Exercise 1

A sawtooth signal has a waveform that ramps up linearly from -1 to 1, then drops to -1 and repeats. See http://en.wikipedia.org/wiki/Sawtoot

Write a class called `SawtoothSignal` that extends `Signal` and provides `evaluate` to evaluate a sawtooth signal.

Compute the spectrum of a sawtooth wave. How does the harmonic structure compare to triangle and square waves?

Solution

My solution is basically a simplified version of TriangleSignal.

In [3]:

```
from thinkdsp import Sinusoid
from thinkdsp import normalize, unbias
import numpy as np

class SawtoothSignal(Sinusoid):
    """Represents a sawtooth signal."""

    def evaluate(self, ts):
        """Evaluates the signal at the given times.

        ts: float array of times

        returns: float wave array
        """
        cycles = self.freq * ts + self.offset / np.pi / 2
        frac, _ = np.modf(cycles)
        ys = normalize(unbias(frac), self.amp)
        return ys
```

Here's what it sounds like:

In [4]:

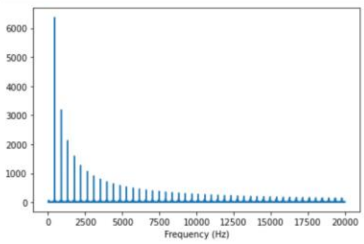
```
sawtooth = SawtoothSignal().make_wave(duration=0.5, framerate=40000)
sawtooth.make_audio()
```

Out[4]: 0:00 / 0:00

And here's what the spectrum looks like:

In [5]:

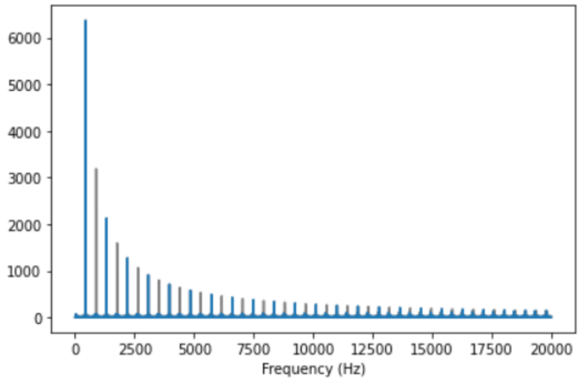
```
sawtooth.make_spectrum().plot()
decorate(xlabel='Frequency (Hz)')
```



Compared to a square wave, the sawtooth drops off similarly, but it includes both even and odd harmonics. Notice that I had to cut the amplitude of the square wave to make them comparable.

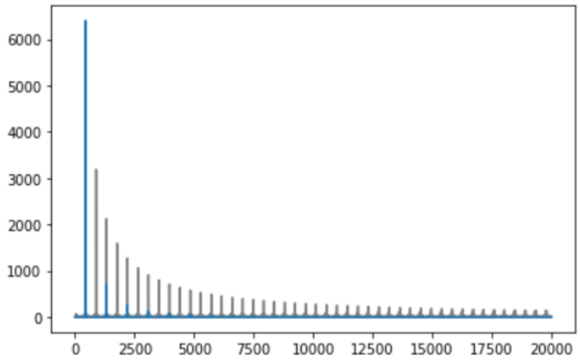
In [6]:

```
from thinkdsp import SquareSignal

sawtooth.make_spectrum().plot(color='gray')
square = SquareSignal(amp=0.5).make_wave(duration=0.5, framerate=40000)
square.make_spectrum().plot()
decorate(xlabel='Frequency (Hz)')
```



Compared to a triangle wave, the sawtooth doesn't drop off as fast.

[7]:
```
from thinkdsp import TriangleSignal

sawtooth.make_spectrum().plot(color='gray')
triangle = TriangleSignal(amp=0.79).make_wave(duration=0.5, framerate=40000)
triangle.make_spectrum().plot()
decorate(xlabel='Frequency (Hz)')
```



Specifically, the harmonics of the triangle wave drop off in proportion to $1/f^2$, while the sawtooth drops off like $1/f$.

Exercise 5

The triangle and square waves have odd harmonics only; the sawtooth wave has both even and odd harmonics. The harmonics of the square and sawtooth waves drop off in proportion to $1/f$; the harmonics of the triangle wave drop off like $1/f^2$. Can you find a waveform that has even and odd harmonics that drop off like $1/f^2$?

Hint: There are two ways you could approach this: you could construct the signal you want by adding up sinusoids, or you could start with a signal that is similar to what you want and modify it.

Solution

One option is to start with a sawtooth wave, which has all of the harmonics we need:

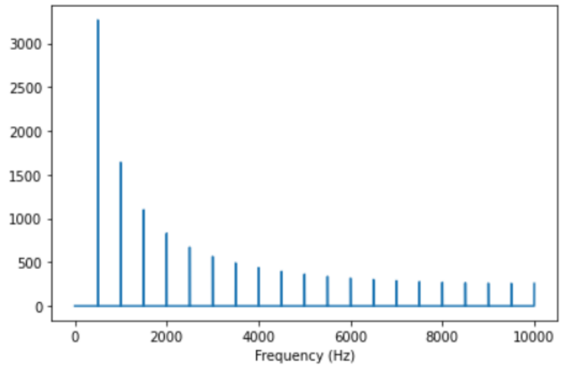
In [22]:
```
freq = 500
signal = SawtoothSignal(freq=freq)
wave = signal.make_wave(duration=0.5, framerate=20000)
wave.make_audio()
```

Out[22]:

0:00 / 0:00

Here's what the spectrum looks like. The harmonics drop off like $1/f$.

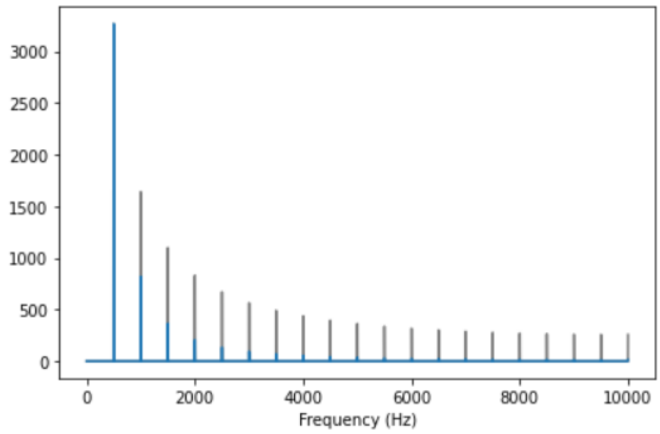
In [23]:
```python
spectrum = wave.make_spectrum()
spectrum.plot()
decorate(xlabel='Frequency (Hz)')
```



If we apply the filter we wrote in the previous exercise, we can make the harmonics drop off like $1/f^2$.

In [24]:
```python
spectrum.plot(color='gray')
filter_spectrum(spectrum)
spectrum.scale(freq)
spectrum.plot()
decorate(xlabel='Frequency (Hz)')
```


In [24]:
```python
spectrum.plot(color='gray')
filter_spectrum(spectrum)
spectrum.scale(freq)
spectrum.plot()
decorate(xlabel='Frequency (Hz)')
```

Here's what it sounds like:

In [25]:
```python
wave = spectrum.make_wave()
wave.make_audio()
```

Out[25]:

0:00 / 0:00