TEMA

FUNCIONES DE VARIAS VARIABLES

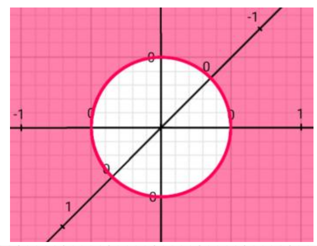
Hallar el dominio y graficar:

$$f(x,y)=\frac{xy}{x^2+y^2}$$

$$x^2+y^2\neq 0$$

$$x^2\neq -y^2$$

$$DF\left\{(x,y);\in R^2; x^2\neq -y^2\right\}$$



$$f(x,y,z)=\frac{x+y+z}{\sqrt{x^2+y^2+z^2}}$$

$$x^2+y^2+z^2\geq 0$$

$$DF\left\{(x,y);\in R^2; x^2+y^2+z^2>0\right\}$$

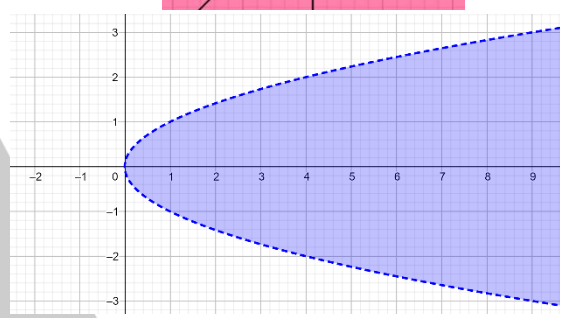
$$f(x,y)=\frac{y}{\sqrt{x-y^2}}$$

$$x-y^2>0$$

$$x>y^2$$

$$DF\left\{(x,y);\in R^2; x>y^2\right\}$$

F	x	y
0	0	0
1	1	± 1
2	2	$\pm\sqrt{2}$
3	3	$\pm\sqrt{3}$



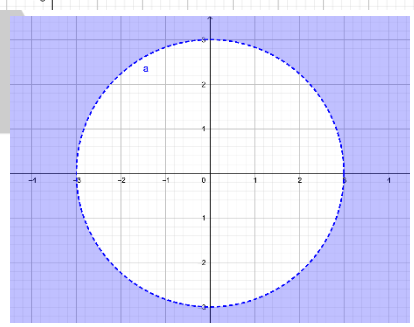
$$f(x,y)=\frac{x}{\sqrt{x^2+y^2-9}}$$

$$x^2+y^2-9>0$$

$$x^2+y^2>9$$

$$x^2+y^2>3^2$$

$$DF\left\{(x,y);\in R^2; x^2+y^2>3^2\right\}$$

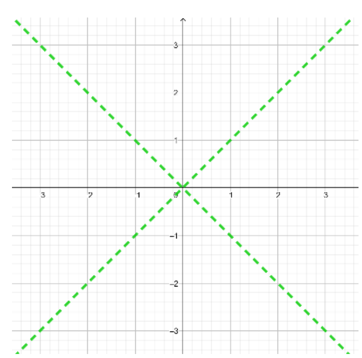


$$f(x,y)=\frac{x^4-y^4}{x^2-y^2}$$

$$x^2-y^2\neq 0$$

$$x^2\neq y^2$$

$$DF\left\{(x,y);\in R^2; x^2\neq y^2\right\}$$



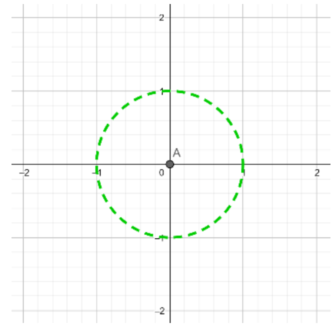
$$f(x,y)=\frac{1}{x^2-y^2-1}$$

$$x^2-y^2-1\neq 0$$

$$x^2-y^2\neq 1$$

$$x^2-y^2\neq 1^2$$

$$DF\left\{(x,y);\in R^2;x^2-y^2\neq 1^2\right\}$$



$$f(x,y)=\sqrt{16-x^2-y^2}$$

$$16-x^2-y^2\geq 0$$

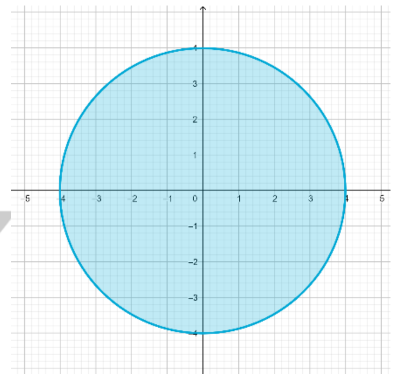
$$-x^2-y^2\geq -16$$

$$-x^2-y^2\geq -16 \;\; *(-1)$$

$$x^2+y^2\leq 16$$

$$x^2+y^2\leq 4^2$$

$$DF\left\{(x,y);\in R^2;x^2+y^2\leq 4^2\right\}$$



$$f(x,y)=\sqrt{1-x^2-y^2}$$

$$1-x^2-y^2\geq 0$$

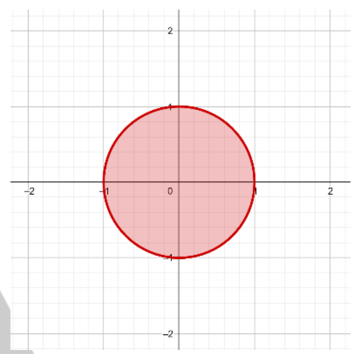
$$-x^2-y^2\geq -1$$

$$-x^2-y^2\geq -1 \;\; *(-1)$$

$$x^2+y^2\leq 1$$

$$x^2+y^2\leq 1^2$$

$$DF\left\{(x,y);\in R^2;x^2+y^2\leq 1^2\right\}$$



$$f(x,y)=\sqrt{16-4x^2-y^2}$$

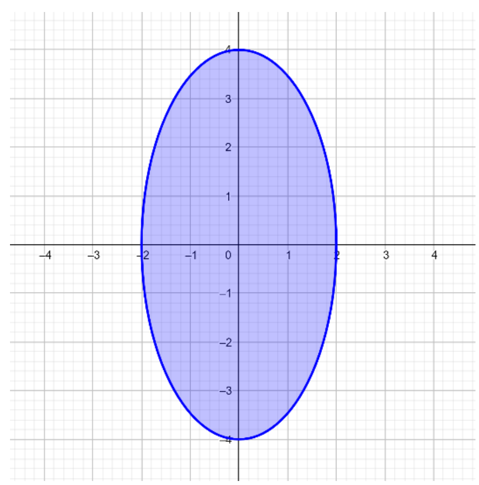
$$16-4x^2-y^2\geq 0$$

$$-4x^2-y^2\geq -16$$

$$-4x^2-y^2\geq -16 \;\; *(-1)$$

$$4x^2+y^2\leq 16$$

$$DF\left\{(x,y);\in R^2;4x^2+y^2\leq 16\right\}$$

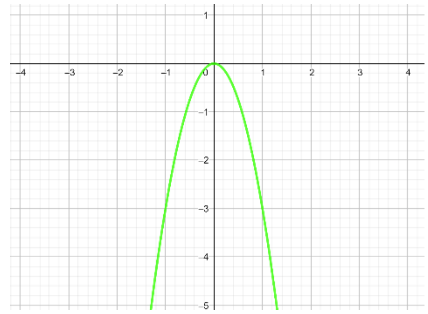


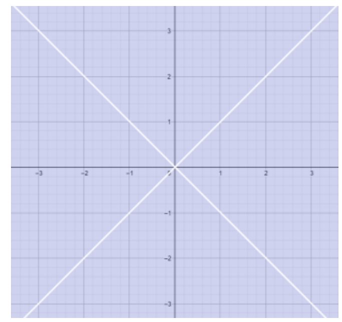
$$f(x,y)=3x^2+y$$

$$3x^2+y=0$$

$$3x^2=-y$$

$$DF\left\{(x,y);\in R^2;3x^2=-y\right\}$$



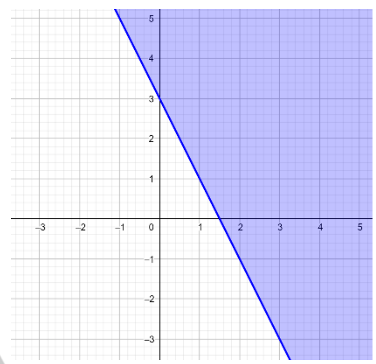


$$f(x,y)=\frac{1}{x^2-y^2}$$

$$x^2-y^2\neq 0$$

$$x^2\neq y^2$$

$$DF\left\{(x,y);\in R^2;x^2\neq y^2\right\}$$

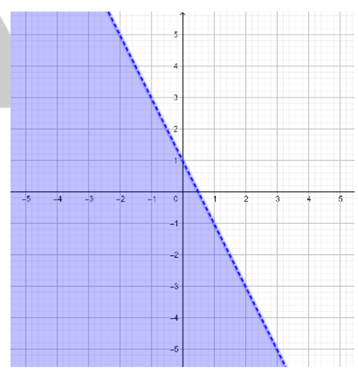


$$f(x,y)=\sqrt{2x+y-3}$$

$$2x+y-3\geq 0$$

$$2x+y\geq 3$$

$$DF\left\{(x,y);\in R^2;2x+y\geq 3\right\}$$



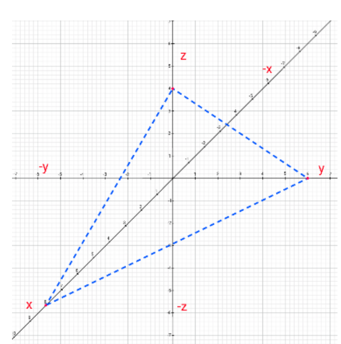
$$f(x,y)=\log(1-2x-y)$$

$$1-2x-y>0$$

$$-2x-y>-1\quad *(-1)$$

$$2x+y<1$$

$$DF\left\{(x,y);\in R^2;2x+y<1\right\}$$



$$f(x,y,z)=\frac{1}{3x+4y+6z-24}$$

$$3x+4y+6z-24\neq 0$$

$$3x+4y+6z\neq 24$$

$$DF\left\{(x,y,z);\in R^3;3x+4y+6z\neq 24\right\}$$

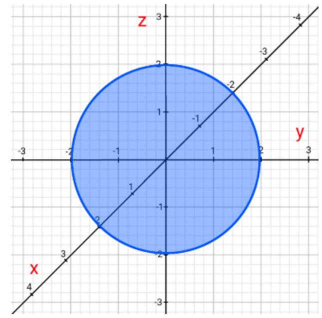
$$f(x,y,z)=\sqrt{2^2-x^2-y^2-z^2}$$

$$2^2-x^2-y^2-z^2\geq 0$$

$$-x^2-y^2-z^2\geq -2^2 \quad *(-1)$$

$$x^2+y^2+z^2\leq 2^2$$

$$DF\left\{(x,y,z);\in R^3;x^2+y^2+z^2\leq 2^2\right\}$$



$$f(x,y,z)=\log\left(z-x^2-y^2\right)$$

$$z-x^2-y^2>0$$

$$-x^2-y^2>-z \quad *(-1)$$

$$x^2+y^2<z$$

$$DF\left\{(x,y,z);\in R^3;x^2+y^2<z\right\}$$

$$f(x,y)=4-x^2-y^2$$

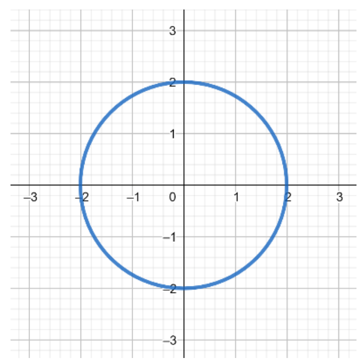
$$4-x^2-y^2=0$$

$$-x^2-y^2=-4 \quad *(-1)$$

$$x^2+y^2=4$$

$$x^2+y^2=2^2$$

$$DF\left\{(x,y);\in R^2;x^2+y^2=2^2\right\}$$



$$f(x,y)=2x^2+y^2$$

$$2x^2+y^2=0$$

$$2x^2=-y^2$$

$$DF\left\{(x,y);\in R^2;2x^2=-y^2\right\}$$

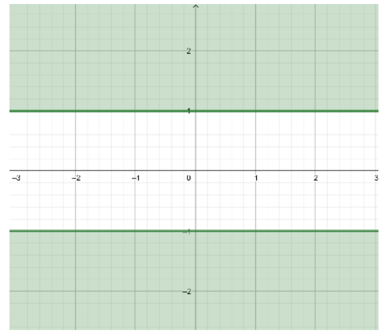
$$f(x,y)=x^2+1$$

$$x^2+1=0$$

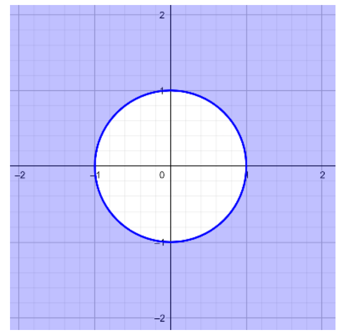
$$x^2=-1$$

$$DF\left\{(x,y);\in R^2;x^2=-1\right\}$$

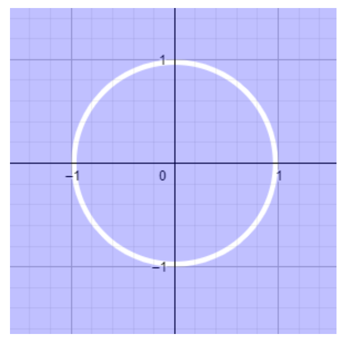
$$f(x,y)=\sqrt{1-y^2}$$
$$1-y^2\geq 0$$
$$-y^2\geq -1 \quad *(-1)$$
$$y^2\geq 1$$
$$y^2\geq 1^2$$
$$DF\left\{(x,y);\in R^2;y^2\geq 1^2\right\}$$



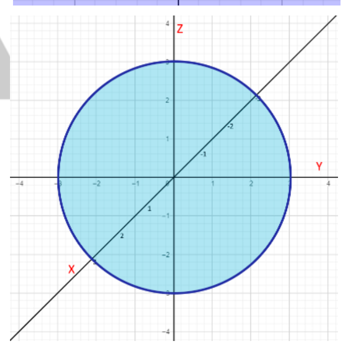
$$f(x,y)=\sqrt{x^2+y^2-1}$$
$$x^2+y^2-1\geq 0$$
$$x^2+y^2\geq 1$$
$$x^2+y^2\geq 1^2$$
$$DF\left\{(x,y);\in R^2;x^2+y^2\geq 1^2\right\}$$



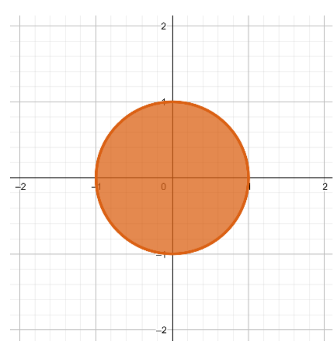
$$f(x,y)=\frac{1}{x^2-y^2-1}$$
$$x^2-y^2-1\neq 0$$
$$x^2-y^2\neq 1$$
$$DF\left\{(x,y);\in R^2;x^2-y^2\neq 1\right\}$$



$$f(x,y,z)=\sqrt{3^2-x^2-y^2-z^2}$$
$$3^2-x^2-y^2-z^2\geq 0$$
$$-x^2-y^2-z^2\geq -3^2 *(-1)$$
$$x^2+y^2+z^2\leq 3^2$$
$$DF\left\{(x,y,z);\in R^3;x^2+y^2+z^2\leq 9\right\}$$



$$f(x,y)=\sqrt{b-x^2-y^2}$$
$$b-x^2-y^2\geq 0$$
$$-x^2-y^2\geq -b^2 \quad *(-1)$$
$$x^2+y^2\leq b^2$$
$$DF\left\{(x,y);\in R^2;x^2+y^2\leq b^2\right\}$$



$$f\left(x,y,z\right)=\sqrt{2x^2+3y^2-8x+12y+3z+23}$$

$$2x^2+3y^2-8x+12y+3z+23\geq 0$$

$$\left(2x^2-8x\right)+\left(3y^2+12y\right)+3z+23\geq 0$$

$$2\left(x^2-4x\right)+3\left(y^2+4y\right)+3z+23\geq 0$$

$$2\left(x-2\right)^2-2^2+3\left(y+2\right)^2-2^2+3z+23\geq 0$$

$$2\left(x-2\right)^2-8+3\left(y+2\right)^2-12+3z+23\geq 0$$

$$2\left(x-2\right)^2+3\left(y+2\right)^2+3z+3\geq 0$$

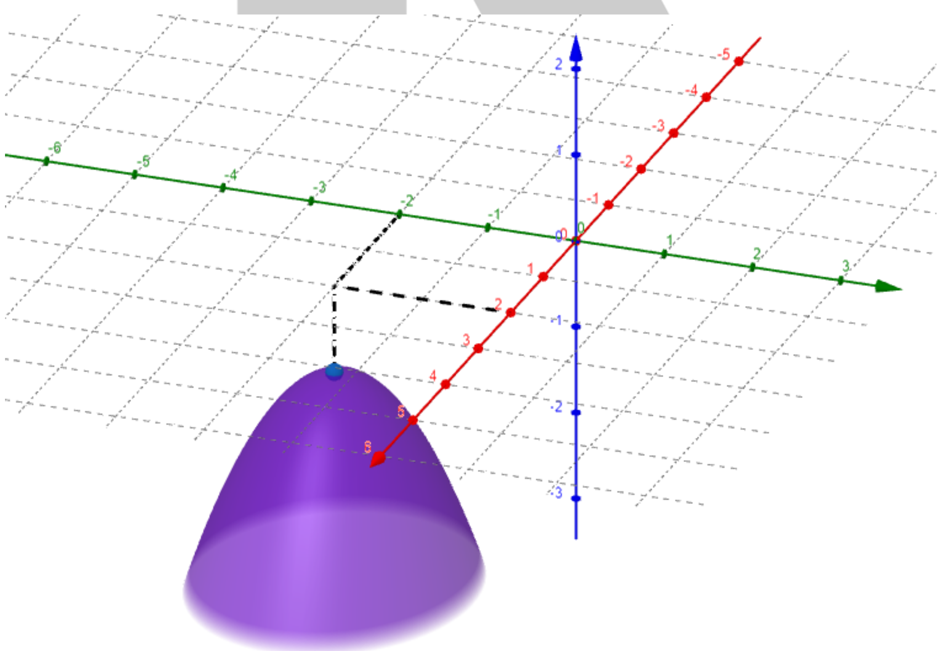
$$2\left(x-2\right)^2+3\left(y+2\right)^2\geq\left(-3z-3\right)$$

$$2\left(x-2\right)^2+3\left(y+2\right)^2\geq -3\left(z+1\right)$$

$$-\frac{2\left(x-1\right)^2}{3}-\frac{3\left(y+2\right)^2}{3}\geq -\frac{3\left(z+1\right)}{-3}$$

$$-\frac{\left(x-2\right)^2}{\frac{3}{2}}-\frac{\left(y+2\right)^2}{1}\geq\left(z+1\right)$$

$$DF\left\{\left(x,y,z\right);\in R^3;-\frac{\left(x-2\right)^2}{\frac{3}{2}}-\frac{\left(y+2\right)^2}{1}\geq\left(z+1\right)\right\}$$



$$f(x,y,z)=\log\left(2x^2+4y^2-4x-y-24z+36\right)$$

$$2x^2+4y^2-4x-y-24z+36>0$$

$$\left(2x^2-4x\right)+\left(4y^2-y\right)-24z+36>0$$

$$2\left(x^2-2x\right)+4\left(y^2-\frac{1}{4}y\right)-24z+36>0$$

$$2(x-1)^2-1^2+4\left(y-\frac{1}{8}\right)^2-\left(\frac{1}{8}\right)^2-24z+36>0$$

$$2(x-1)^2-2+4\left(y-\frac{1}{8}\right)^2-\frac{1}{16}-24z+36>0$$

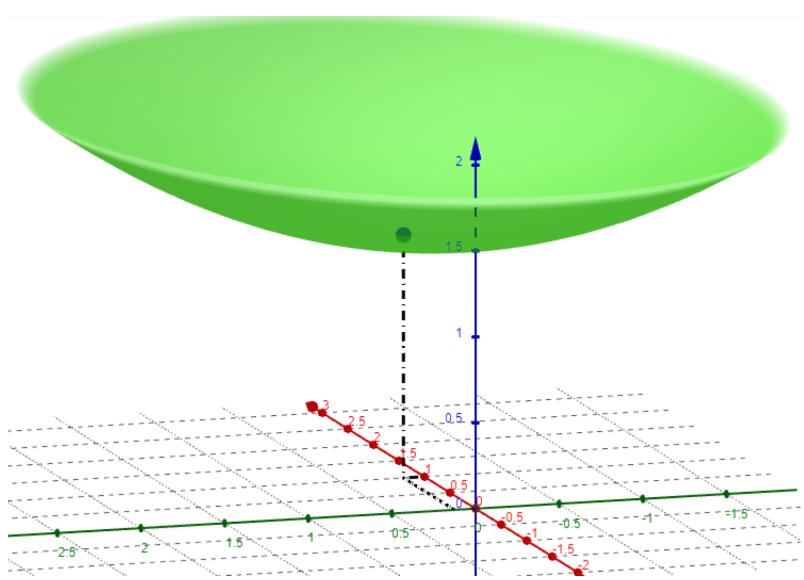
$$2(x-1)^2+4\left(y-\frac{1}{8}\right)^2>24z-\frac{543}{16}$$

$$2(x-1)^2+4\left(y-\frac{1}{8}\right)^2>24\left(z-\frac{181}{128}\right)$$

$$\frac{2(x-1)^2}{24}+\frac{4\left(y-\frac{1}{8}\right)^2}{24}>\frac{24\left(z-\frac{181}{128}\right)}{24}$$

$$\frac{(x-1)^2}{12}+\frac{\left(y-\frac{1}{8}\right)^2}{6}>\left(z-\frac{181}{128}\right)$$

$$DF\left\{(x,y,z);\in R^3;\frac{(x-1)^2}{12}+\frac{\left(y-\frac{1}{8}\right)^2}{6}>\left(z-\frac{181}{128}\right)\right\}$$



$$f(x,y,z)=\frac{1}{\sqrt{3x^2+4y^2-2z^2+6x-16y+8z-13}}$$

$$3x^2+4y^2-2z^2+6x-16y+8z-13>0$$

$$\left(3x^2+6x\right)+\left(4y^2-16y\right)+\left(-2z^2+8z\right)-13>0$$

$$3\left(x^2+2x\right)+4\left(y^2-4y\right)-2\left(z^2-4z\right)-13>0$$

$$3(x+1)^2-1^2+4(y-2)^2-2^2-2(z-2)^2-2^2-13>0$$

$$3(x+1)^2-3+4(y-2)^2-16-2(z-2)^2+8-13>0$$

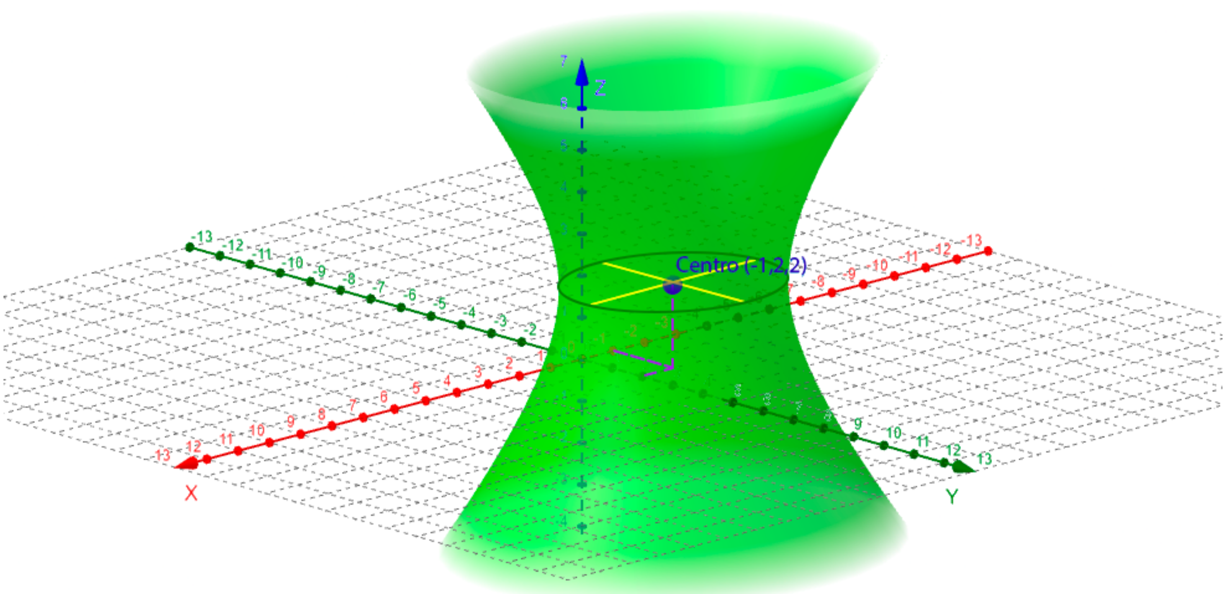
$$3(x+1)^2+4(y-2)^2-2(z-2)^2-3-16+8-13>0$$

$$3(x+1)^2+4(y-2)^2-2(z-2)^2>24$$

$$\frac{3(x+1)^2}{24}+\frac{4(y-2)^2}{24}-\frac{2(z-2)^2}{24}>\frac{24}{24}$$

$$\frac{(x+1)^2}{8}+\frac{(y-2)^2}{6}-\frac{(z-2)^2}{12}>1$$

$$DF\left\{(x,y,z);\in R^3;\frac{(x+1)^2}{8}+\frac{(y-2)^2}{6}-\frac{(z-2)^2}{12}>1\right\}$$

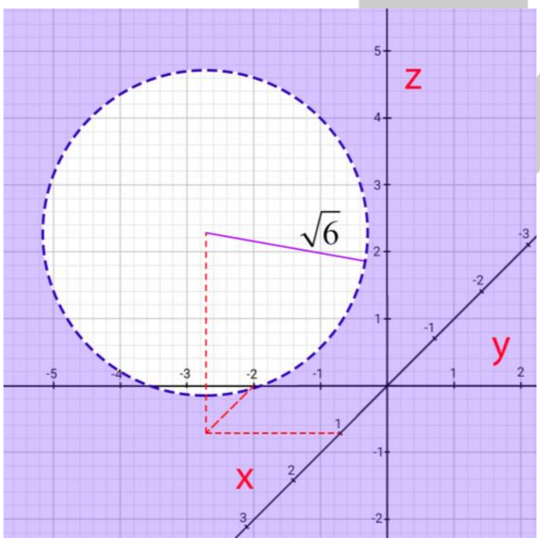


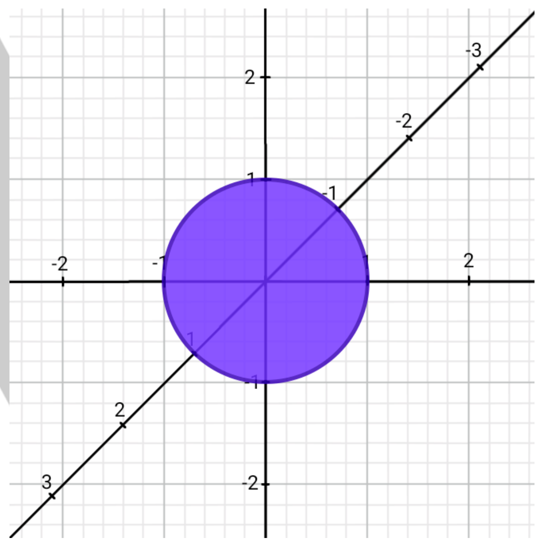
$$f(x,y,z)=\frac{\sqrt{1-x^2-y^2-z^2}}{\log\left(x^2+y^2+z^2-2x+4y-6z+8\right)}$$

$$x^2+y^2+z^2-2x+4y-6z+8>0$$
$$\left(x^2-2x\right)+\left(y^2+4y\right)+\left(z^2-6z\right)+8>0$$
$$(x-1)^2-1^2+(y+2)^2-2^2+(z-3)^2-3^2+8>0$$
$$(x-1)^2-1+(y+2)^2-4+(z-3)^2-9+8>0$$
$$(x-1)^2+(y+2)^2+(z-3)^2-1-4-9+8>0$$
$$(x-1)^2+(y+2)^2+(z-3)^2>6$$
$$(x-1)^2+(y+2)^2+(z-3)^2>\left(\sqrt{6}\right)^2$$

$$1-x^2-y^2-z^2$$
$$-x^2-y^2-z^2\geq -1 \quad *(-1)$$
$$x^2+y^2+z^2\leq 1^2$$

$$DF\left\{(x,y,z);\in R^3;\left\{(x-1)^2+(y+2)^2+(z-3)^2>\left(\sqrt{6}\right)^2\right\}\ \wedge\ \left\{x^2+y^2+z^2\leq 1^2\right\}\right\}$$





$$f\left(x,y,z\right)=\frac{3x^2+3y^2+3z^2-8x+12y-10z+10}{\sqrt{4-x^2-y^2-z^2}}$$

$$3x^2+3y^2+3z^2-8x+12y-10z+10=0 \qquad\qquad 4-x^2-y^2-z^2>0$$

$$x^2+y^2+z^2-\frac{8}{3}x+4y-\frac{10}{3}z+\frac{10}{3}=0 \qquad\qquad -x^2-y^2-z^2>-4 \quad *\left(-1\right)$$

$$\left(x^2-\frac{8}{3}x\right)+\left(y^2+4y\right)+\left(z^2-\frac{10}{3}z\right)+\frac{10}{3}=0 \qquad\qquad x^2+y^2+z^2<4$$

$$\left(x-\frac{4}{3}\right)^2-\left(\frac{4}{3}\right)^2+\left(y+2\right)^2-2^2+\left(z-\frac{5}{3}\right)^2-\left(\frac{5}{3}\right)^2+\frac{10}{3}=0$$

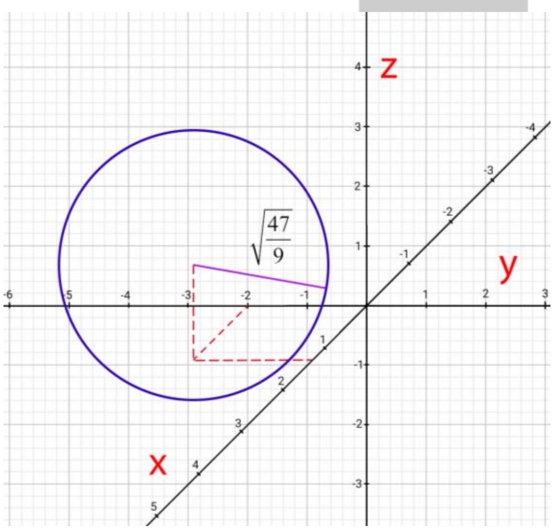
$$\left(x-\frac{4}{3}\right)^2-\frac{16}{9}+\left(y+2\right)^2-4+\left(z-\frac{5}{3}\right)^2-\frac{25}{9}+\frac{10}{3}=0$$

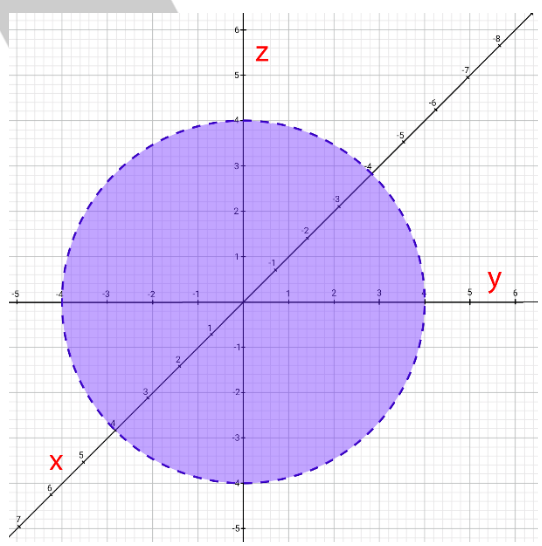
$$\left(x-\frac{4}{3}\right)^2+\left(y+2\right)^2+\left(z-\frac{5}{3}\right)^2-\frac{16}{9}-4-\frac{25}{9}+\frac{10}{3}=0$$

$$\left(x-\frac{4}{3}\right)^2+\left(y+2\right)^2+\left(z-\frac{5}{3}\right)^2=\frac{47}{9}$$

$$\left(x-\frac{4}{3}\right)^2+\left(y+2\right)^2+\left(z-\frac{5}{3}\right)^2=\left(\sqrt{\frac{47}{9}}\right)^2$$

$$DF\left\{\left(x,y,z\right);\in R^3;\left\{\left(x-\frac{4}{3}\right)^2+\left(y+2\right)^2+\left(z-\frac{5}{3}\right)^2=\left(\sqrt{\frac{47}{9}}\right)^2\right\}\wedge\left\{x^2+y^2+z^2<4\right\}\right\}$$





$$f\left(x,y,z\right)=\frac{x^2+y^2-4}{3x^2+5y^2-2x+10y-12z+21}$$

$$3x^2+5y^2-2x+10y-12z+21\neq 0$$

$$x^2+y^2-4=0$$

$$\left(3x^2-2x\right)+\left(5y^2+10y\right)-12z+21\neq 0$$

$$x^2+y^2=4$$

$$3\left(x^2-\frac{2}{3}x\right)+5\left(y^2+2y\right)-12z+21\neq 0$$

$$3\left(x-\frac{1}{3}\right)^2-\left(\frac{1}{3}\right)^2+5\left(y+1\right)^2-1^2-12z+21\neq 0$$

$$3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}+5\left(y+1\right)^2-5-12z+21\neq 0$$

$$3\left(x-\frac{1}{3}\right)^2+5\left(y+1\right)^2-12z+21-\frac{1}{3}-5\neq 0$$

$$3\left(x-\frac{1}{3}\right)^2+5\left(y+1\right)^2\neq 12z-21+\frac{1}{3}+5$$

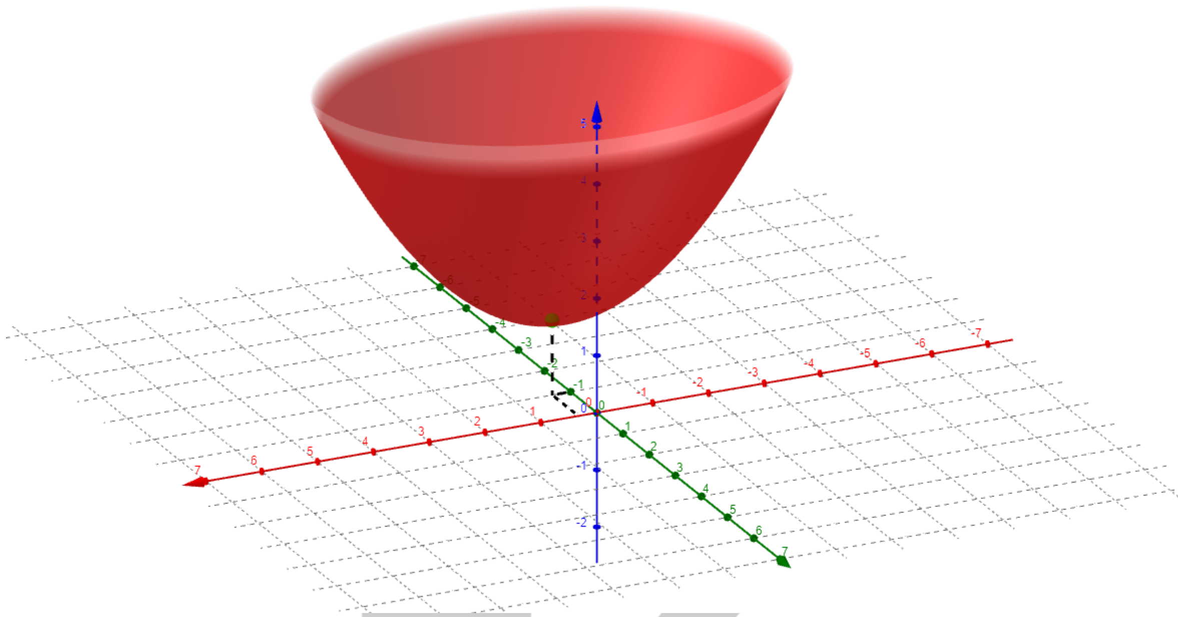
$$3\left(x-\frac{1}{3}\right)^2+5\left(y+1\right)^2\neq 12z-\frac{47}{3}$$

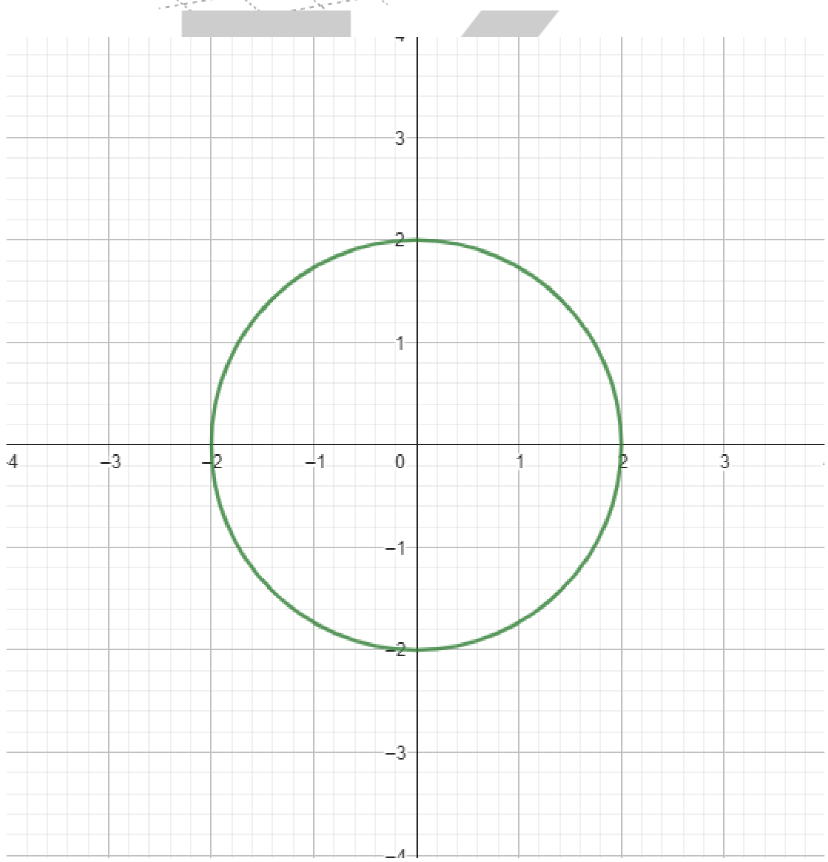
$$3\left(x-\frac{1}{3}\right)^2+5\left(y+1\right)^2\neq 12\left(z-\frac{47}{36}\right)$$

$$\frac{3\left(x-\frac{1}{3}\right)^2}{12}+\frac{5\left(y+1\right)^2}{12}\neq\frac{12\left(z-\frac{47}{36}\right)}{12}$$

$$\frac{\left(x-\frac{1}{3}\right)^2}{4}+\frac{\left(y+1\right)^2}{\frac{12}{5}}\neq\left(z-\frac{47}{36}\right)$$

$$DF\left\{\left(x,y,z\right);\in R^3;\left\{\frac{\left(x-\frac{1}{3}\right)^2}{4}+\frac{\left(y+1\right)^2}{\frac{12}{5}}\neq\left(z-\frac{47}{36}\right)\right\}\wedge\left\{x^2+y^2=4\right\}\right\}$$





3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

$$f\left(x,y,z\right)=\log\left(3x^2+4y^2-2z^2+6x-16y+8z-13\right)$$

$$3x^2+4y^2-2z^2+6x-16y+8z-13>0$$

$$\left(3x^2+6x\right)+\left(4y^2-16y\right)+\left(-2z^2+8z\right)-13>0$$

$$3\left(x^2+2x\right)+4\left(y^2-4y\right)-2\left(z^2-4z\right)-13>0$$

$$3\left(x+1\right)^2-1^2+4\left(y-2\right)^2-2^2-2\left(z-2\right)^2-2^2-13>0$$

$$3\left(x+1\right)^2-3+4\left(y-2\right)^2-16-2\left(z-2\right)^2+8-13>0$$

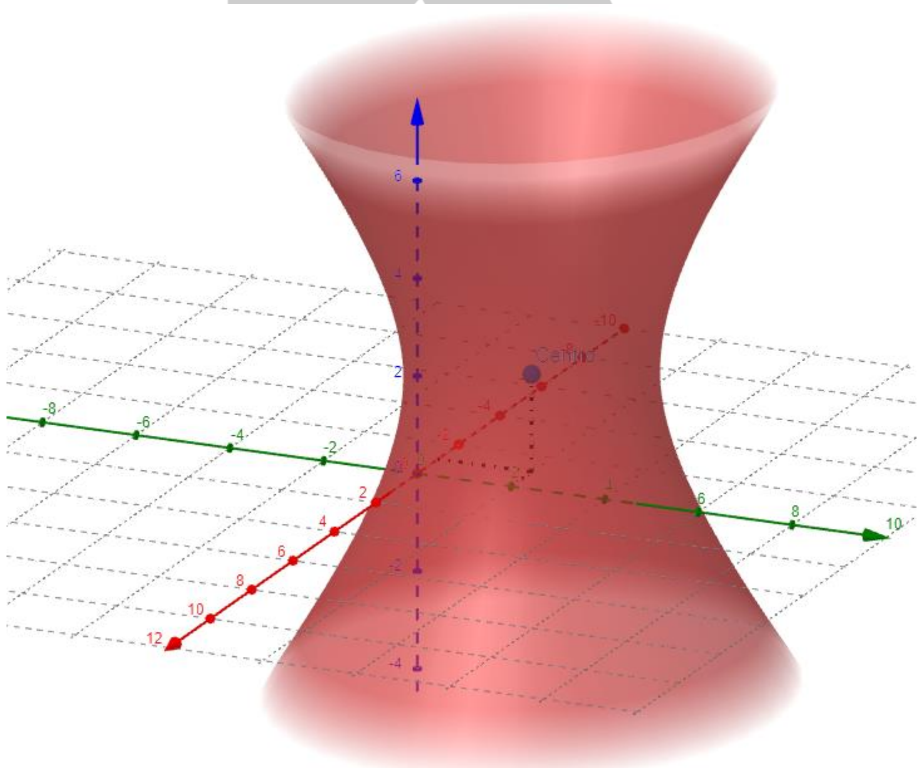
$$3\left(x+1\right)^2+4\left(y-2\right)^2-2\left(z-2\right)^2-3-16+8-13>0$$

$$3\left(x+1\right)^2+4\left(y-2\right)^2-2\left(z-2\right)^2>24$$

$$\frac{3\left(x+1\right)^2}{24}+\frac{4\left(y-2\right)^2}{24}-\frac{2\left(z-2\right)^2}{24}>\frac{24}{24}$$

$$\frac{\left(x+1\right)^2}{8}+\frac{\left(y-2\right)^2}{6}-\frac{\left(z-2\right)^2}{12}>1$$

$$DF\left\{\left(x,y,z\right);\in R^3;\frac{\left(x+1\right)^2}{8}+\frac{\left(y-2\right)^2}{6}-\frac{\left(z-2\right)^2}{12}>1\right\}$$



$$f(x,y,z)=\sqrt{2x^2-3y^2-2z^2-8x+6y-12z-21}$$

$$2x^2-3y^2-2z^2-8x+6y-12z-21\geq 0$$

$$\left(2x^2-8x\right)+\left(-3y^2+6y\right)+\left(-2z^2-12z\right)-21\geq 0$$

$$2\left(x^2-4x\right)-3\left(y^2-2y\right)-2\left(z^2+6z\right)-21\geq 0$$

$$2(x-2)^2-2^2-3(y-1)^2-1^2-2(z+3)^2-3^2-21\geq 0$$

$$2(x-2)^2-8-3(y-1)^2+3-2(z+3)^2+18-21\geq 0$$

$$2(x-2)^2-3(y-1)^2-2(z+3)^2-8+3+18-21\geq 0$$

$$2(x-2)^2-3(y-1)^2-2(z+3)^2\geq 8$$

$$\frac{2(x-2)^2}{8}-\frac{3(y-1)^2}{8}-\frac{2(z+3)^2}{8}\geq\frac{8}{8}$$

$$\frac{(x-2)^2}{4}-\frac{(y-1)^2}{\frac{8}{3}}-\frac{(z+3)^2}{4}\geq 1$$

$$DF\left\{(x,y,z);\in R^3;\frac{(x-2)^2}{4}-\frac{(y-1)^2}{\frac{8}{3}}-\frac{(z+3)^2}{4}\geq 1\right\}$$