



# GrabCut论文复现及分析



## 1. What is GrabCut?

**Graph Cut + GMM + Iterative energy**

## 2. What matters in GrabCut?

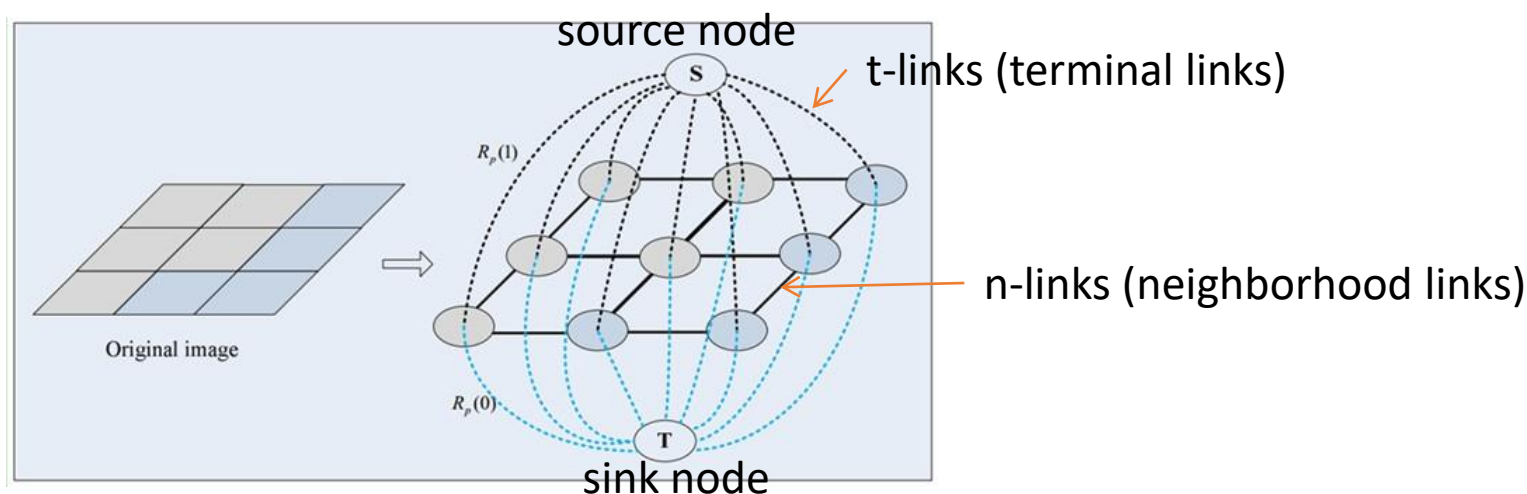
$\gamma$  and  $K$

## 3. GMM vs. Colorful Histogram?

**User Interaction and Apply Restrictions**

# 1.1 Graph cut

## 1. s-t graph



## 2.maxflow


最大流最小割定理：在一个网络流中，从源点到汇点的最大的流量，等于它的最小割中每一条边的容量之和

## 1.2 Energy function

$$E(\alpha, \theta, z) = U(\alpha, \theta, z) + V(\alpha, z)$$


$$U(\alpha, \theta, z) = \sum_n -\log h(z_n, \alpha_n)$$

区域能量项（全局分析）


$$V(\alpha, z) = \gamma \sum_{(m,n) \in C} dis(m,n)^{-1} [\alpha_n \neq \alpha_m] \exp(-\beta(z_m - z_n)^2)$$

边界平滑项（局部分析）

目标：将图像分为前景和背景两个不相交的部分，运用图像分割方法来实现。

建图：两类顶点 + 两类带权边（分别由区域能量项和边界平滑项计算得到）

分割：min-cut 计算权值和最小的边的集合——对应能量最小化

# 1.3 Gaussian Mixture Model

## 1. d维高斯分布

$$p(x|\{\mu, \Sigma\}) = \frac{1}{\sqrt{(2\pi)^d |\Sigma|}} \exp[(x - \mu)^T \Sigma^{-1} (x - \mu)]$$

## 2. 混合模型（加权和）

$$P(x|\{\mu, \Sigma\}) = \sum_{k=1}^K \pi_k \cdot p(x|\{\mu_k, \Sigma_k\})$$

## 3. Energy function

$$U(\alpha, \theta, z) = \sum_n -\log h(z_n, \alpha_n)$$



$$U(\alpha, k, \theta, z) = \sum_n D(\alpha_n, k_n, \theta, z_n)$$

$$D(\alpha_n, k_n, \theta, z_n) = -\log(\pi(\alpha_n, k_n) \cdot p(z_n|\alpha_n, k_n, \theta))$$

```

24 //3维高斯分布: 计算color像素属于第k个高斯分量的概率
25 double GMM::Possibility(int k, const Vec3d color) const
26 {
27     double res = 0;
28     if(Pi[k] > 0){
29         Vec3d diff = color;
30         double* m = mean + 3 * k;
31         // X - \mu
32         diff[0] -= m[0]; diff[1] -= m[1]; diff[2] -= m[2];
33         /* (X - \mu)^T * cov^-1 * (X - \mu)
34          = [diff{0}, diff{1}, diff{2}] * [covInv[k][0][0], covInv[k][0][1], covInv[k][0][2]] * [diff{0}]
35          [covInv[k][1][0], covInv[k][1][1], covInv[k][1][2]] |diff{1}|
36          [covInv[k][2][0], covInv[k][2][1], covInv[k][2][2]] |diff{2}|
37          */
38         double mult = (diff[0] * covInv[k][0][0] + diff[1] * covInv[k][1][0] + diff[2] * covInv[k][2][0]) * diff[0]
39             + (diff[0] * covInv[k][0][1] + diff[1] * covInv[k][1][1] + diff[2] * covInv[k][2][1]) * diff[1]
40             + (diff[0] * covInv[k][0][2] + diff[1] * covInv[k][1][2] + diff[2] * covInv[k][2][2]) * diff[2];
41         res = 1.0f / sqrt(covDet[k]) * exp(-0.5f * mult);
42     }
43     return res;
44 }

```

# 1.4 GrabCut流程

## Initialisation

- User initialises trimap  $T$  by supplying only  $T_B$ . The foreground is set to  $T_F = \emptyset$ ;  $T_U = \bar{T}_B$ , complement of the background.
- Initialise  $\alpha_n = 0$  for  $n \in T_B$  and  $\alpha_n = 1$  for  $n \in T_U$ .
- Background and foreground GMMs initialised from sets  $\alpha_n = 0$  and  $\alpha_n = 1$  respectively.

## Iterative minimisation

1. Assign GMM components to pixels: for each  $n$  in  $T_U$ ,

$$k_n := \arg \min_{k_n} D_n(\alpha_n, k_n, \theta, z_n).$$

2. Learn GMM parameters from data  $\mathbf{z}$ :

$$\underline{\theta} := \arg \min_{\underline{\theta}} U(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z})$$

3. Estimate segmentation: use min cut to solve:

$$\min_{\{\alpha_n: n \in T_U\}} \min_{\mathbf{k}} E(\underline{\alpha}, \mathbf{k}, \underline{\theta}, \mathbf{z}).$$

4. Repeat from step 1, until convergence.

5. Apply border matting (section 4).

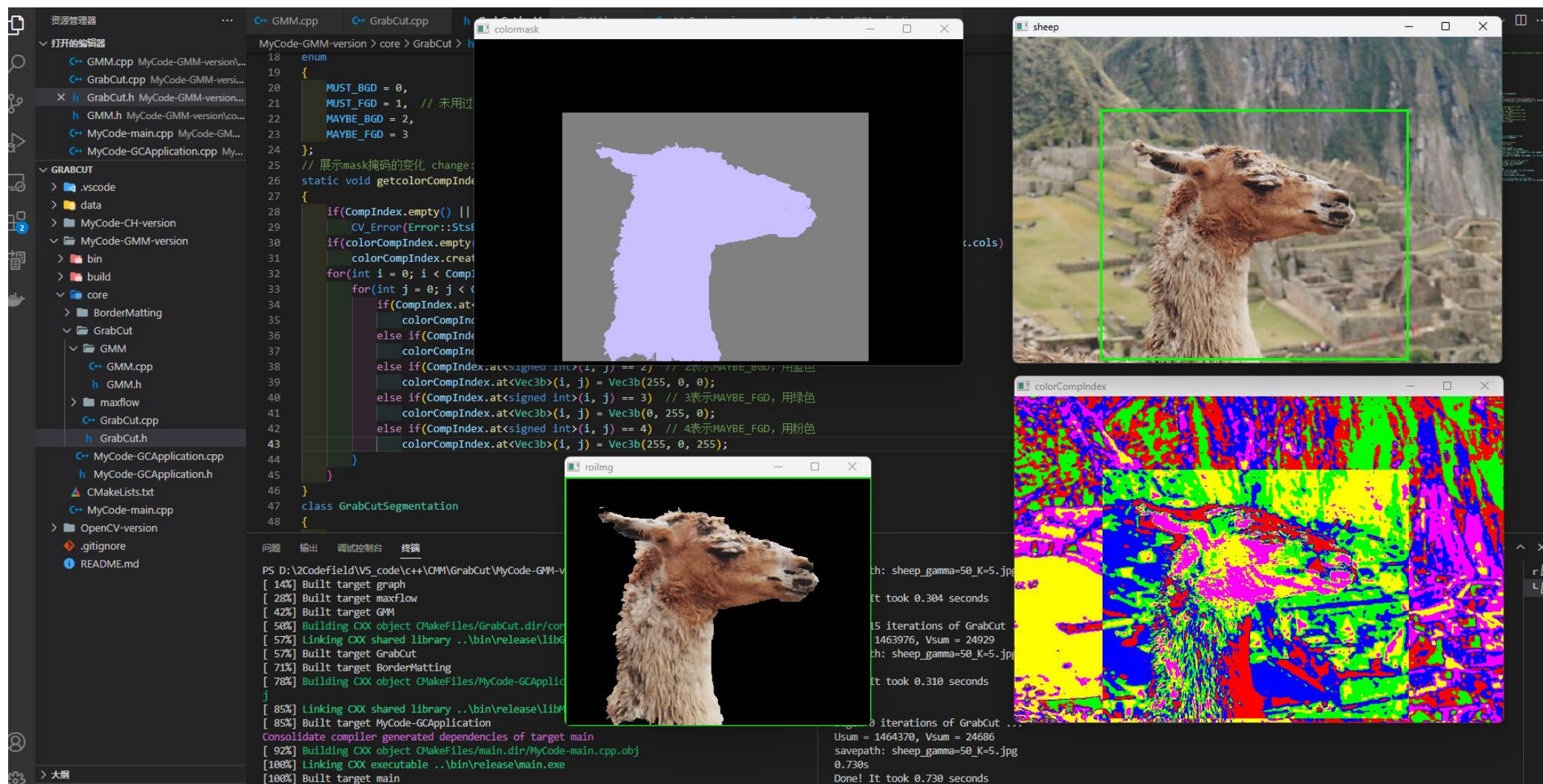
## User editing

- *Edit*: fix some pixels either to  $\alpha_n = 0$  (background brush) or  $\alpha_n = 1$  (foreground brush); update trimap  $T$  accordingly. Perform step 3 above, just once.
- *Refine operation*: [optional] perform entire iterative minimisation algorithm.

```
//GrabCut 主函数
void GrabCutSegmentation::GrabCut(InputArray arrayimg, InputOutputArray arraymask, Rect rect,
                                  InputOutputArray _bgdModel, InputOutputArray _fgdModel,
                                  int iterCount, int mode)
{
    Mat img = arrayimg.getMat();
    Mat& mask = arraymask.getMatRef();
    Mat& bgdModel = _bgdModel.getMatRef();
    Mat& fgdModel = _fgdModel.getMatRef();
    GMM backgroundGMM(bgdModel), foregroundGMM(fgdModel); // 构建背景和前景的GMM模型
    if(mode == GC_WITH_RECT){
        // 初始化mask
        initMaskWithRect(mask, img.size(), rect);
        // 初始化GMM模型
        initGMMs(img, mask, backgroundGMM, foregroundGMM);
    }
    if(iterCount <= 0) return;

    // 计算Beta的值
    const double beta = CalcBeta(img);
    // 计算平滑项(边界能量项V)
    Mat leftWeight, upleftWeight, upWeight, uprightWeight;
    CalcSmoothness(img, beta, gamma, leftWeight, upleftWeight, upWeight, uprightWeight);
    // 存储每个像素属于哪个高斯模型
    Mat ComponentIndex(img.size(), CV_32SC1);
    const double lambda = 8 * gamma + 1;
    for(int i = 0; i < iterCount; i++){
        int vCount = img.cols*img.rows;
        int eCount = 2 * (4 * vCount - 3 * img.cols - 3 * img.rows + 2); // 无向图=双向图
        Graph<double, double, double> graph(vCount, eCount); // 建图
        AssignGMMComponents(img, mask, backgroundGMM, foregroundGMM, ComponentIndex);
        LearnGMMParameters(img, mask, backgroundGMM, foregroundGMM, ComponentIndex);
        getGraph(img, mask, backgroundGMM, foregroundGMM, leftWeight, upleftWeight, upWeight, uprightWeight, lambda, graph);
        EstimateSegmentation(graph, mask);
        // CalcEnergyFunction(graph, mask, leftWeight, upleftWeight, upWeight, uprightWeight);
    }
}
```





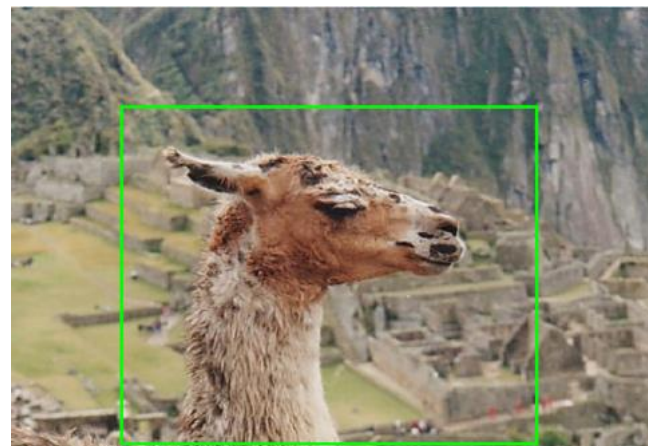
## 2.1 $\gamma$ and $K$

控制变量分析：相同图片，相同位置的交互框，控制迭代2次

分析变量1:  $E = U + \gamma \cdot V$

$$V(\alpha, z) = \gamma \sum_{(m, n) \in C} \text{dis}(m, n)^{-1} [\alpha_n \neq \alpha_m] \exp(-\beta(z_m - z_n)^2)$$

分析变量2:  $K$





2.1  $\gamma$  & K

K=2

K=5

K=10

K=20

 $\gamma = 12$  $\gamma = 50$  $\gamma = 200$ 

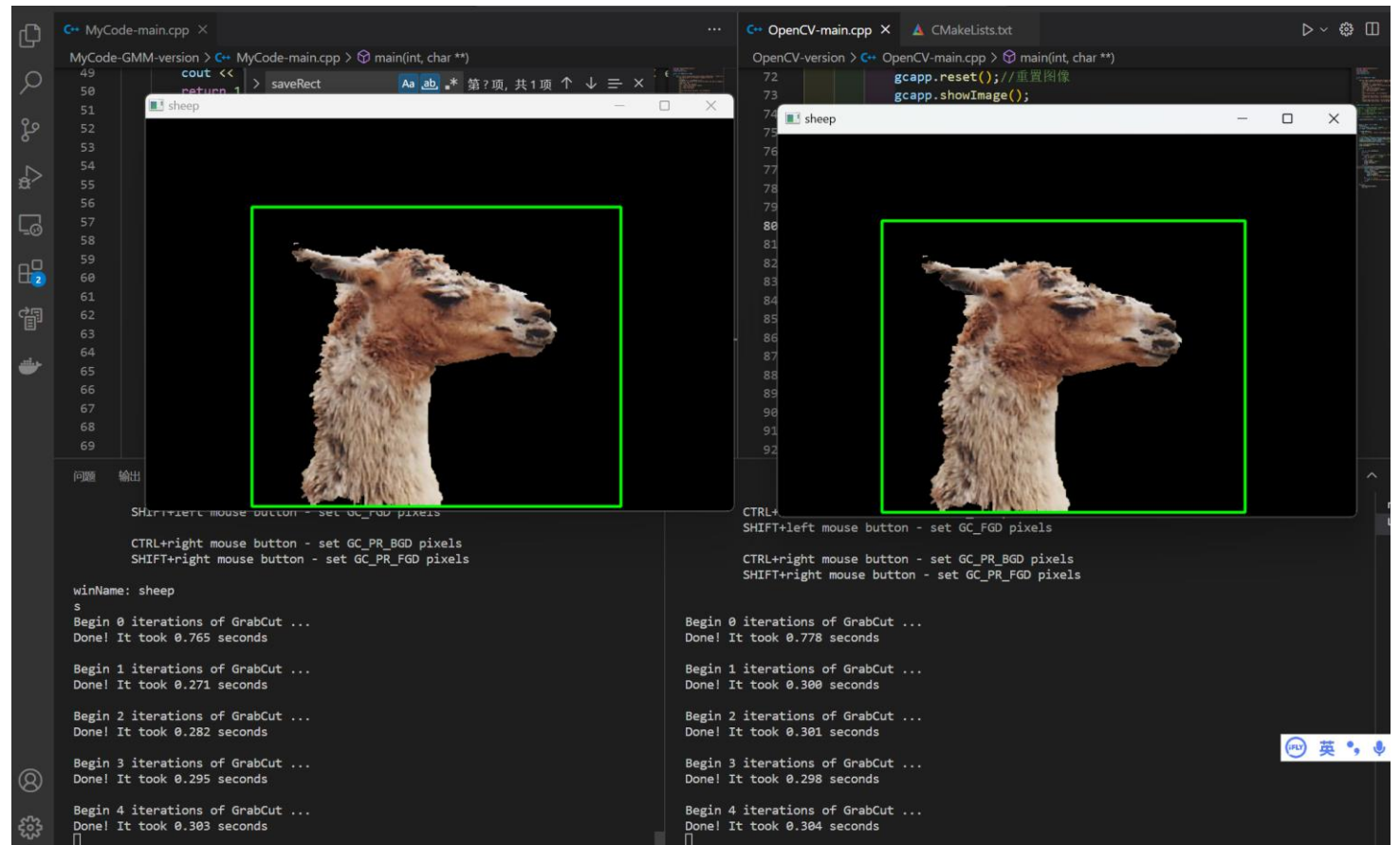
## 2.2 Analysis

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1.  $\gamma$  作为平衡区域能量项和边界平滑项的权重，在分割效果上至关重要
2.  $K$  作为GMM划分特征的“超参数”，在小范围变动时影响不大。

## 2.3 Time-consuming

	自己实现	调用opencv包
第1次迭代	0.765s	0.778s
第2次迭代	0.271s	0.300s
第3次迭代	0.282s	0.301s
第4次迭代	0.295s	0.298s



## 3.1 GMM vs. Colorful Hist

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彩色直方图加速：量化颜色，平滑处理，保留颜色

1. 对于目标物体对比程度&显著性
2. 对于目标物体占比

# Reference

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- [1] BOYKOV, Y., AND JOLLY, M.-P, "Interactive Graph Cuts for Optimal Boundary & Region Segmentation of Objects in N-D Images, " In Proc. IEEE Int. Conf. on Computer Vision, CD-ROM, 2001.
- [2] C. Rother, V. Kolmogorov, and A. Blake, ""GrabCut"— Interactive foreground extraction using iterated graph cuts," ACM TOG, vol. 23, no. 3, pp. 309–314, 2004.
- [3] M.-M. Cheng, N. J. Mitra, X. Huang, P. H. Torr, and S.-M. Hu, "Global contrast based salient region detection," IEEE Trans. Pattern Anal. Mach. Intell., vol. 37, no. 3, pp. 569–582, 2015.

Thanks