R=T|S=T) = \frac{1}{2} \quad P(R=T|S=F) = \frac{1}{18}$

$$I(S:R) = H(R) - H(R|S)$$

$$P(R=T) = \sum_S P(R=T, S)$$

$$= P(R=T|S=T)P(S=T) + P(R=T|S=F)P(S=F)$$

$$= \frac{1}{2} \times 0.1 + \frac{1}{18} \cdot \frac{9}{10} = 0.1$$

$H(R) = 0.47$

$$H(R|S) = \sum_S P(S) H(R|S)$$

$$= -\sum_S P(S) \sum_R P(R|S)\log_2 P(R|S)$$

$$= -\sum_S\sum_R P(S)\, P(R|S)\log_2 P(R|S)$$

$$= -\big(P(S=T)P(R=T|S=T)\log_2 P(R=T|S=T) + P(S=T)P(R=F|S=T)\log_2 P(R=F|S=T)$$
$$+ P(S=F)P(R=F|S=F)\log_2 P(R=F|S=F) + P(S=F)P(R=T|S=F)\log_2 P(R=T|S=F) \big)$$

$$= -\left(0.1 \times \frac{1}{2}\log_2\frac{1}{2} + 0.1 \times \frac{1}{2}\log_2\frac{1}{2} + 0.9 \times \frac{17}{18}\log_2\frac{17}{18} + 0.9 \times \frac{1}{18}\log_2\frac{1}{18}\right)$$

$$= 0.38$$

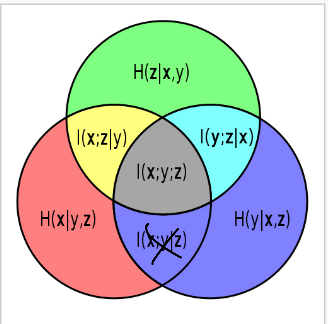
$$I(S:R) = H(R) - H(R|S) = 0.47 - 0.38 = 0.09$$

2.* Invent an example of two process that have non-zero mutual information, but 0 transfer entropy. (Juts an idea of how to construct such example is enough) *(3 points)*

From Wikipedia:

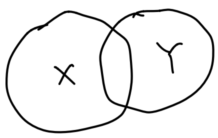
Transfer entropy is conditional mutual information,[5][6] with the history of the influenced variable $Y_{t-1:t-L}$ in the condition:

$$T_{X\to Y} = I(Y_t; X_{t-1:t-L} \mid Y_{t-1:t-L}).$$

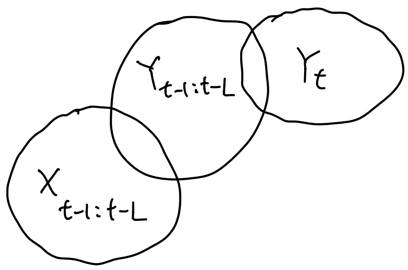


Venn diagram of information theoretic measures for three variables x, y, and z, represented by the lower left, lower right, and upper circles, respectively. The conditional mutual informations $I(x;z|y)$, $I(y;z|x)$ and $I(x;y|z)$ are represented by the yellow, cyan, and magenta regions, respectively.

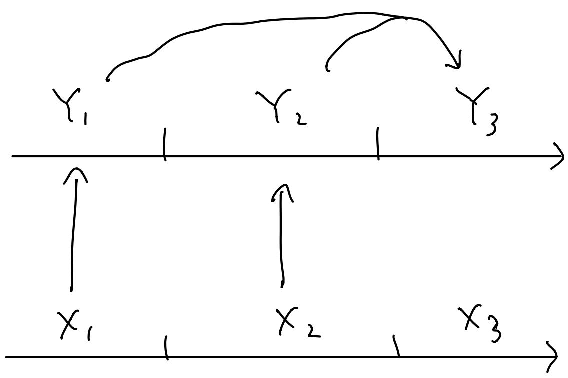
non-zero mutal information $\Rightarrow$ X, Y has overlap



zero transfer entropy $\Rightarrow$ conditioning on $Y_{t-1:t-L}$
$X_{t-1:t-L}$ & Y_t has no overlap



ex.



An example would be history of Y depends on X but current value of Y_t only depends on history of Y.