

1. (30%) Consider a population of all 20- to 74-year-old males living in the United States. The serum cholesterol levels have a mean value  $\mu=211$  and a standard deviation  $\sigma=46$ . (a) For a sample size of  $n=25$  people having an average cholesterol level of 235, determine the 95% confidence interval (CI) of it and show that the CI does not contain the actual mean value  $\mu=211$ . (b) Knowing a smaller  $n$  would allow you to have bigger CI, what is the largest possible integer  $n$  you would need in order to let the 95% CI contain the actual mean value of 211? (c) If I'd like to limit the CI within 10, what is the smallest possible integer  $n$  needed for the sample?

Answer:

(a)

```
>> mu=211;n=25;sigma=46;
>> SEM=sigma/sqrt(n) =    9.2000
>> Z=norminv(0.975) =    1.9600
>> Lower_bound=235-Z*SEM =  216.9683
>> Upper_bound=235+Z*SEM =  253.0317
The CI is from 216.9683 to 253.0317. It does not contain the actual mean of 211.
```

(b) In order to have the 95% CI contain 211, we must have half of the CI as  $Z*SEM=235-211=24$ . It is also known that  $SEM=sigma/sqrt(n)$ . Therefore we have  $sqrt(n)=sigma/(24/Z)$

```
>> sqrt_N=sigma/(24/Z) =    3.7566
>> sqrt_N^2 =    14.1120
```

The largest integer  $n = 14$  to ensure SEM big enough to extend the lower bound of the CI to cover 211.

(c) CI = 10 means  $Z*SEM=Z*sigma/sqrt(n)=5$ . Therefore,  $sqrt(n)=(Z*sigma)/5$ .

```
>> Z*sigma/5
ans = 18.0317
>> ans^2
ans =  325.1411
```

The smallest integer is  $n = 326$ .

2. (30%) Percentages of ideal body weight were determined for 18 randomly selected insulin-dependent diabetics (需要施打胰島素的糖尿病患者) and are shown below. A percentage of 120 means one weighs 20% more than ideal; a percentage of 95 means one weighs 5% less than ideal. 107, 119, 99, 114, 120, 104, 88, 114, 124, 116, 101, 121, 152, 100, 125, 114, 95, 117
- (a) Determine the mean value and standard deviation of these percentages.  
(b) Compute a two-sided 95% confidence interval for the true mean percentage of ideal body weight for this population.  
(c) Does this CI contain the value 100%? What does the answer tell you?

Answer:

```
>> X=[107, 119, 99, 114, 120, 104, 88, 114, 124, 116, 101, 121, 152, 100, 125, 114, 95, 117]';
```

```
>> PD=fitdist(X,'normal')
```

```
PD =
```

```
NormalDistribution
```

```
Normal distribution
```

```
mu = 112.778 [105.605, 119.951]
```

```
sigma = 14.4245 [10.8239, 21.6244]
```

```
>> CI=paramci(PD, 0.05)
```

```
CI =
```

```
105.6047 10.8239
```

```
119.9509 21.6244
```

```
>>
```

The 95% CI is from 105.6047 to 119.9509. It does not contain 100%, meaning that insulin-dependent diabetics are overweight.

3. (20%) In the class we performed a **two-sided** test to determine whether a coin flip is fair or not. Suppose we had heads 15 times out of 20 flips. (a) What is the null hypothesis for this test? (b) Determine the p-value for this test, and decide to reject or not to reject the null hypothesis for  $\alpha=0.05$ .

Answer:

(a)  $H_0$ : This coin is a fair one (it lands 50%/50% on either head or tail)

(b)

```
>> 1-binocdf(14, 20, 0.5)
```

```
ans =
```

```
0.0207
```

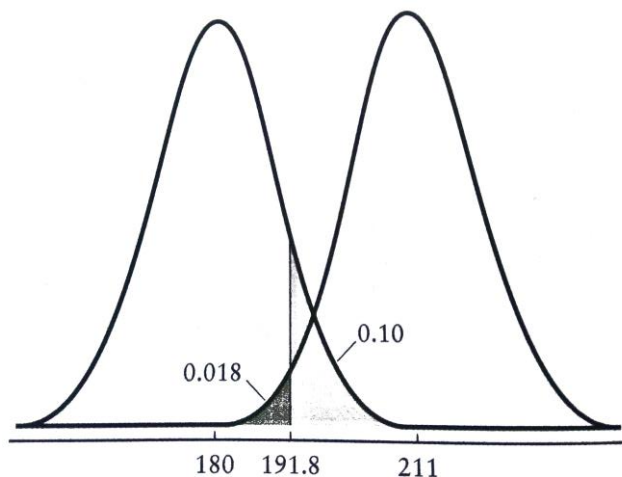
```
>> ans*2
```

```
ans =
```

```
0.0414
```

Since this p-value is smaller than 0.05, we'd reject the null hypothesis (that the coin is fair). On other words, this is not a fair coin.

4. (20%) We talked about the two distributions of mean values of  $n=25$  samples in class – the left bell centered at 180, the right bell centered at 211. Both have population standard deviation  $\sigma=46$ . We showed in class that for choosing  $\alpha=0.1$  from testing  $H_0=180$ , the cut-off value would be 191.8, the type II error  $\beta$  would be 0.018 (the black area) and the power would be  $1-0.018=0.982$ . (a) Determine the new cut-off value if I reduce  $\alpha=0.01$  from testing  $H_0=180$ . (b) Determine the new  $\beta$  and power of the test.



Answer:

```
>> norminv(0.99, 180, 46/5)
```

ans =

```
201.4024
```

```
>> beta=normcdf(ans,211,46/5)
```

beta =

```
0.1484
```

```
>> power=1-beta
```

power =

```
0.8516
```

```
>>
```