旋转矩阵推导

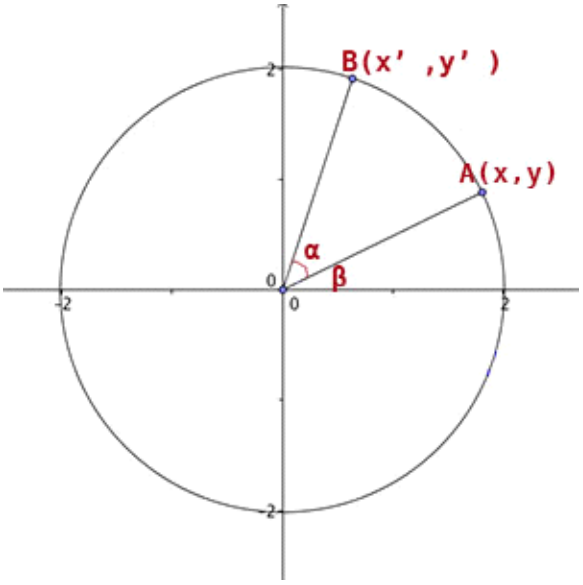
2018年5月8日 14:02

cos(α+β) = cosα*cosβ - sinα*sinβ

cos(α-β) = cosα*cosβ + sinα*sinβ

sin(α+β) = sinα*cosβ + sinaβ*cosα

sin(α-β) = sinα*cosβ - sinaβ*cosα



X = r*cosβ

Y = r*sinβ

x' = r*cos(α+β)

=r(cosα*cosβ - sinα*sinβ)

=r*cosβ*cosα - r*sianβ*sinα

=x*cosα - y*sinα

y' = r*sin(α+β)

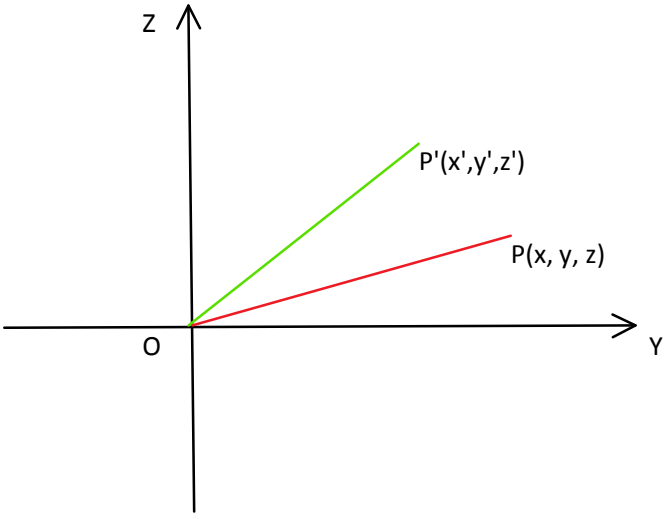
=r*cosβ*sinα + r*sinaβ*cosα

=x*sinα+y*cosα

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin a \\ \sin\alpha & \cos\alpha \end{bmatrix} * \begin{bmatrix} x \\ y \end{bmatrix}$$

对于三维坐标：

绕x轴旋转即是在yz平面绕x轴(原点)旋转



β = ∠P'OP

α = ∠POY

y = r*cosβ

z = r*sinβ

x' = x

y' = r*cos(α+β) = y*cosα - z*sinα

z' = r*sin(α+β) = y*sinα + z*cosα

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

绕y轴同理图中Y改为X

x = r*cosβ

z = r*sinβ

y' = y

x' = r*cos(α+β) = x*cosα - z*sinα

z' = r*sin(α+β) = x*sinα + z*cosα

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

绕z轴同理图中Y改为X, Z改为Y

x = r*cosβ

y = r*sinβ

z' = z

x' = r*cos(α+β) = x*cosα - y*sinα

y' = r*sin(α+β) = x*sinα + y*cosα

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

为了和缩放平移一起操作将上面3*3矩阵扩展成4*4齐次矩阵即可。

rotate_matrix(angle_x, angle_y, angle_z) = z_matrix(angle_z) * y_matrix(angle_y) * x_matrix(angle_x)

Tip:矩阵括号方括号和圆括号有什么区别？

英美式的教材多用方括号，苏联式的教材多用圆括号