IT3030 midterm exam#2 5/11/2021 ID: ______________ Name: ___________

1. (20%) A BMI (Body-Mass-Index) is defined as the weight (in kilogram) divided by the square of height (in meter). A BMI between 18 to 24 is considered normal. Larger BMI means you are too fat. Consider the BMI for all 10-year-old elementary school students, the mean value is 22.5, and the standard deviation is 9.8. We now have a sample of 25 students randomly chosen from this population, and there are many such n=25 selections. (a) What would be the mean value for these sampling means? Explain your reason? (b) What is the SEM (standard error of the means) for these samples? (c) Determine the proportion of these samples (each with n=25) having BMI between 18 and 24. (d) What will be the answer from (c) if n=100?

Answer: (a) According to central limit theorem, the mean value of sampling means would be the same as the population mean, therefore the answer is 22.5. (b) SEM=9.8/sqrt(25)=1.96. (c) The Z value for these sampling means is defined as (x-22.5)/SEM=(x-22.5)/1.96. For x of 18 and 24, the corresponding Z values are (18-22.5)/1.96 and (24-22.5)/1.96 respectively, which are -2.2959 and 0.7653. Finally the proportion of sample means between these 2 Z-values would be normcdf(0.7653)-normcdf(-2.296) =0.7671. Alternatively, we can omit computing the two Z values and getting the answer directly by normcdf(24, 22.5, 9.8/sqrt(25))- normcdf(18, 22.5, 9.8/sqrt(25)) = 0.7671.
(d) For n=100, the SEM becomes 0.98, two Z-values are (18-22.5)/0.98 and (24-22.5)/0.98, which are two times the values we earlier computed: -4.5918 and 1.5306. The new proportion would be normcdf(1.5306)-normcdf(--4.5918) =0.9371. The same answer can be obtained by normcdf(24, 22.5, 9.8/sqrt(100))-normcdf(18, 22.5, 9.8/sqrt(100)).

2. (20%) We have a sample of n limited observations. There are two approaches in evaluating the mean value of these n observations – one is Point Estimation, and the other is Interval Estimation. (a) Describe each of these two, with their pros and cons (advantages and disadvantages). (b) Following Problem #1 above, for n = 25, determine the 90%, 95% and 99% 2-sided confidence interval (CI) of the mean BMI value 22.5.

Answer: (a) Refer to course PPT.
(b) CI is 22.5 ± z*SEM, where SEM = 9.8/sqrt(25)=1.96. For these three CIs, z can be obtained by norminv(0.05)=-1.6449, norminv(0.025)=-1.96, norminv(0.005)=-2.5758. Half of the interval for 90%, 95% and 99% would be z*9.8/sqrt(25) for these 3 z values would be D1=-norminv(0.05)*SEM=3.2239, D2=-norminv(0.025)*SEM=3.8415, D3=-norminv(0.005)*SEM=5.0486. Finally these three CIs would be (19.276, 25.7239), (18.6585, 26.3415) and (17.4514, 27.5486) for each of 90%, 95% and 99% confidence level.

3. (20%) Weight limit for an elevator is 20 persons with each weighing 166 pounds. Men have weights that are normally distributed with a mean of 172 pounds and a standard deviation of 29 pounds. (a) Determine the probability for one man weighing over 166 pounds? (b) Determine the probability for an average weight from a random sample of 20 men over 166 pounds?

Answer: (a) 1-normcdf(166, 172, 29)= 0.5820 (b) 1-normcdf(166, 172, 29/sqrt(20)) = 0.8226.

4. (20%) The efficacy(有效性) for new vaccine(疫苗) was estimated by measuring the antibody(抗體) value in the blood. There are 30 people taking this vaccine (group M), and another 30 taking placebo (安慰劑) (group P). A higher antibody value means that the vaccine is more useful. The mean and standard deviation for group M are 232.5 and 18.9. For group P they are 189.3 and 16.2. (a) Determine the 95 % CI of the mean antibody value for group M, as well as for group P. (b) How would you conclude whether the vaccine is useful or not based on the confidence intervals you computed above?

Answer:

(a) the T value for locating 95% CI for DF=n-1=29 is T=tinv(0.975,29) = 2.0452.

For group M, SEM1=18.9/sqrt(30)=3.4507. For group P, SEM2= 16.2/sqrt(30) = 2.9577.

Finally, for group M half CI is D1=T*SEM1=7.0574 , and for group P half CI is D2=T*SEM2=6.0492. Thus CI for group M is (232.5- D1, 232.5+D1) = (225.4426, 239.5574). For group P, it is (189.3-D2, 189.3+D2) = (183.2508, 195.3492).

(b) Note that the upper bound 195.3492 for group P is still much lower than the lower bound 225.4426 of group M. This means the two CIs are well-separated, thus proving the vaccine to be useful in increasing the antibody value.

5. (20%) Upon tossing a coin that might be defected, we hypothesized that the coin is fair, and perform a test based on the observation that tossing the same coin 100 times and found head showing up 63 times. (a) Compute the p-value of this test and decide whether the coin is fair or not, using 2-sided test with α=0.03. You must explicitly answer either to reject or not to reject the null hypothesis. (b) What would be the smallest number of tosses appearing head what would barely reject the test with α=0.05?

(a)

We want to get the probability for 63 heads and beyond (right tail). This is 2-sided test. Therefore twice the right tail wound be the answer.

```
>> 2*(1-binocdf(62,100,0.5))
ans =      0.012032975725363
```

This is smaller than 0.03, so we'd reject the hypothesis that this is a fair coin.

(b) One could do a try-and-error and found:

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>> 2*(1-binocdf(59,100,0.5))        #probability for 60 heads and beyond
ans =      0.056887933640981
>> 2*(1-binocdf(60,100,0.5))        #probability for 61 heads and beyond
ans =      0.035200200217705
```

So the answer would be tossing 60 heads or more results in a p-value of 0.057 and tossing 61 heads or more results in a p-value of 0.035. Therefore, the latter (tossing 61 heads) would barely reject the null hypothesis that we have a fair coin. Tossing 60 heads would not be sufficient to call it unfair.

One could also use the following MATLAB function to get the answer:

```
>> binoinv(0.975, 100, 0.5)     # n that cuts off the right tail (0.025)
ans =       60
```

This accumulates the probability from X=0 to X=60 to limit the remaining (X=61 and above) within 0.025. So tossing **61 heads** would be statistically significant to reject the null hypothesis.