Exercise 1

If you did the exercises in the previous chapter, you downloaded the historical price of BitCoins and estimated the power spectrum of the price changes. Using the same data, compute the autocorrelation of BitCoin prices. Does the autocorrelation function drop off quickly? Is there evidence of periodic behavior?

In [3]:

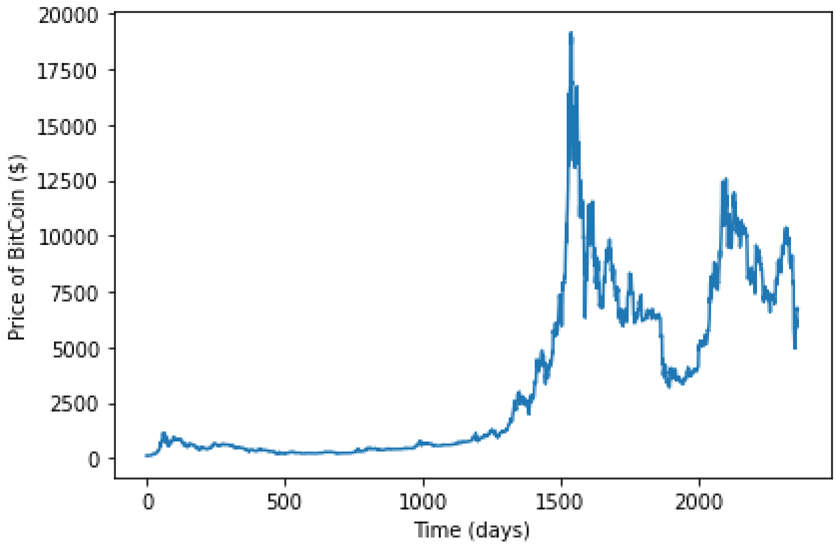
```
if not os.path.exists('BTC_USD_2013-10-01_2020-03-26-CoinDesk.csv'):
    !wget https://github.com/AllenDowney/ThinkDSP/raw/master/code/BTC_USD_2013-10-01
```

In [4]:

```
import pandas as pd

df = pd.read_csv('BTC_USD_2013-10-01_2020-03-26-CoinDesk.csv',
                 parse_dates=[0])

ys = df['Closing Price (USD)']
ts = df.index
```

In [5]:

```
from thinkdsp import Wave

wave = Wave(ys, ts, framerate=1)
wave.plot()
decorate(xlabel='Time (days)',
         ylabel='Price of BitCoin ($)')
```



In [6]:

```
def autocorr(wave):
    """Computes and plots the autocorrelation function.

    wave: Wave
    """
    lags = np.arange(len(wave.ys)//2)
    corrs = [serial_corr(wave, lag) for lag in lags]
    return lags, corrs
```

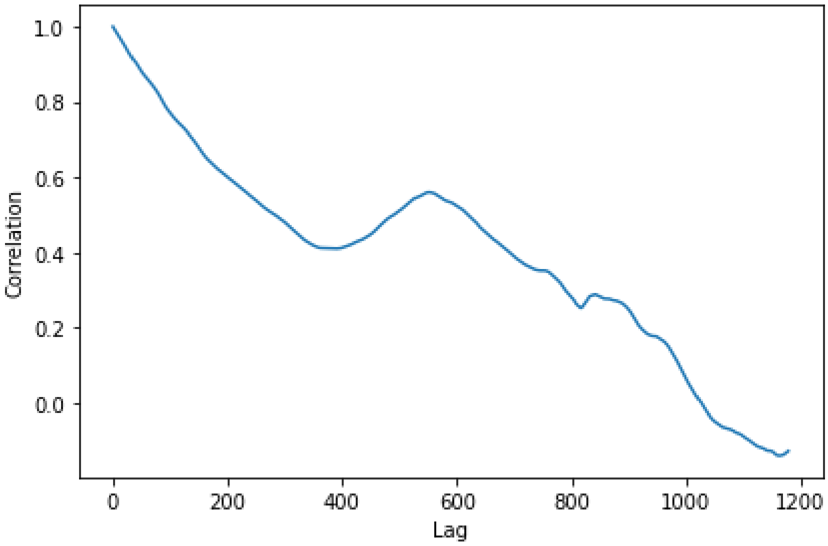
In [7]:

```
def serial_corr(wave, lag=1):
    """Computes serial correlation with given lag.
```

```
    wave: Wave
    lag: integer, how much to shift the wave

    returns: float correlation coefficient
    """
    n = len(wave)
    y1 = wave.ys[lag:]
    y2 = wave.ys[:n-lag]
    corr_mat = np.corrcoef(y1, y2)
    return corr_mat[0, 1]
```

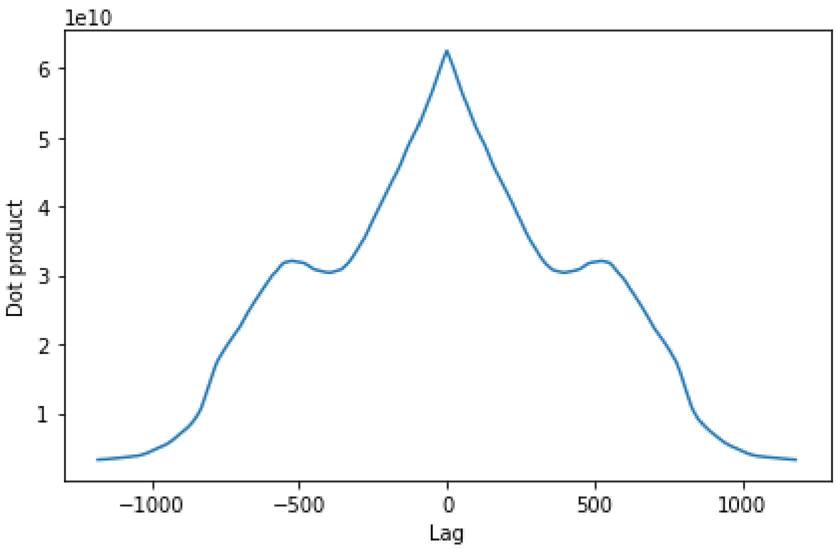
In [8]:

```
lags, corrs = autocorr(wave)
plt.plot(lags, corrs)
decorate(xlabel='Lag',
         ylabel='Correlation')
```



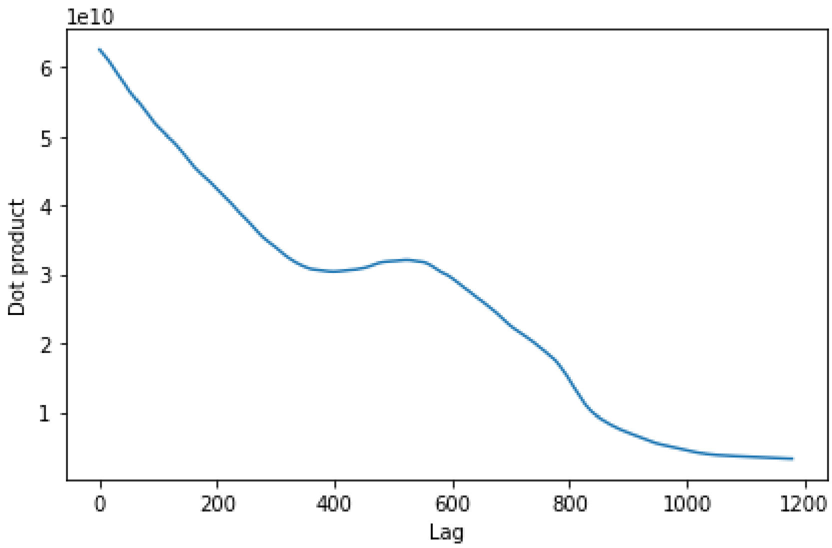
In [9]:

```
N = len(wave)
corrs2 = np.correlate(wave.ys, wave.ys, mode='same')
lags = np.arange(-N//2, N//2)
plt.plot(lags, corrs2)
decorate(xlabel='Lag',
         ylabel='Dot product')
```



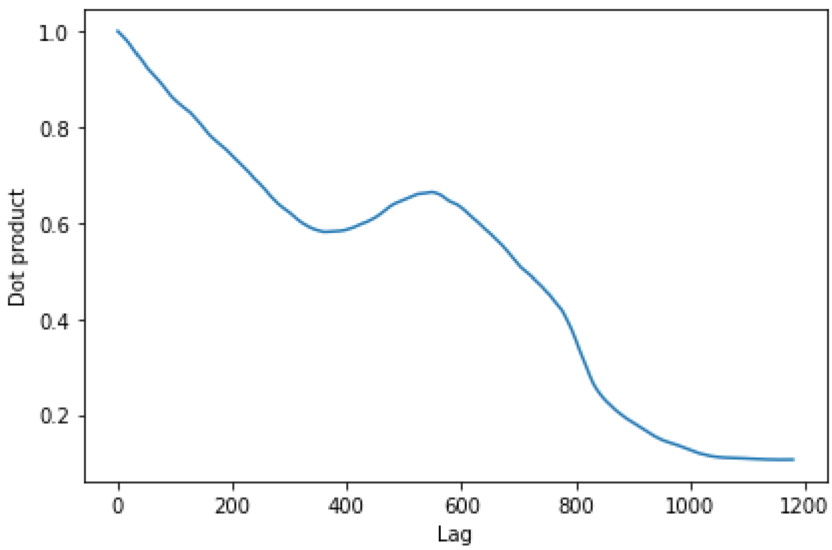
In [10]:

```
N = len(corrs2)
half = corrs2[N//2:]
plt.plot(half)
decorate(xlabel='Lag',
         ylabel='Dot product')
```



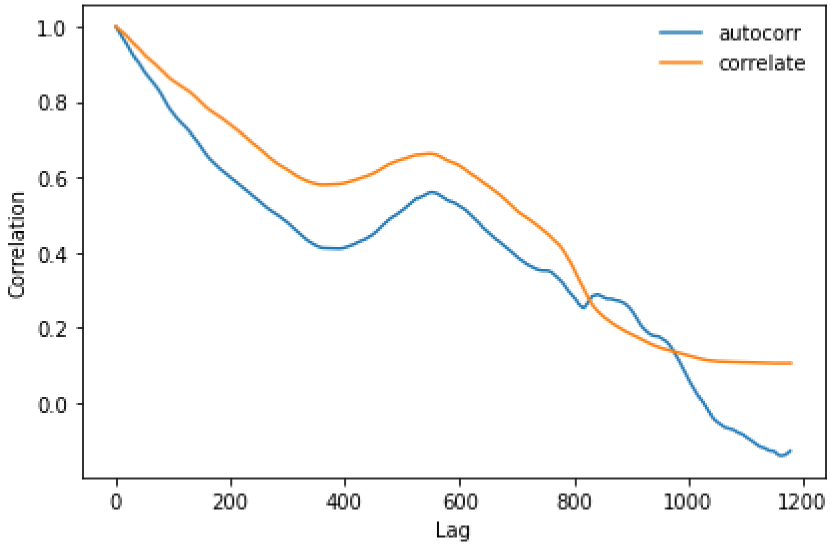
In [11]:

```
lengths = range(N, N//2, -1)
half /= lengths
half /= half[0]
plt.plot(half)
decorate(xlabel='Lag',
         ylabel='Dot product')
```



In [12]:

```
plt.plot(corrs, label='autocorr')
plt.plot(half, label='correlate')
decorate(xlabel='Lag', ylabel='Correlation')
```



Exercise 2

The example code in `chap05.ipynb` shows how to use autocorrelation to estimate the fundamental frequency of a periodic signal. Encapsulate this code in a function called `estimate_fundamental`, and use it to track the pitch of a recorded sound.

To see how well it works, try superimposing your pitch estimates on a spectrogram of the recording.

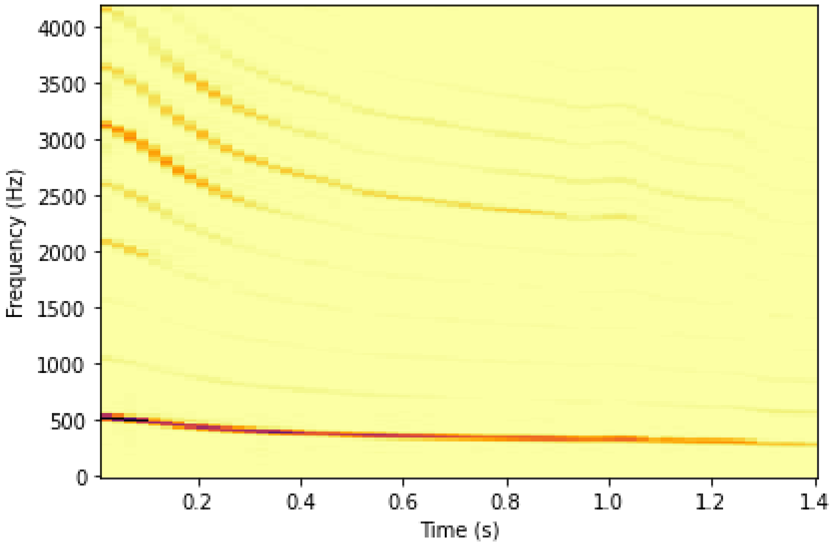
In [13]:
```
if not os.path.exists('28042__bcjordan__voicedownbew.wav'):
    !wget https://github.com/AllenDowney/ThinkDSP/raw/master/code/28042__bcjordan__v
```

In [14]:
```
from thinkdsp import read_wave

wave = read_wave('28042__bcjordan__voicedownbew.wav')
wave.normalize()
wave.make_audio()
```

Out[14]:

0:00 / 0:01

In [15]:
```
wave.make_spectrogram(2048).plot(high=4200)
decorate(xlabel='Time (s)',
         ylabel='Frequency (Hz)')
```



In [16]:

```
def estimate_fundamental(segment, low=70, high=150):
    lags, corrs = autocorr(segment)
    lag = np.array(corrs[low:high]).argmax() + low
    period = lag / segment.framerate
    frequency = 1 / period
    return frequency
```

In [17]:

```
duration = 0.01
segment = wave.segment(start=0.2, duration=duration)
freq = estimate_fundamental(segment)
freq
```

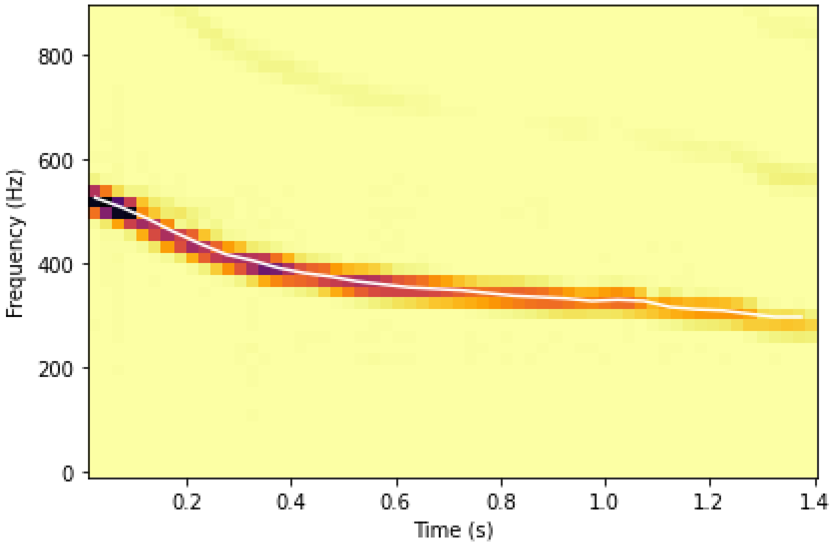
Out[17]: 436.63366336633663

In [18]:

```
step = 0.05
starts = np.arange(0.0, 1.4, step)

ts = []
freqs = []

for start in starts:
    ts.append(start + step/2)
    segment = wave.segment(start=start, duration=duration)
    freq = estimate_fundamental(segment)
    freqs.append(freq)
```

In [19]:

```
wave.make_spectrogram(2048).plot(high=900)
plt.plot(ts, freqs, color='white')
decorate(xlabel='Time (s)',
                     ylabel='Frequency (Hz)')
```


800
600
400
200
0
Frequency (Hz)
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Time (s)