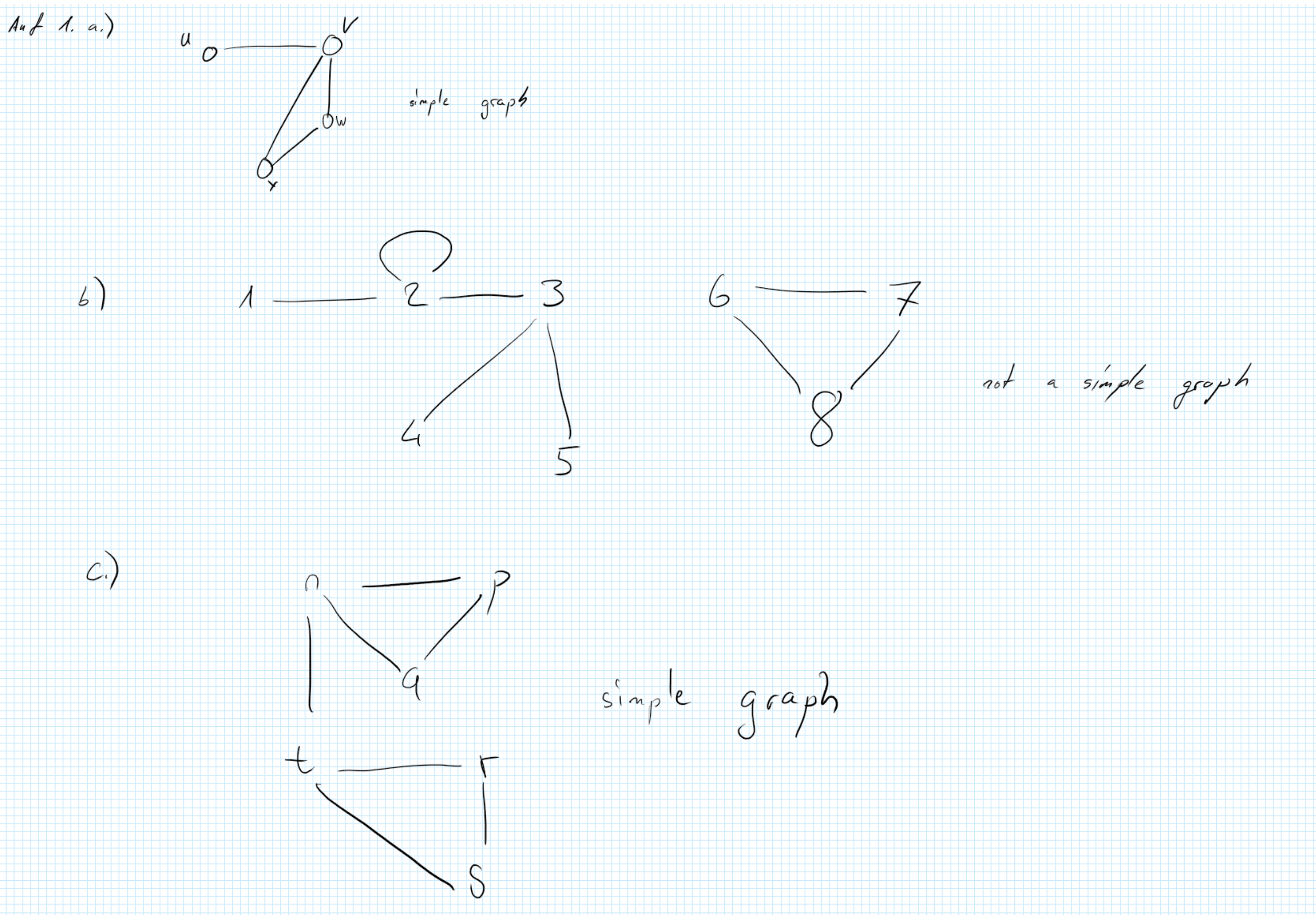
EI Assignment 10

Vithusan Ramalingam (21-105-515)

Jan Ellenberger (21-103-643)

Aufgabe 1.)



Aufgabe 2.)

Auf 2. Two graphs are isomorphic if they're identical except for their node names

$A \cong B$? $A = \{a, b, d, d\}$, $E = \{ab, ac, ad, bc, cd\}$

B
1 = c
2 = a
3 = b
4 = d

$\{23, 21, 24, 31, 14\}$ ✓

$A \cong C$? can't be isomorphic because C has 2 nodes with 4 edges and A only has one node with 4 edges so it's impossible that they're isomorphich

$A \cong D$? We can rename the vertices of D as following

1 = b
2 = d
3 = c
4 = a

so the Edges are like this $E\{bc, ab, ad, ac, dc\}$

so D is isomorphic to A

Vithusan Ramalingam (21-105-515) Jan Ellenberger (21-103-643)

Aufgabe 3.)

- **Q, T, V are not subgraphs**

Aufgabe 4.)

Auf 4.)

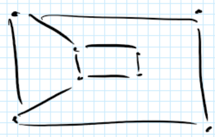
A.) Degree Sequence = (2, 3, 3, 4)

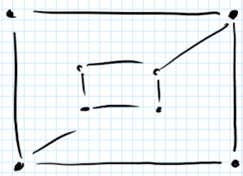
B.) (3, 3, 3, 3, 4)

C.) (3, 3, 5, 5)

D.) (1, 1, 1, 1, 1, 1, 2, 4, 4)

Aufgabe 5.)

Auf 5.) No, G_1 could look like this:  (2, 2, 2, 2, 3, 3, 3, 3)

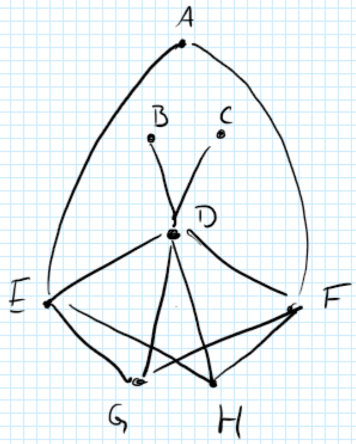
and G_2 like this:  (2, 2, 2, 2, 3, 3, 3, 3)

Aufgabe 6.)

- **Yes, because if the degree sequence is different, they can't be isomorphic anymore.**

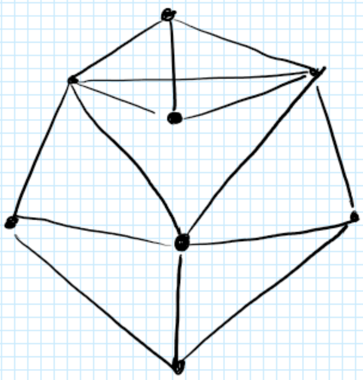
Aufgabe 7.)

Auf 7. a.)

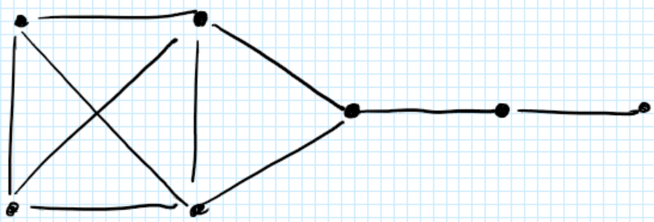


A: 2
B, C: 1
D: 6
E, F: 4
G, H: 3

b)



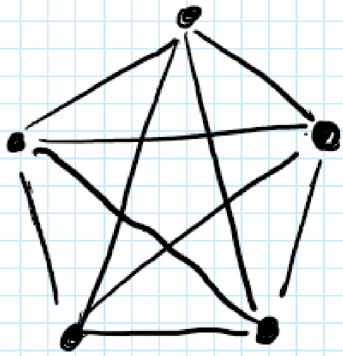
c.)



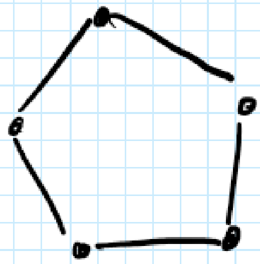
Aufgabe 8.)

Auf 8.)

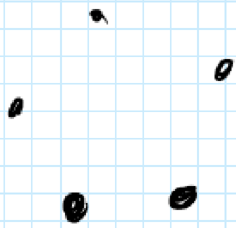
K_5



C_5

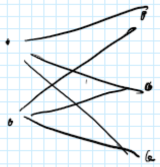


N_5



Aufgabe 9.)

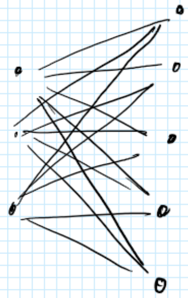
Auf 9.) $K_{2,3}$



5 vertices

6 edges

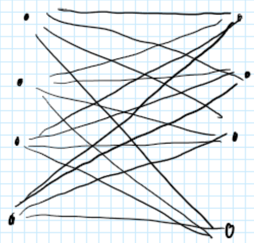
$K_{3,5}$



8 vertices

15 edges

$K_{4,4}$



8 vertices

16 edges

In complete bipartite graphs $K_{r,s}$ we would expect $r+s$ vertices and $r \cdot s$ edges.

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Aufgabe 10.)

Auf 10.)

We want to proof that there are no odd cycles in a bipartite graph

We know: the number of edges in a cycle is the number of edges

If we proof that there are no odd cycle in a bipartite graph we know there are only odd cycle which have an odd number of edges

$V_1 = X$, $Y = V_2$ ← sets of vertices

$G = (v_1, v_2, v_3, v_4 \dots v_n, v_1)$ We make the assumption: n is odd

Rule A: In a bipartite graph are no edges within the set of vertices. So only connections from Set V_1 to V_2 or V_2 to V_1 are allowed

$v_1 \in X$, $v_2 \in Y$, $v_3 \in X$, $v_4 \in Y$ ......

$v_i \in X$: i is odd for every odd i we assign it to Vertices set X

every i which is even is assigned to vertices set Y

As mentioned above we make the assumption that n is odd

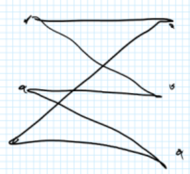
with our assumption we'll come to the conclusion that $\boxed{v_n \in X \;↯}$

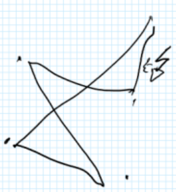
v_n we'll be connected with v_1 and this is also an Element from X. This wouldn't be a bipartite Graph after Rule A □

No we know that if a bipartit graph has a cycle it only can be if n the number of vertices is even and so the number of edges of the cyle will also be even. □

Examples:

$K_{3,3}$ has C_6 with 6 edges $K_{2,3}$ C_5 ↯





can't make a cycle for this bipartite graph

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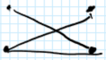
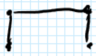
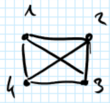
Aufgabe 11.)

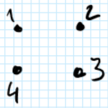
Auf 11.)

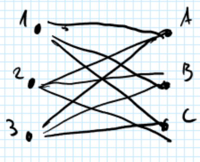
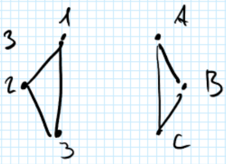
a.) P_4 $\overline{P^4}$ 

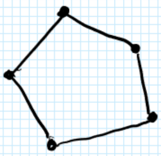
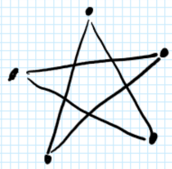
($\overline{P_4}$  P_4 ) ← This example is only to show the visualize the solution

P_4 and $\overline{P_4}$ are isomorph, because it's a self complementary graph

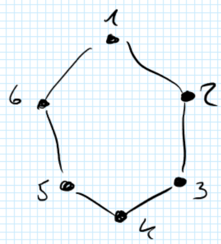
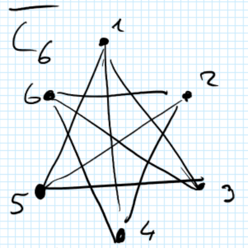
b.) K_4 

$\overline{K_4}$ 

$K_{3,3}$  $\overline{K_{3,3}}$ 

C_5  $\overline{C_3}$ 

c.) If the graph has n vertices an one vertex is joined with x vertices, the vertex in the complement will be joined with $(n-1)-x$ vertices.

Example: C_6  $\overline{C_6}$ 

$(2,2,2,2,2,2)$ $(3,3,3,3,3,3)$

Vertex 1 is joined with two vertices → so $\overline{V}_1$ will be joined wit $(6-1)-2=3$ vetices.

d.) If a graph G and it's complement $\overline{G}$ are isomorphic, they must have the same number of edges.

So the number of edges of G and the number of edges of $\overline{G}$ together are the number of edges are equal to the number of edges of the complete graph G.

The number of edges for a complete graph are $\frac{n(n-1)}{2}$ but as there are only half of these edges in each graph the number is $\frac{n(n-1)}{4}$.

So n has to be $4k$ or $4k+1$ for a $k \in \mathbb{N}$.

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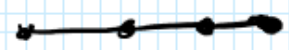
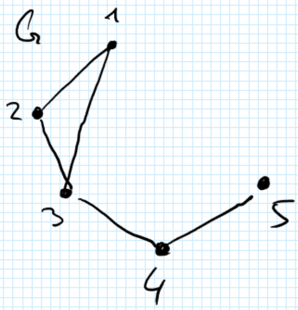
e.) With 4 vertices it's only P_4 

with 5 vertices there's:

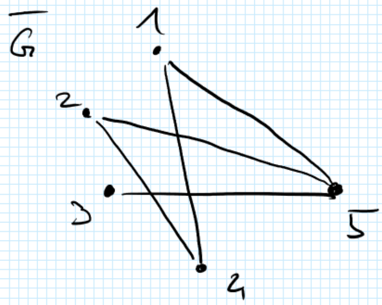
$(2,2,2,2,2) \rightarrow C_5$  $\overline{C_5}$ 

$(2,2,2,3,1) \rightarrow$

G

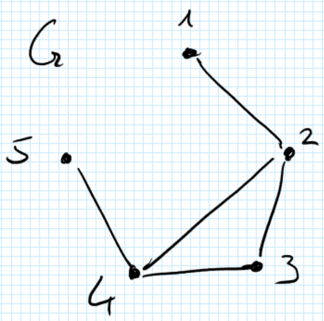


$\overline{G}$

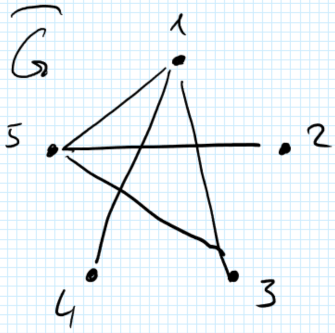


$(2,3,1,3,1)$

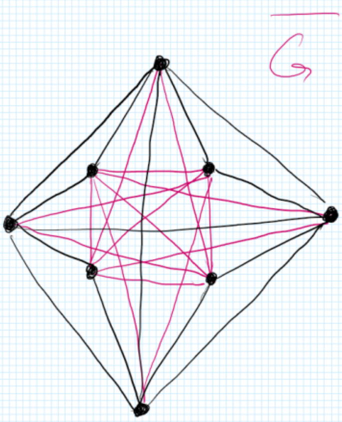
G



$\overline{G}$



f.) G $\overline{G}$



Black ist the graph G which is isomorphic to it's Complement $\overline{G}$ so G is self complementary

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Dynamic Programming Aufgabe 1.)

Dynamic Programming

Aufg 1

Items

Limit	0	1	2	3	4
0	0	0	0	0	0
1	0	0	0	0	0
2	0	0	0	0	0
3	0	0	0	0	50
4	0	0	40	40	50
5	0	10	40	40	50
6	0	10	40	40	50
7	0	10	40	40	90
8	0	10	40	40	90
9	0	10	50	50	90
10	0	10	50	70	90

i	1	2	3	4
value	10	40	30	50
size	5	4	6	3
limit	10			

We take Item 4 & Item 2 and have a value of 90

And a seize of 7

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Dynamic Programming Aufgabe 2.)

Auf 2.)

	0	A	B	C	B	D	A	B
0	0	0	0	0	0	0	0	0
B	0	0	1	1	1	1	1	1
D	0	0	1	1	1	2	2	2
C	0	0	1	2	2	2	2	2
A	0	1	1	2	2	2	3	3
B	0	1	2	2	3	3	3	4
A	0	1	2	2	3	3	4	4

The LCS is 4 (BDAB)