Analysis of Assets and Portfolios

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Part One: Summary

- Assets from similar sectors always have strong positive linear associations
- Stable assets are more likely to have larger weights in the MVP, and volatile assets should follow a short sale
- Efficient portfolio contains 13.75% for TWTR, 19.8% for FB, 32.02% for AAL, 4.68% for DAL, and 29.76% WBD. The Tangency portfolio contains 48.3% for MSFT, 6.86% for TSLA, 9.03% for AAPL, 4.52% for AMZN, and 31.29% for NVDA.
- When a risk-free asset is added, the combination of tangency portfolio and risk-free with weighting 12.76% and 87.24% is a better choice in our case
- Diversification should be a good tool for reducing risk but is not very helpful for us
- Using around eight or more Principal Components would explain the majority (90%) of the variance in our data, but four PCs are enough under factor analysis
- Using parametric estimation, the highest VaR is from Tesla, $23648.3; the lowest VaR is from Microsoft, $7239.2. The highest ES is from Tesla, $30876; the lowest ES is from Microsoft, $9712
- Using nonparametric estimation the highest VaR is from Twitter, $22137.6; the lowest VaR is from Microsoft, $6918.2. The highest ES is from American Airline, $24467; the lowest ES is from Microsoft, $9183
- Both AIC and log-likelihood show that t-copula would be the best one to represent the joint distribution of returns

Part Two: Descriptive Statistics

2.1 Sample Statistics (Mean, SD, Skewness, Kurtosis, Beta) (Appx.Table 1)

By calculating the means of both prices and returns of the 12 assets, we found out that, over the years, AMZN always has the highest average monthly price—$1550.23 and is much higher than the other assets we chose. In contrast, BAC has the lowest average monthly price—$23.76. Compared to S&P 500 Index, all the assets have lower prices. In the perspective of returns, NVDA has the highest average monthly return—5.15%, from 2014 till now, and WBD has the lowest average monthly return—0.11%. MSFT, TSLA, AAPL, AMZN, FB, NFLX, BAC, and NVDA have higher average monthly returns than S&P 500 Index does. From the standard deviations of prices., we can see AMZN has the highest standard deviation, meaning that it has the highest volatility in prices, and WBD has the lowest volatility in prices. TSLA has the highest standard deviation in its monthly returns, showing the most significant fluctuation in returns, and MSFT has the lowest standard deviation in its monthly returns, showing that it is the most stable asset among these 12 assets. Compared to the S&P500, every asset has a higher standard deviation.

Then we calculated the skewness and the kurtosis coefficients. We found that most monthly prices of the assets have a slightly right-skewed distribution, except AAL and DAL have left-skewed distribution. By checking the skewness in their returns, AAPL, FB, AAL, DAL, and BAC show negative coefficients, indicating a left-skewed distribution in their monthly returns. From the kurtosis coefficients of their prices, we found that TSLA, AAPL, NVDA, and WBD have positive kurtosis, and NVDA has the highest kurtosis, exhibiting a higher peak and fatter tail and larger changes from means. However, only the kurtosis of AAPL's monthly return is negative, meaning that only AAPL has more minor changes in monthly returns relative to its means. However, since all the skewness and kurtosis coefficients are generally small—close to 0, they do not show too many deviations from a normal distribution.

Using S&P 500 Index as the market index (which gives a beta of 1), we found TSLA, AAPL, AMZN, FB, AAL, DAL, BAC, NVDA, and WBD have betas larger than 1, indicating they are more aggressive assets compared to the market. Only TWTR and NFLX show betas smaller than 1, showing that they are less aggressive. MSFT has a beta value of 1.004, which is almost 1, meaning that MSFT is an asset that has almost the same changes as the market.

2.2 Time Plots

By checking the plots of monthly prices versus time (Appx.Figure 2), we found that prices of MSFT, TSLA, AAPL, AMZN, FB, NFLX, BAC, and NVDA, in general, increased continuously. On the contrary, TWTR, AAL, DAL, and WBD have much more fluctuations in their monthly prices. Because of the COVID-19, AAL and DAL had the largest losses and had the severest crashes in their monthly prices, followed by BAC and WBD also had a drop in their prices.

However, we can see that MSFT and NVDA still had a steady increase; TSLA, AAPL, TWTR, AMZN, FB, and NFLX even had sharper increases in their prices. This can be explained by the increase in the consumption of these companies' products during the lockdown. We should notice the WBD price had a massive increase, to a new peak, followed by a dip to its original level. By checking the news, we found that probably it was because, in December of 2020, Warner Bros. announced to release all 17 of its 2021 films in a "hybrid model." By further checking the returns plots (Appx.Figure 2), we noticed sharp decreases in FB's and NFLX's returns at the beginning of 2022. As for Facebook, we thought it was because of the outage in October 2021, and since then, FB's price has kept decreasing. As for Netflix, a rise in standard plan price and a weaker content slate in 2022 both stand to lose its subscriber base.

2.3 Equity Curve

By comparing the asset's equity curves and the S&P 500 equity curve (Appx.Figure 3), we noticed that MSFT, TSLA, AAPL, AMZN, FB, NFLX, and NVDA are the assets that always outperform the market. Comparing the two airline companies, AAL and DAL, we found before COVID-19 and around the mid of 2018, AAL began to show poorer performance. But for DAL, only until COVID-19 does it show performance worse than the market. What is more, WBD and TWTR always show lower equity curves than S&P500.

2.4 Stationary Test

In this section, we first draw the ACF plots (Appx.Figure 4). The plots show that almost all assets' autocorrelations are in the 95% limit threshold, except only one or two lags exceed the limits. Thus, we can conclude that the residuals behave like white noise. We then conduct the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test to test the stationary. All the assets show a p-value of 0.1 or larger, meaning that all of them are stationary and there is no specific trend.

2.5 Histograms, Boxplots, and QQ-Plots

Let us then look at the histograms (Appx.Figure 5). Combining with the density plots, MSFT, AMZN, FB, AAL, DAL, BAC, and NVDA have almost normal distributions in their monthly returns. TSLA, TWTR, and WBD show obvious right skewness. AAPL shows a lower peak and fatter tail, and NFLX shows a slight right-skewed distribution. Further checking the QQ-plots (Appx.Figure 5), the information is consistent with what we found in the histograms. From the boxplots (Appx.Figure 5), we found that all assets have outliers in their returns, and most of them are in the right tails except for AAPL. We also noticed that TSLA and TWTR have larger ranges in their monthly returns.

2.6 Fit Distributions

We choose three distributions: t-distribution, normal distribution, and generalized error distribution to fit the assets. By comparing which gives the smallest AIC by fitting them individually to each asset, we decide which distribution we should apply. Therefore, the AIC results (Appx.Table 6) conclude that MSFT, FB, NFLX, AAL, DAL, BAC, NVDA, and WBD should fit with generalized error distribution, and TSLA AAPL, TWTR, and AMZN should fit with normal distribution. No asset should fit with t-distribution.

2.7 Sharpe's Slope

According to the Sharpe's slopes (Appx.Table 7), we know that NVDA has the highest Sharpe's slope, indicating the best investment performance, given the risk; WBD has the lowest Sharpe's slope, with a negative number. DAL has the smallest Sharpe's slope among all the positive data.

2.8 Annualization (Appx. Table 8)

By comparing the monthly statistics and the annual statistics, we found the annual statistics have larger ranges: the lowest annual mean of 1.27% is from the WBD, and the highest 61.75% is from NVDA; MSFT has the lowest annual standard deviation, and TSLA has the highest annual standard deviation, in which are the same as the monthly data.

2.9 Pairwise Scatter Plots and Covariance Matrix

The pairwise scatter plots on assets' monthly returns (Appx.Figure 9) demonstrate that there is, in general, no obvious relationship or positive relationship between any two assets. We noticed a strong positive linear association between DAL and AAL, which is apparent as they are both airline companies. AAPL and FB also show a positive relationship. Thus, we can conclude that companies from the same sector are more likely to have a strong positive linear relationship. In addition, we can see some tail dependency in these plots, such as the plot of AAL and BAC and the plot of AMZN and NVDA.

By checking the covariance matrix (Appx.Table 10), we can ensure that there are no negative linear associations between either two of the 12 assets.

Part Three: Portfolio Theory

This part will discuss the MVPs and tangency portfolios with and without a short sale. Graphs below show how the portfolios are distributed under these two situations. The solid red curve is the efficient frontier, and the dashed curve is the line of efficient portfolios. The blue line is tangent to the efficient frontier, connecting the risk-free asset (asterisk at bottom left) and the tangency portfolio with the 12 assets (asterisk on the solid red curve). The plus sign on the curve is the MVP with the 12 assets. When a short sale is allowed, we can see that all 12 assets are to the right of the frontier, showing higher standard deviations. They have either higher or lower returns compared to the MVP. But when a short sale is restricted, only NVDA has higher monthly returns than the tangency portfolio.

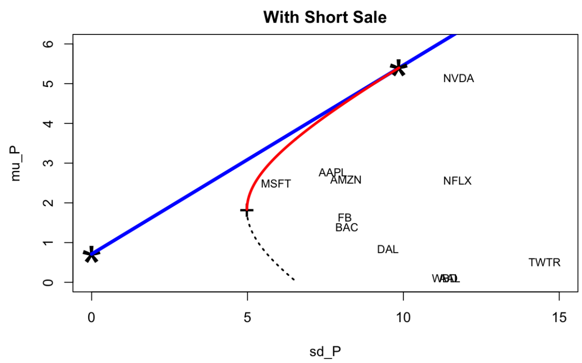


Figure 1

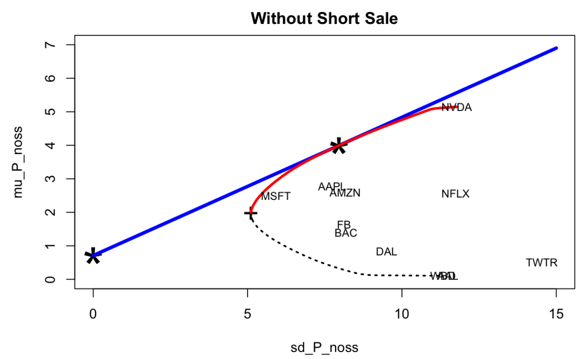


Figure 2

3.1 With Short Sale

3.1.1 Minimum Variance Portfolio (MVP)

The MVP with short sales has a monthly return of 1.82% and a corresponding standard deviation of 4.98; it has an annual return of 21.8% and a standard deviation of 17.25. We noticed that MSFT, TSLA, AAPL, AMZN, NFLX, and NVDA have higher annual returns than the MVP, but the MVP has the lowest annual standard deviation, showing the slightest volatility. The table below shows the weights (in the percentage) of the 12 assets in the MVP. We can see that we should invest over half of our funds to MSFT and sell short for TSLA, AAL, and NVDA.

	MSFT	TSLA	AAPL	TWTR	AMZN	FB	NFLX	AAL	DAL	BAC	NVDA	WBD
weights_mvp	50.17	-4.98	13.48	4.89	3.85	13.16	4.31	-9.16	21.82	8.07	-5.96	0.34

Table 3

Because of the tail dependency between the assets, we choose to fit the MVP with a multivariate t-distribution. When holding $100,000 to invest, the MVP's 5% value-at-risk (VaR) is $6285.691, and the expected shortfall (ES) is $8957.941. According to the best-fit distributions figured out above, we use either parametric or non-parametric methods to calculate the VaR for each asset according to their distributions (Appx.Table 11). The MVP has the smallest VaR of $6285.69, followed by MSFT has a VaR of $6918.18.

3.1.2 Tangency Portfolio

The tangency portfolio with short sales has a monthly return of 5.4%, a variance of 97.24, and a standard deviation of 9.86. The tangency portfolio has a Sharpe's ratio of 0.475, higher than Sharp's ratios of any of the 12 assets.

3.2 Without Short Sale

3.2.1 MVP

The MVP without short sales has a monthly return of 1.98% and a standard deviation of 5.11. The annual return is 23.72%, and the standard deviation is 17.73. The statistics are higher than what we got when the short sale was allowed.

Below is the table indicating how we should invest when the short sale is not allowed. We should still invest the most in MSFT and not in TSLA, AAL, and NVDA.

	MSFT	TSLA	AAPL	TWTR	AMZN	FB	NFLX	AAL	DAL	BAC	NVDA	WBD
weight	48.7966	0	6.5926	5.6099	3.2476	13.4688	2.2567	0	14.6045	4.4533	0	0.97

Table 4

Using identical distributions, the MVP's 5% VaR holding $100,000 without a short sale is $6291.464, and the ES is $9034.092. Both numbers are higher than what we got when the short sale is allowed.

3.2.2 Tangency Portfolio

Without the short sale, the monthly return of the tangency portfolio is 4%, the variance is 63.52, and the standard deviation is 7.97. The Sharpe's ratio is 0.4125, and all of the four statistics are smaller than what we got when the short sale was allowed. Compared to each asset individually, the tangency portfolio still has the highest Sharpe's ratio.

Part Four: Asset Allocation

After computing the minimum variance portfolio, in this part, we are going to dig further into the asset allocation with an efficient portfolio and tangency portfolio.

In order to achieve a target expected return of 6% per year with only risky assets and no short sales allowed, we carry out an efficient portfolio with a weight of 13.75% for TWTR, 19.8% for FB, 32.02% for AAL, 4.68% for DAL, and 29.76% WBD. With that efficient portfolio, our monthly risk is 2.13%. With an initial investment of $100,000, our monthly 5% value-at-risk is $3011.654 (3.01%) and expected shortfall is $3903.775.

```
efficient.portfolio(er = return_year, cov.mat = cov_mat, target.return = 0.06,
    shorts = FALSE)

Portfolio expected return:     0.05999993
Portfolio standard deviation:  0.07395627
Portfolio weights:
  MSFT   TSLA   AAPL   TWTR   AMZN     FB   NFLX    AAL    DAL    BAC   NVDA    WBD
0.0000 0.0000 0.0000 0.1375 0.0000 0.1980 0.0000 0.3202 0.0468 0.0000 0.0000 0.2976
```

Figure 5

A tangency portfolio is a portfolio of risky assets that has the highest Sharpe ratio. In order to create a tangency portfolio, we need to invest 48.3% for MSFT, 6.86% for TSLA, 9.03% for AAPL, 4.52% for AMZN, and 31.29% for NVDA. By combining this portfolio with the Treasury Bill, we need to weigh 12.76% for the tangency portfolio, and 87.24% for the risk-free.

With an initial investment of $100,000, our monthly 5% value-at-risk is $2852.626 (2.852%) and expected shortfall is $3683.158.

Comparing the VaR of these two portfolios, the combination of the tangency portfolio and Treasury bill performed better with a monthly 5% value-at-risk equaling 2.852%.

```
tangency.portfolio(er = return_year, cov.mat = cov_mat, risk.free = risk_free,
    shorts = FALSE)

Portfolio expected return:     0.4214667
Portfolio standard deviation:  0.06885209
Portfolio weights:
  MSFT   TSLA   AAPL   TWTR   AMZN     FB   NFLX    AAL    DAL    BAC   NVDA    WBD
0.4830 0.0686 0.0903 0.0000 0.0452 0.0000 0.0000 0.0000 0.0000 0.0000 0.3129 0.0000
```

Figure 6

Part Five: Principal Component Analysis

In order to focus more on the relationships between the returns of the twelve assets, we exhibit a variety of methods that show their patterns. We first display a correlation matrix of the returns, where the upper right panel shows the correlation between the variables, while the lower-left displays the smoothed regression lines. As shown in Figure 7, AAL and DAL are the assets that are most highly correlated with a correlation = 0.78, while NFLX and DAL have the lowest correlation = 0.0089.

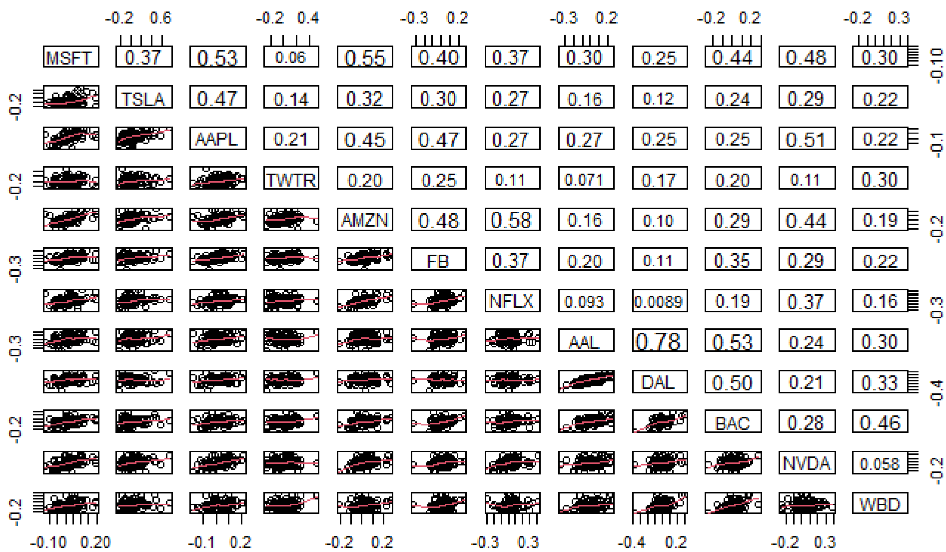


Figure 7, Correlation Matrix

By understanding the correlation between the asset, we could determine whether the allocating investments could make the risk management. According to the theory, diversification will reduce the risk for the assets that are negatively correlated. For example, if the correlation between two assets is -1, which is the perfect value for diversification, if one asset goes up 5%, the other asset will go down 5%. So combining these 2 assets would minimize our risk to 0. In our twelve assets, we did not see any negative correlation. As a result, diversification will not reduce risk.

Another way to dig into the correlation between the return of the assets, we looked at PCA and its plots. PCA helps to reduce the dimension of the dataset and compute principal components that contain most of the information in the data. From the PCA Analysis (Figure 8), we found that the first eight components capture almost 90% of the total variance, while nine components could explain 93% of the variance.

Importance of components:

	PC1	PC2	PC3	PC4	PC5	PC6	PC7	PC8	PC9	PC10	PC11	PC12
Standard deviation	2.0761	1.3586	1.06307	0.93727	0.8879	0.80884	0.78736	0.70899	0.67221	0.56977	0.54539	0.44294
Proportion of Variance	0.3592	0.1538	0.09418	0.07321	0.0657	0.05452	0.05166	0.04189	0.03766	0.02705	0.02479	0.01635
Cumulative Proportion	0.3592	0.5130	0.60717	0.68038	0.7461	0.80060	0.85227	0.89415	0.93181	0.95886	0.98365	1.00000

Figure 8

The Scree Plot (Figure 9) shows that the unexplained part drops quickly by adding PC1 to PC4, and gradually slower after that. Therefore, using around four to eight PCs would explain the majority of the variance in our data.

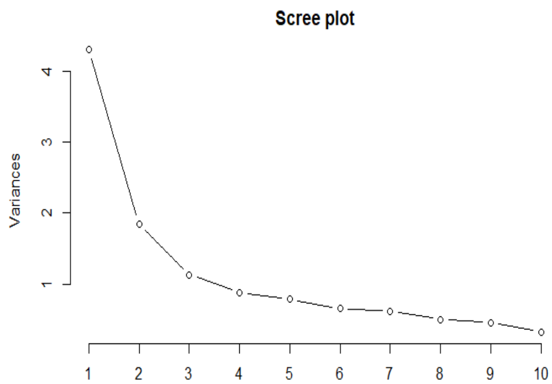


Figure 9

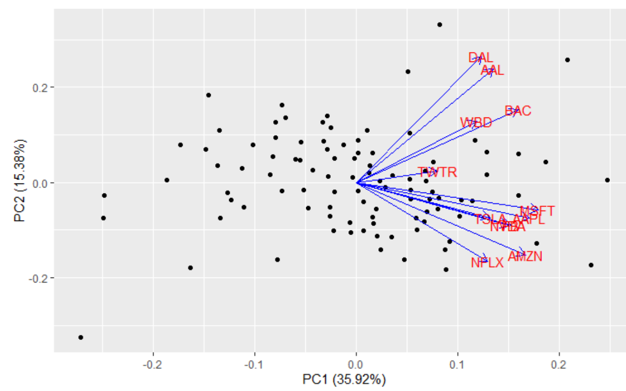


Figure 10

To further develop the correlations, we also draw a biplot (Figure 10). According to the principle of the biplots, the variables are positively correlated if their vectors form an angle smaller than 90 degrees. From our plot, all the assets are having an acute angle, so we conclude that they all have a positive relationship.

We also run a factor analysis with 4 PCs, the result showing that four factors are sufficient to explain our data set. By reducing the factor to three, the p-value of the test drops sharply to 0.185, so we reject the hypothesis and conclude that four or more PCs are sufficient.

```
Call:
factanal(x = df, factors = 4, rotation = "varimax")

Uniquenesses:
 MSFT  TSLA  AAPL  TWTR  AMZN    FB  NFLX   AAL   DAL   BAC  NVDA   WBD 
0.487 0.728 0.005 0.834 0.325 0.613 0.519 0.192 0.225 0.468 0.584 0.394 

Loadings:
     Factor1 Factor2 Factor3 Factor4
MSFT  0.565   0.217   0.316   0.217 
TSLA  0.308           0.356   0.214 
AAPL  0.303   0.132   0.927   0.165 
TWTR                  0.122   0.374 
AMZN  0.784           0.196   0.137 
FB    0.479           0.296   0.254 
NFLX  0.685                         
AAL   0.106   0.880   0.108   0.106 
DAL           0.855   0.122   0.170 
BAC   0.295   0.513           0.425 
NVDA  0.492   0.191   0.369         
WBD           0.237           0.735 

               Factor1 Factor2 Factor3 Factor4
SS loadings      2.178   1.940   1.394   1.114
Proportion Var   0.181   0.162   0.116   0.093
Cumulative Var   0.181   0.343   0.459   0.552

Test of the hypothesis that 4 factors are sufficient.
The chi square statistic is 19.62 on 24 degrees of freedom.
The p-value is 0.718
```

Figure 11

Part Six: Risk Management

To have a basic understanding of risk management strategies, we considered the Value at Risk (VaR) and Expected Shortfall (ES) to analyze the risk of getting an extreme loss in one month. By definition, VaR is a bound such that the loss over the horizon is less than this bound with probability equal to the confidence coefficient. By knowing the VaR, on the other hand, we will get to know what is the cutoff of investment loss that may occur with a certain probability or more extreme. ES is the expected loss given a tail event, tail loss, and shortfall. In other words, ES will let us know the average loss once we fall into an extreme situation. In our analysis, we will set alpha = 0.05, which means that we are evaluating the monthly extreme losses within 5% probability. Our investment amount is $100,000. There are two ways to do VaR and ES estimation - parametric and nonparametric.

6.1.1 Parametric Estimation for VaR (Appx. Table 12)

To calculate the VaR for each of the 12 assets in a parametric way, we are going to first assume the distribution of asset monthly returns. In this case, we assume that all returns are normally distributed. Then, by knowing the standard deviation and mean of each asset return, we can further define their normal distributions. Calculating the quantile of normal returns is easy, and we can know the 0.05 quantile of return for each asset, as we defined alpha equals 0.05. VaR is the product of negative 100,000 times the 0.05 quantile of return.

For example, from the data of Microsoft return from Jan 2014 to Mar 2022, we calculated the VaR to be $7239.2, which means that there is only a 5% chance of a loss exceeding $7239.2 over the next month if we invest $100,000 in Microsoft.

6.1.2 Parametric Estimation for ES (Appx. Table 12, 13)

We can also calculate the ES in a parametric way. Assuming that the returns are normally distributed and are continuous variables, we can further calculate the ES.

For example, we have calculated the Expected Shortfall in Microsoft return from Jan 2014 to Mar 2022, which is $9712. The ES is telling us that, the expected loss given that the loss exceeds VaR is $9712. The highest VaR is from Tesla, $23648.3; the lowest VaR is from Microsoft, $7239.2. The highest ES is from Tesla, $30876; the lowest ES is from Microsoft, $9712.

6.2 Nonparametric Estimation for VaR and ES (Appx. Table 12)

Similarly, we can use a nonparametric method to calculate the two values. Instead of assuming a specific distribution of returns, we simply look at the quantile of the sample itself and determine the VaR. The nonparametric estimation of VaR for Microsoft is $6918.2, and of ES is $9183. Notice that here ES calculation is different since we no longer assume the return to be continuous and normally distributed. Instead, we sum up all the returns that exceed the 0.05 quantile and calculate the average to be ES.

The nonparametric result is slightly different from the parametric one. It indicates that an MLE estimation of normal distribution might be appropriate for estimating the VaR and ES of Microsoft. In some other stocks, however, the conclusion would be different. For example, there is a large gap between the two estimation methods for Tesla. The highest VaR is from Twitter, $22137.6; the lowest VaR is from Microsoft, $6918.2. The highest ES is from American Airline, $24467; the lowest ES is from Microsoft, $9183.

6.3 Bootstrapping (Appx. Table 13)

We would like to have a further investigation on the confidence interval of VaR and ES. The challenge here is that, since we can only calculate one VaR and ES from one sample, we cannot get a general distribution of VaR and ES by calculating the quantiles. That’s the reason why we would like to use bootstrapping techniques. By generating different samples of returns multiple times, we can get a distribution of VaR and ES, and then further calculate the confidence intervals.

As we have introduced parametric and nonparametric estimation in the previous content, we would like to use both methods here to get the CI. The lower and upper quantiles of the bootstrap estimates of VaR(α) and ES(α) are the limits of the basic percentile method confidence intervals.

Part Seven: Copulas

The dependency between each asset return is also important. Usually, we use the correlation to know how a distribution is related to the other. However, it only implies linear relationships. Copula is a more flexible tool to understand the dependency between various distributions. The copulas allow us to model the dependence structure independently from the marginal distributions. For example, a pair of assets may have weakly correlated returns, but their largest losses may tend to occur in the same periods.

There are many candidate copulas that might be a good fit for the distribution of asset returns. Hence we are going to fit parameters for various copulas first using maximum likelihood. Our copulas include Gaussian, Frank, Clayton, Gumbel, and Joe. After fitting copulas, we need to select the best-fitted copula by log-likelihood and AIC (figure 12). Both criteria show that t-copula would be the best one to represent the joint distribution of returns. T-copulas have the tail dependence and the dependence increases with correlation and decreases with the number of degrees of freedom. Hence it indicates that we would be able to know about the association of extreme values between two asset returns by looking at the correlation and degrees of freedom.

Copula families	AIC	Log-Likelihood
t-distribution	-338.93	236.47
Gaussian	-230.92	116.46
Frank	-165.58	83.79
Clayton	-221.52	111.76
Gumbel	-170.91	86.46
Joe	-104.77	53.38

Figure 12

Part Eight: Conclusion

In this project, we used the monthly statistics, from January 2014 to March 2022, of MSFT, TSLA, AAPL, TWTR, AMZN, FB, NFLX, AAL, DAL, BAC, NVDA, and WBD. We found that different assets have different fitted distributions, which will in turn affect our parametric estimates of their VaR. Assuming different distributions, the MVP's 5% VaR holding $100,000 without a short sale is $6291.464, and the ES is $9034.092. Both numbers are higher than what we got when the short sale is allowed. The result of VaR and ES when assuming all distributions are normal is different (see part six).

Our efficient portfolio includes 13.75% for TWTR, 19.8% for FB, 32.02% for AAL, 4.68% for DAL, and 29.76% WBD. With that efficient portfolio, our monthly risk is 2.13%. With an initial investment of $100,000, our monthly 5% value-at-risk is $3011.654 (3.01%) and expected shortfall is $3903.775.

Our tangency portfolio contains 48.3% for MSFT, 6.86% for TSLA, 9.03% for AAPL, 4.52% for AMZN, and 31.29% for NVDA. When a risk-free asset is added, the combination of tangency portfolio and risk-free with weighting 12.76% and 87.24%, our monthly 5% value-at-risk is $2852.626 (2.852%), and the expected shortfall is $3683.158. That portfolio is a better choice in our case.

The Correlation Matrix shows that the returns are all having a positive relationship. By using PCA, we found that unexplained errors drop quickly by adding PC1 to PC4, and gradually slower after that. Therefore, using around four to eight PCs would explain the majority of the variance in our data, as the first eight components capture almost 90% of the total variance.

Then we used parametric (assuming normally distributed) and nonparametric estimation to get the VaR and ES of each asset. We found that for some assets, these two estimates give quite different results, probably because of the discrepancy between their actual return distribution and the assumed normal distribution. We also used a bootstrapping method to get confidence intervals of VaR and ES for each asset.

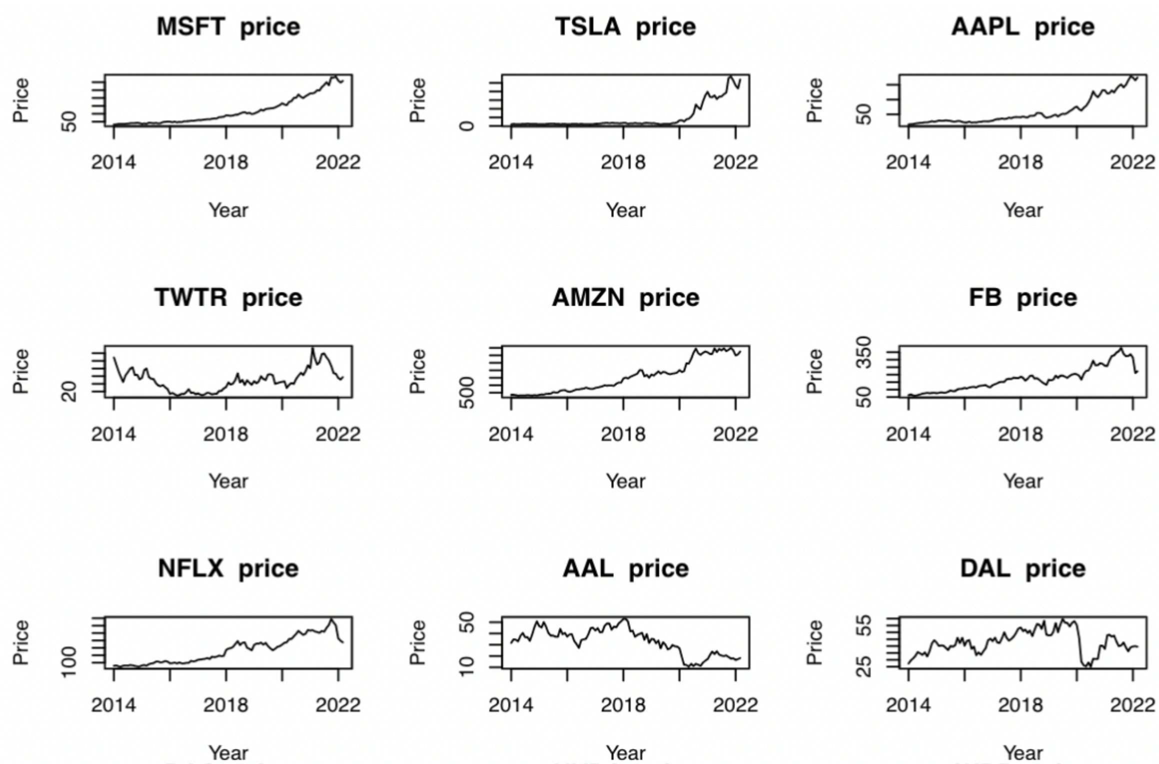
To know more about the dependency between assets, we used several copulas to fit the joint distribution of asset return data. We found that the t-copula fits better than any other copula, which gives us some implications for the tailed dependency of returns by knowing their correlations and the model's degrees of freedom.

Appendix

Descriptive statistics

	Mean (price)	Mean(return %)	SD (price)	SD(return %)	Skewness (price)	Skewness (return %)	Kurtosis (price)	Kurtosis (return %)	Beta
MSFT	117.508	2.495	86.817	5.918	1.028	0.228	-0.139	0.616	1.004
TSLA	203.202	4.802	298.245	17.296	1.796	1.273	1.874	2.234	1.993
AAPL	58.870	2.774	45.183	7.744	1.251	-0.230	0.225	-0.272	1.247
TWTR	34.921	0.506	14.229	14.535	0.640	0.410	-0.004	0.376	0.869
AMZN	1550.228	2.595	1062.254	8.163	0.541	0.395	-1.091	0.651	1.175
FB	171.007	1.636	80.593	8.123	0.702	-0.330	-0.295	2.773	1.219
NFLX	264.865	2.569	176.639	11.741	0.447	0.465	-1.037	1.031	0.941
AAL	33.150	0.112	11.580	11.493	-0.350	-0.095	-0.941	0.217	1.559
DAL	42.931	0.840	8.403	9.510	-0.056	-0.184	-0.648	2.629	1.183
BAC	23.756	1.398	9.369	8.191	0.685	-0.157	-0.320	1.025	1.438
NVDA	64.032	5.146	74.026	11.778	1.673	0.070	2.157	0.418	1.452
WBD	29.026	0.106	6.327	11.332	1.077	0.841	1.363	1.107	1.244
S.P500	2770.638	1.036	799.620	4.002	0.951	-0.387	-0.111	1.343	1.000

Table 1, Sample Statistics



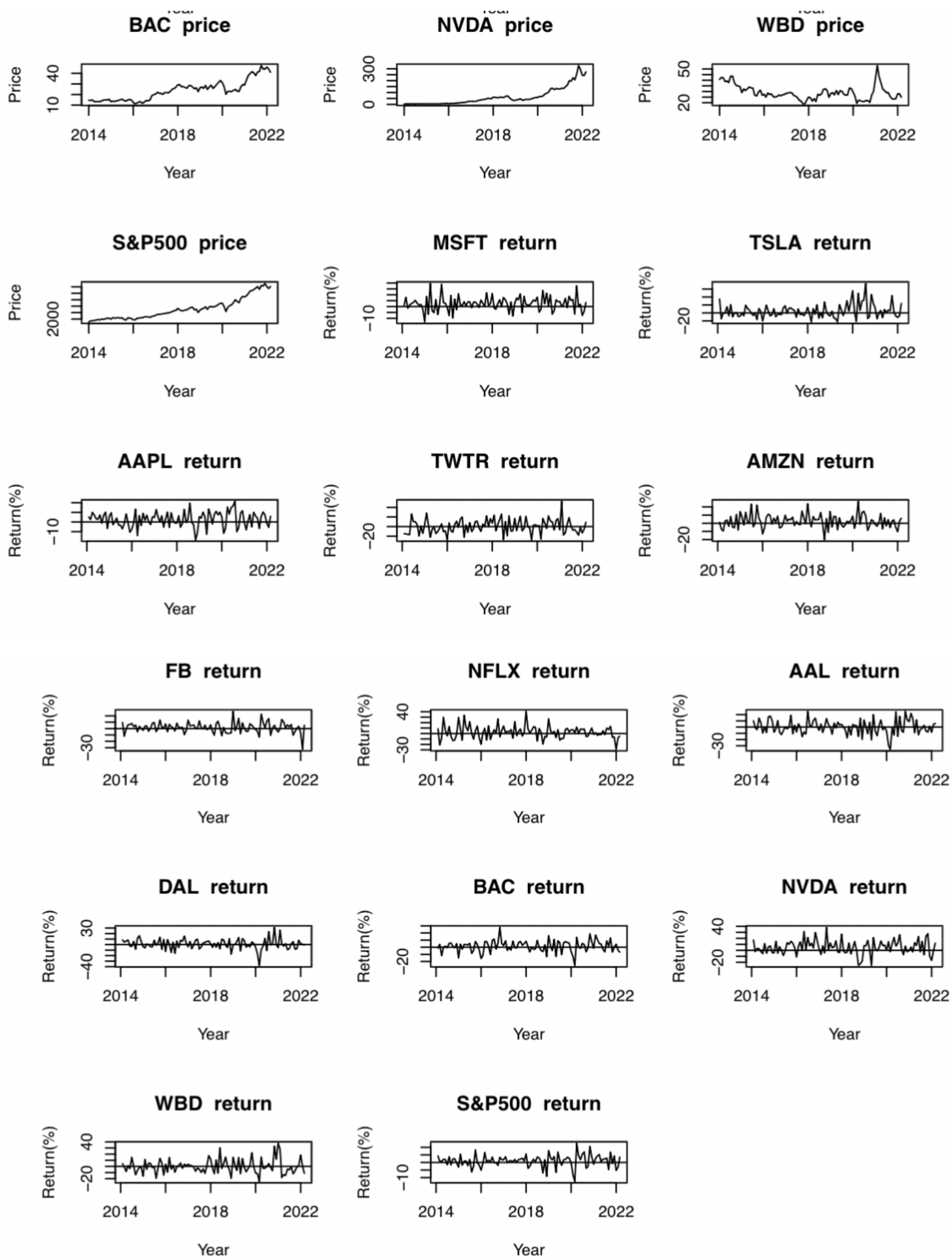


Figure 2, Time Plots on Prices and Returns

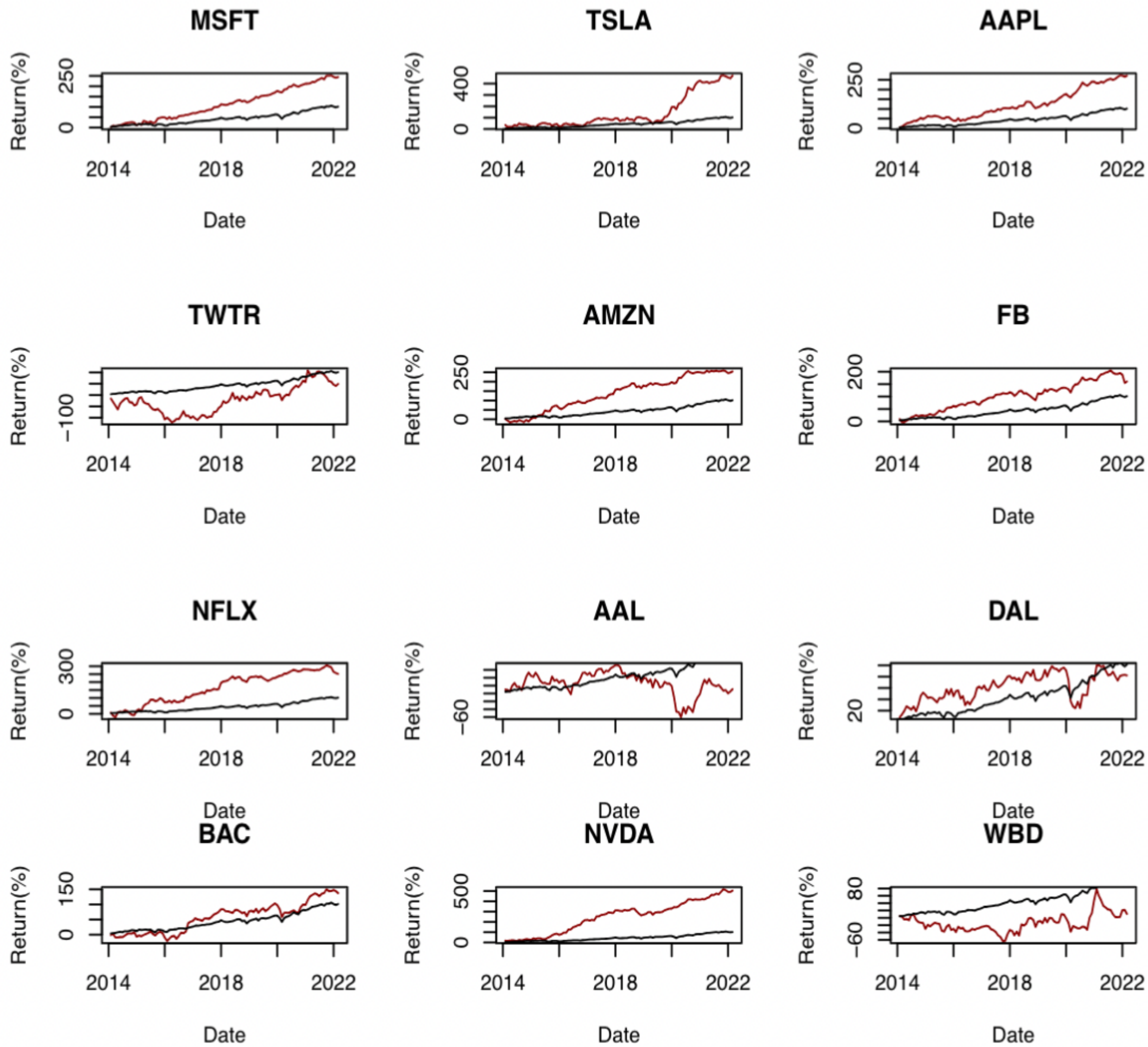


Figure 3, Equity Curve (Red: asset, Black: S&P 500)

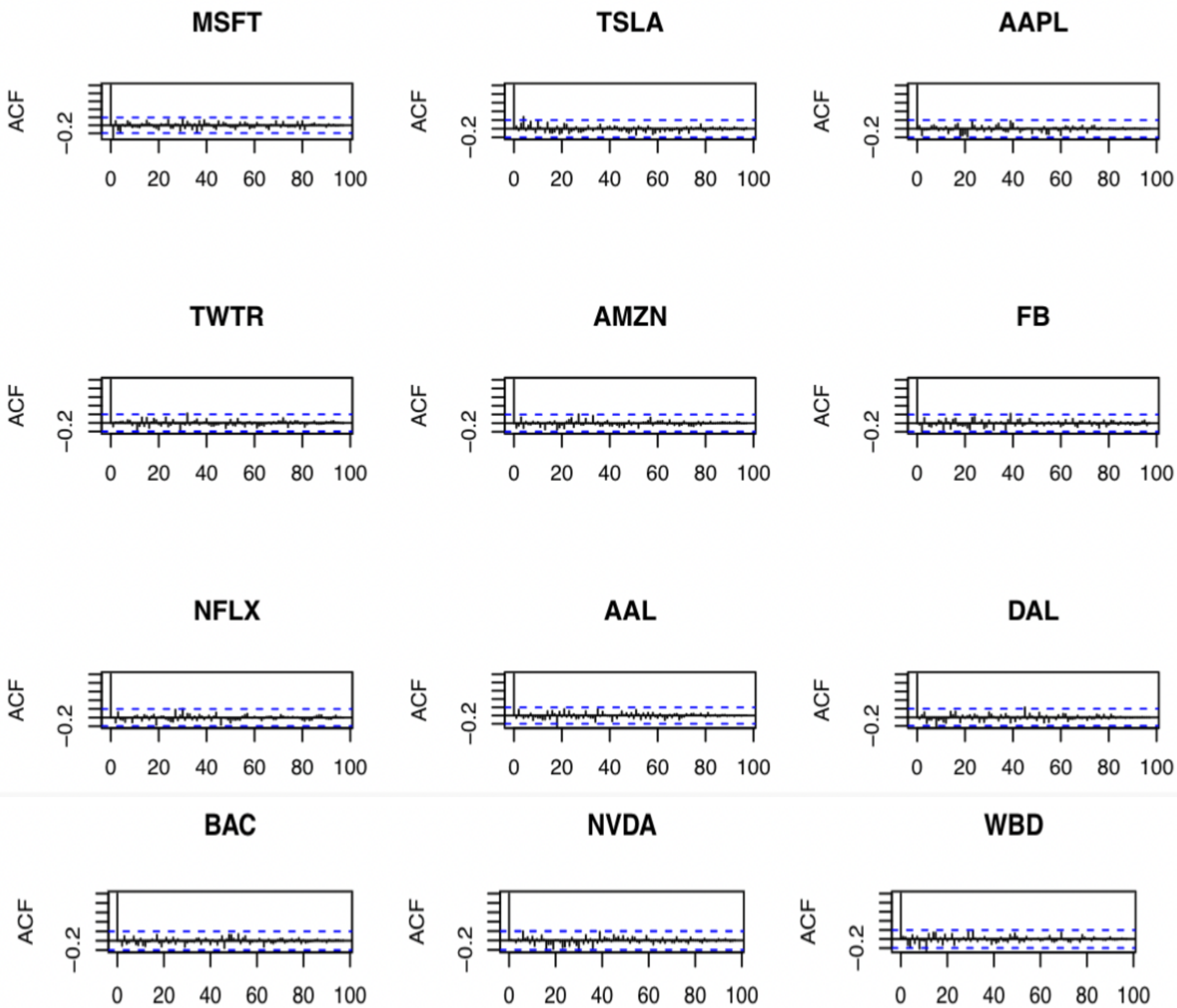
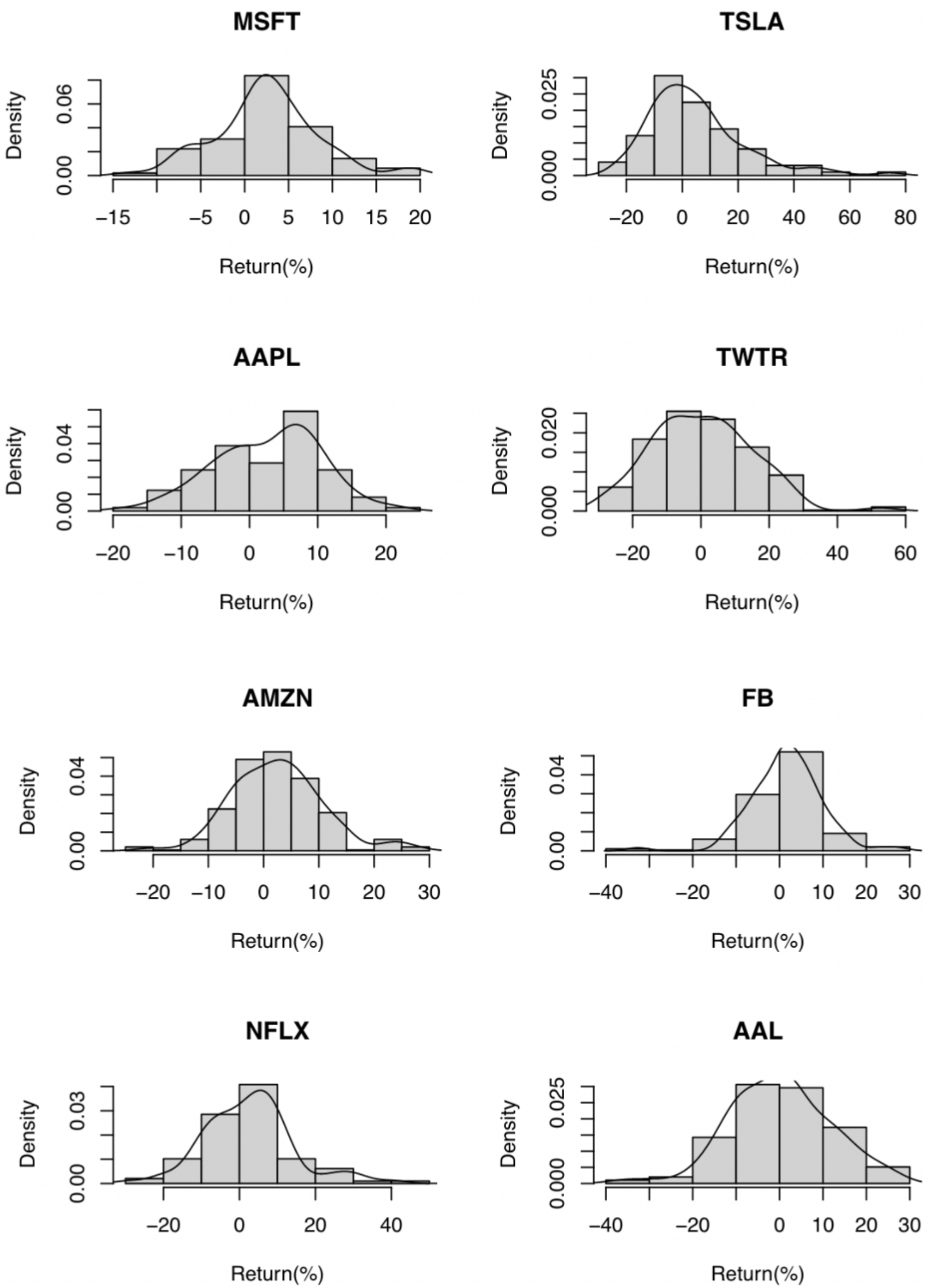
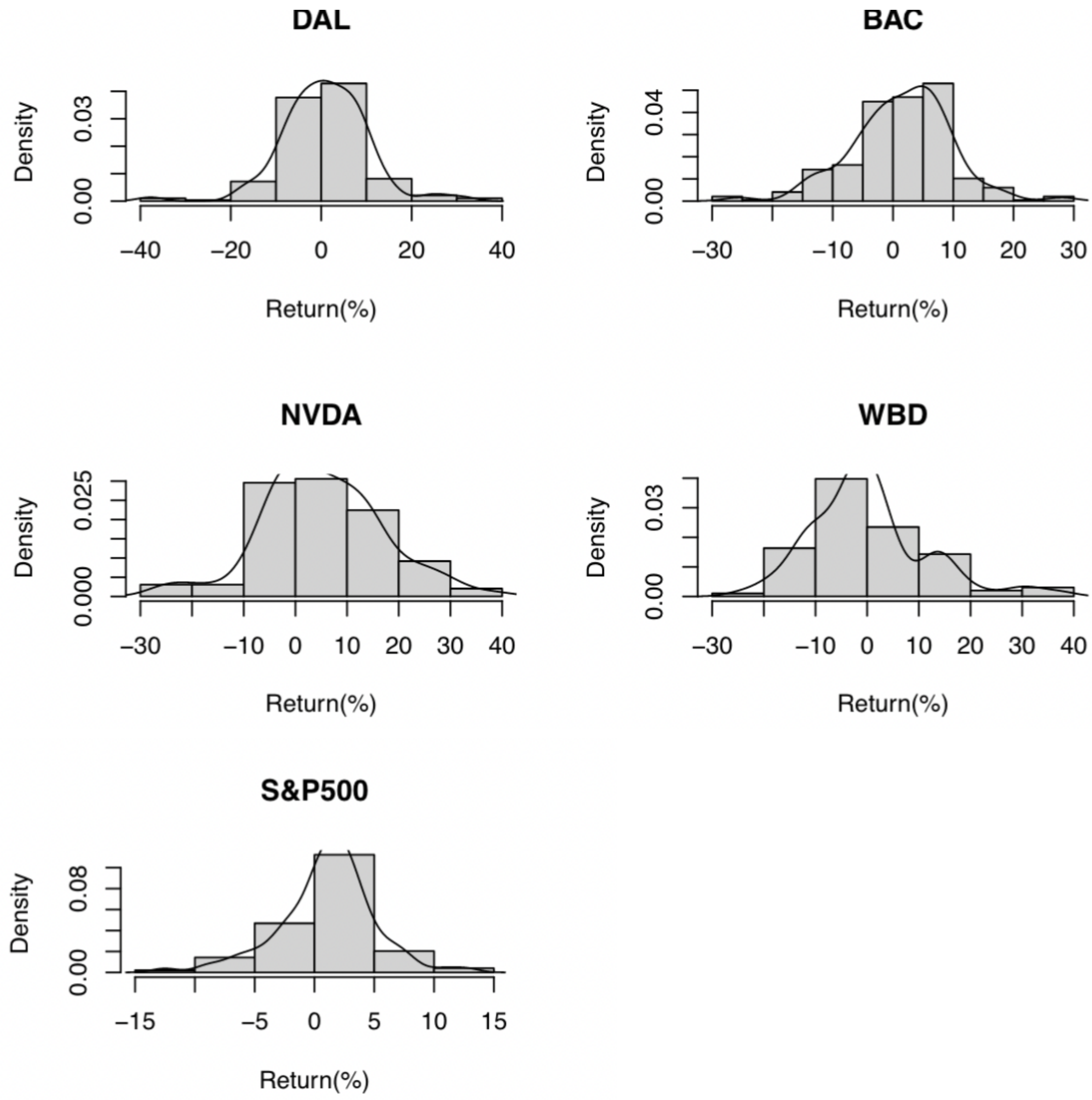
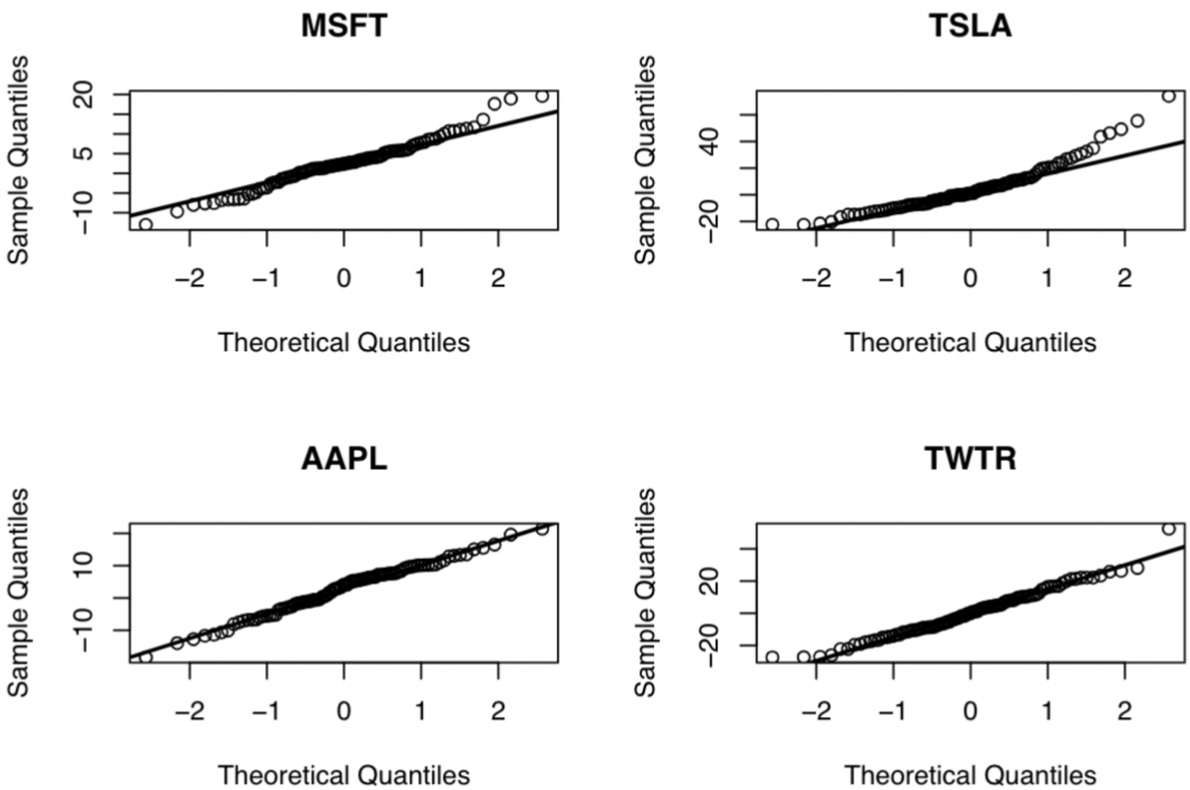
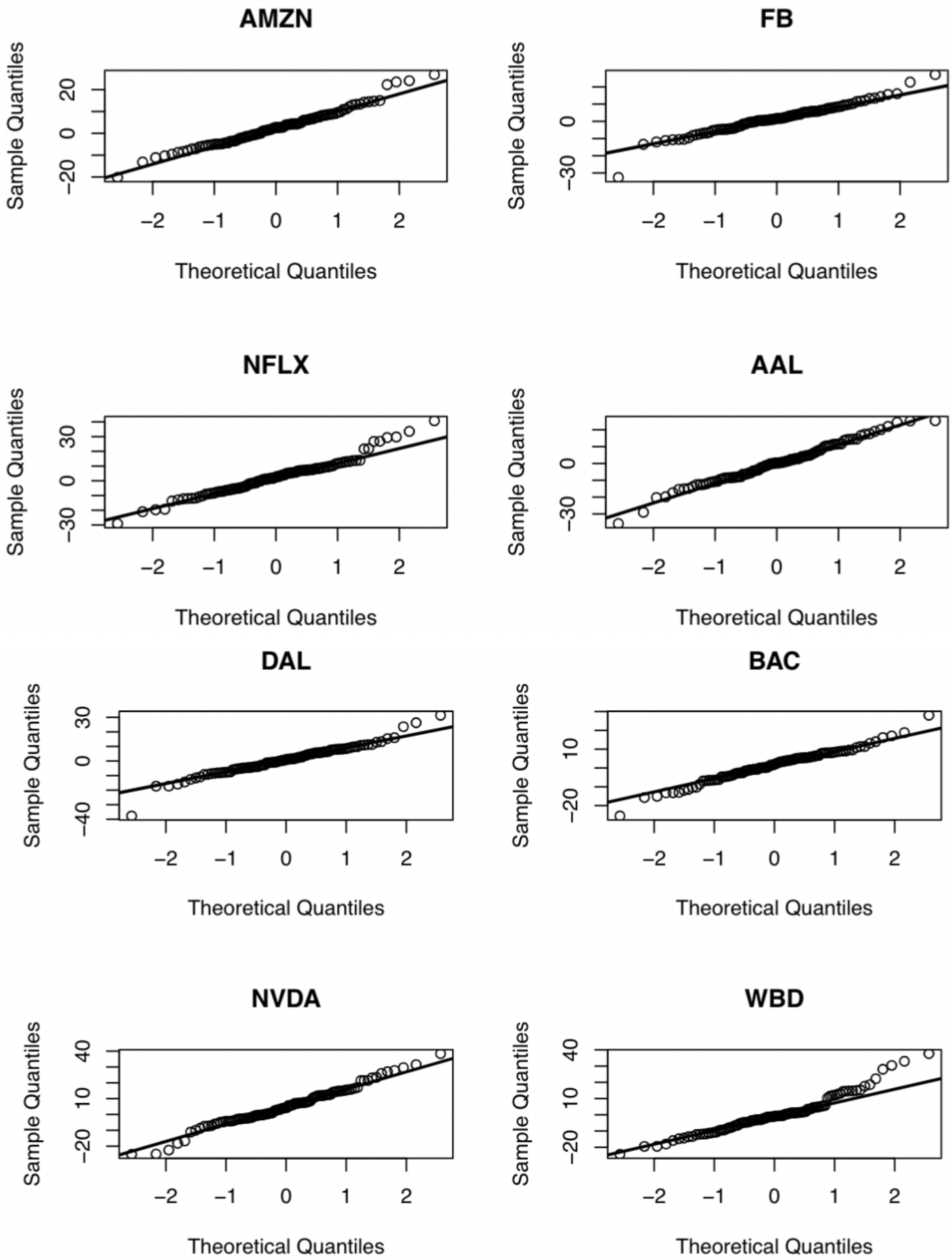


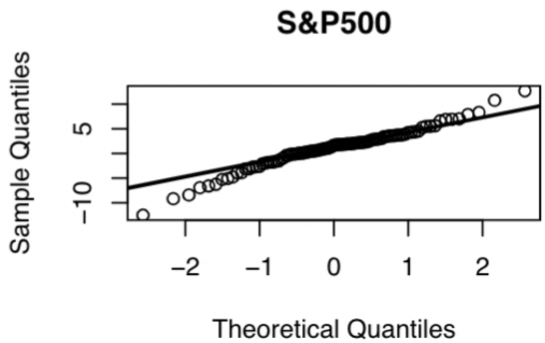
Figure 4, ACF Plot


MSFT
Density
0.00
0.06
−15
−5
0
5
10
15
20
Return(%)
TSLA
Density
0.000
0.025
−20
0
20
40
60
80
Return(%)
AAPL
Density
0.00
0.04
−20
−10
0
10
20
Return(%)
TWTR
Density
0.000
0.020
−20
0
20
40
60
Return(%)
AMZN
Density
0.00
0.04
−20
−10
0
10
20
30
Return(%)
FB
Density
0.00
0.04
−40
−20
0
10
20
30
Return(%)
NFLX
Density
0.00
0.03
−20
0
20
40
Return(%)
AAL
Density
0.000
0.025
−40
−20
0
10
20
30
Return(%)


DAL
Density
0.00
0.03
−40
−20
0
20
40
Return(%)
BAC
Density
0.00
0.04
−30
−10
0
10
20
30
Return(%)
NVDA
Density
0.000
0.025
−30
−10
0
10
20
30
40
Return(%)
WBD
Density
0.00
0.03
−30
−10
0
10
20
30
40
Return(%)
S&P500
Density
0.00
0.08
−15
−5
0
5
10
15
Return(%)


MSFT
TSLA
AAPL
TWTR
Sample Quantiles
Theoretical Quantiles
−10
5
20
−20
40
10
−2
−1
0
1
2


AMZN
FB
NFLX
AAL
DAL
BAC
NVDA
WBD
Sample Quantiles
Theoretical Quantiles



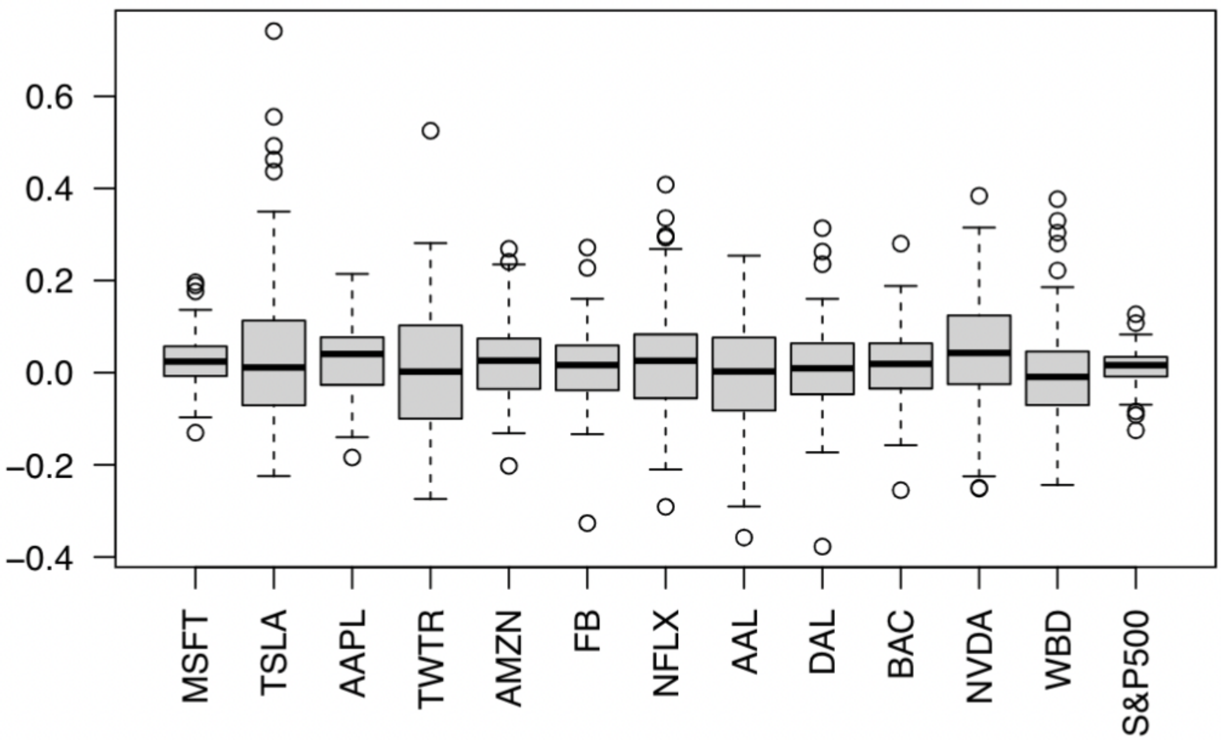


Figure 5, Histograms, QQ-Plots, Boxplots

Asset	t-distribution	Normal Distribution	GED
MSFT	-139.66549	-271.25573	-276.23424
TSLA	-38.58013	-80.34437	-69.20914
AAPL	-112.15671	-220.30961	-218.90939
TWTR	-50.63192	-97.29781	-94.90164
AMZN	-107.95913	-209.86197	-209.57404
FB	-111.98310	-209.20876	-217.11195
NFLX	-73.72148	-138.40822	-140.89449
AAL	-73.60630	-140.92136	-141.25665
DAL	-96.76172	-178.09420	-184.85237
BAC	-108.18352	-207.92323	-209.41384
NVDA	-71.69179	-136.35263	-137.27907
WBD	-77.72453	-151.06318	-153.27377

Table 6, Fit of Distributions

Assets	Sharpe's Slope
MSFT	0.3012400
TSLA	0.2364437
AAPL	0.2662491
TWTR	-0.0141628
AMZN	0.2307159
FB	0.1137256
NFLX	0.1581712
AAL	-0.0522252
DAL	0.0134156
BAC	0.0838003
NVDA	0.3764550
WBD	-0.0534724

Table 7, Sharpe's Slope

	Monthly Mean	Monthly SD	Annual Mean	Annual SD
MSFT	2.4946532	5.917747	29.935838	20.49968
TSLA	4.8015901	17.296293	57.619081	59.91612
AAPL	2.7737763	7.743821	33.285316	26.82538
TWTR	0.5061345	14.535005	6.073614	50.35073
AMZN	2.5953512	8.163112	31.144214	28.27785
FB	1.6357391	8.122604	19.628869	28.13753
NFLX	2.5691372	11.741366	30.829647	40.67329
AAL	0.1117527	11.493264	1.341033	39.81384
DAL	0.8395770	9.510284	10.074924	32.94459
BAC	1.3984216	8.191269	16.781059	28.37539
NVDA	5.1459413	11.778168	61.751296	40.80077
WBD	0.1060357	11.332111	1.272428	39.25558
S&P500	1.0358383	4.002103	12.430059	13.86369

Table 8, Annualized Data

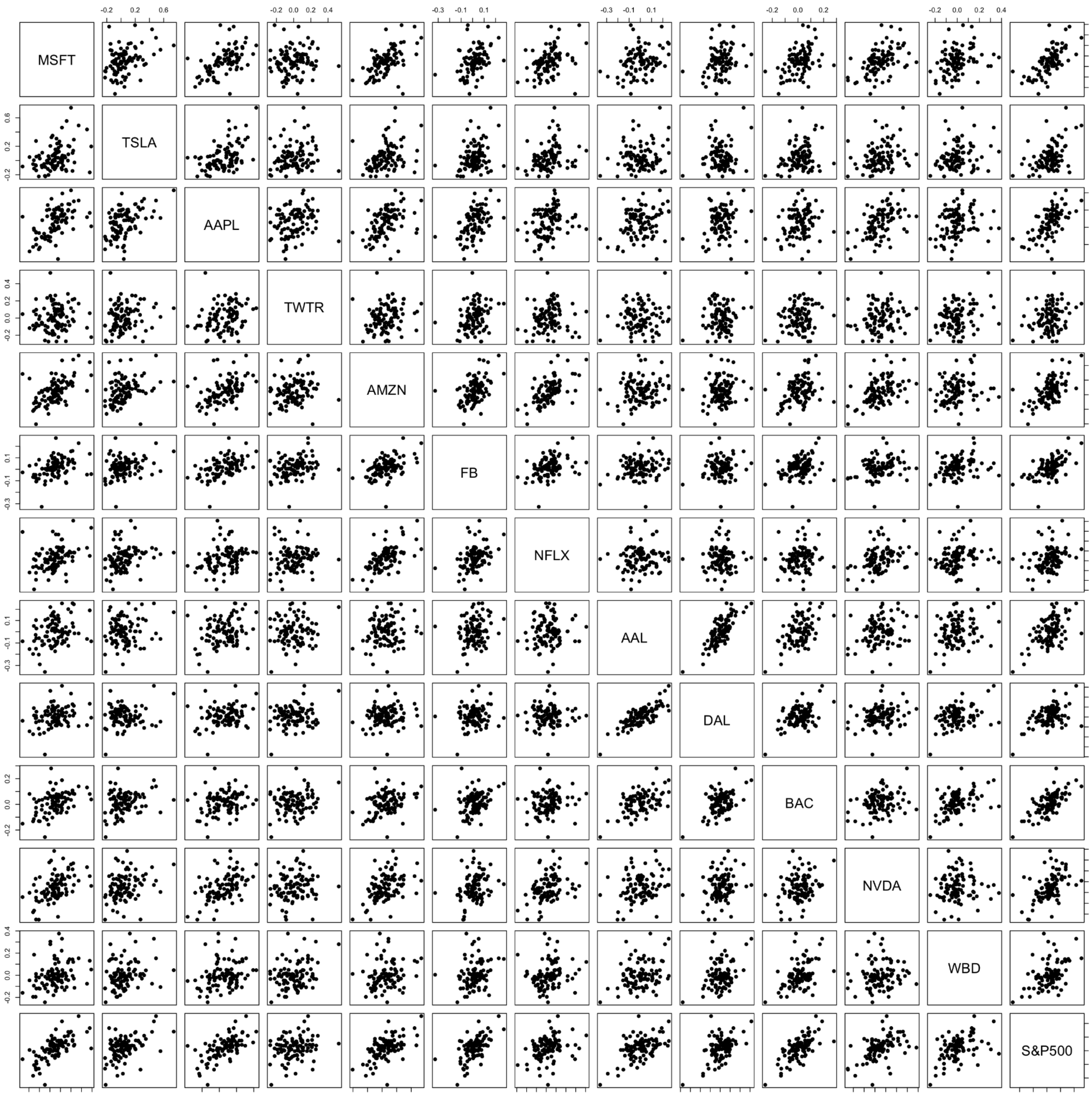


Figure 9, Pairwise Scatter Plots

	MSFT	TSLA	AAPL	TWTR	AMZN	FB	NFLX	AAL	DAL	BAC	NVDA	WBD	S&P500
MSFT	0.0035	0.0038	0.0024	0.0005	0.0027	0.0019	0.0026	0.0020	0.0014	0.0021	0.0034	0.0020	0.0016
TSLA	0.0038	0.0299	0.0063	0.0036	0.0045	0.0041	0.0055	0.0032	0.0020	0.0034	0.0059	0.0043	0.0032
AAPL	0.0024	0.0063	0.0060	0.0024	0.0028	0.0030	0.0025	0.0024	0.0019	0.0016	0.0047	0.0019	0.0020
TWTR	0.0005	0.0036	0.0024	0.0211	0.0023	0.0029	0.0018	0.0012	0.0023	0.0024	0.0019	0.0050	0.0014
AMZN	0.0027	0.0045	0.0028	0.0023	0.0067	0.0032	0.0056	0.0015	0.0008	0.0020	0.0042	0.0017	0.0019
FB	0.0019	0.0041	0.0030	0.0029	0.0032	0.0066	0.0036	0.0019	0.0008	0.0023	0.0028	0.0021	0.0019
NFLX	0.0026	0.0055	0.0025	0.0018	0.0056	0.0036	0.0138	0.0013	0.0001	0.0018	0.0051	0.0021	0.0015
AAL	0.0020	0.0032	0.0024	0.0012	0.0015	0.0019	0.0013	0.0132	0.0086	0.0050	0.0032	0.0039	0.0025
DAL	0.0014	0.0020	0.0019	0.0023	0.0008	0.0008	0.0001	0.0086	0.0090	0.0039	0.0024	0.0036	0.0019
BAC	0.0021	0.0034	0.0016	0.0024	0.0020	0.0023	0.0018	0.0050	0.0039	0.0067	0.0027	0.0043	0.0023
NVDA	0.0034	0.0059	0.0047	0.0019	0.0042	0.0028	0.0051	0.0032	0.0024	0.0027	0.0139	0.0008	0.0023
WBD	0.0020	0.0043	0.0019	0.0050	0.0017	0.0021	0.0021	0.0039	0.0036	0.0043	0.0008	0.0128	0.0020
S&P500	0.0016	0.0032	0.0020	0.0014	0.0019	0.0019	0.0015	0.0025	0.0019	0.0023	0.0023	0.0020	0.0016

Table 10, Covariance Matrix of Returns

Portfolio Theory

Assets	Var
MSFT	6918.17869693084
TSLA	23502.7562034031
AAPL	9898.52221650451
TWTR	23279.5289061133
AMZN	10763.0915501661
FB	10561.0446590248
NFLX	13072.710950181
AAL	15706.0583956349
DAL	12891.9240002285
BAC	13041.1530377474
NVDA	11974.7969532429
WBD	14903.9099722124
MVP	6285.69101812641

Table 11, VaR(5%) For Each Asset

Risk Management

Stocks	***VaR(parametric)***	***ES(parametric)***	***VaR(nonparametric)***	***ES(nonparametric)***
MSFT	*7239.2*	*9712*	*6918.2*	*9183*
TSLA	*23648.3*	*30876*	*15147.2*	*20688*
AAPL	*9963.7*	*13200*	*10749.9*	*13639*
TWTR	*23401.8*	*29475*	*22137.6*	*25981*
AMZN	*10831.8*	*14243*	*8881.9*	*12875*
FB	*11724.8*	*15119*	*10561.0*	*15968*
NFLX	*16743.7*	*21650*	*13072.7*	*20587*
AAL	*18793.0*	*23596*	*15706.1*	*24467*
DAL	*14803.4*	*18777*	*12891.9*	*20512*
BAC	*12075.0*	*15498*	*13041.2*	*16550*
NVDA	*14227.4*	*19149*	*11974.8*	*21517*
WBD	*18533.6*	*23269*	*14903.9*	*19432*

Table 12, VaR(5%) For Each Asset, assuming normal distributed for parametric estimation

Stocks	***VaR 2.5% quantile***	***VaR 97.5% quantile***	***ES 2.5% quantile***	***ES 97.5% quantile***
MSFT	***6610.1***	***7660.4***	***8068.5***	***10898***
TSLA	***14710.9***	***20397.6***	***18708.6***	***22288***
AAPL	***8108.5***	***11733.6***	***12292.1***	***15722***
TWTR	***19101.8***	***26024.1***	***24159.1***	***27276***

AMZN	***7861.3***	***10283.0***	***11024.2***	***16212***
FB	***9596.2***	***11187.7***	***12223.0***	***21899***
NFLX	***12068.5***	***19338.2***	***17473.6***	***24695***
AAL	***14919.7***	***19849.6***	***20429.1***	***30878***
DAL	***10980.4***	***15907.4***	***16102.6***	***26975***
BAC	***11456.9***	***13330.9***	***14640.1***	***20460***
NVDA	***8707.3***	***18231.9***	***17829.6***	***24633***
WBD	***13540.7***	***18046.4***	***17477.9***	***21724***

Table 12, Confidence Interval for VaR(5%) and ES (both nonparametric)

Stocks	***VaR 2.5% quantile***	***VaR 97.5% quantile***	***ES 2.5% quantile***	***ES 97.5% quantile***
MSFT	***6243.7***	***8110.5***	***8582.5***	***10749***
TSLA	***21287.5***	***25686.8***	***27890.2***	***33223***
AAPL	***8663.4***	***11118.4***	***11676.3***	***14471***
TWTR	***21153.5***	***25298.8***	***26890.8***	***31662***
AMZN	***9580.0***	***11948.4***	***12738.9***	***15605***
FB	***9905.3***	***13331.9***	***12943.8***	***17094***
NFLX	***14840.0***	***18385.1***	***19489.2***	***23566***
AAL	***16865.8***	***20529.1***	***21465.9***	***25606***
DAL	***12743.9***	***16725.6***	***16461.7***	***21057***
BAC	***10496.6***	***13421.2***	***13634.7***	***17125***
NVDA	***12246.1***	***16012.4***	***16901.6***	***21207***
WBD	***16914.2***	***19930.5***	***21351.0***	***25099***

Table 13, Confidence Interval for VaR(5%) and ES (both parametric)