

(a) The operations can be categorized into the following types:

- “Short” ENQUEUE: A simple PUSH which costs \$2.
- “Long” ENQUEUE: A PUSH following the migration of 12 elements from  $S_1$  to  $S_2$ , whose cost is  $5 \times 12 + 2 = \$62$ .
- “Short” DEQUEUE: A simple POP which costs \$3.
- “Long” DEQUEUE: A POP following the migration of all elements from  $S_1$  to  $S_2$ , whose cost is  $5k + 3$  (with  $k$  being the number of elements to migrate).

The sequence of 50 ENQUEUE and 50 DEQUEUE operations then looks as follows:

- 12 short ENQUEUEES,
- 1 long ENQUEUE,
- 37 short ENQUEUEES,
- 12 short DEQUEUEES,
- 1 long DEQUEUE with the migration of 38 elements,
- 37 short DEQUEUEES.

The total cost is:

$$12 \cdot \$2 + 1 \cdot \$62 + 37 \cdot \$2 + 12 \cdot \$3 + 1 \cdot (\$5 \cdot 38 + 3) + 37 \cdot \$3 = \$500.$$

Therefore the amortized cost per operation is:

$$\$500/100 = \$5.$$

- (b) Using the accounting method, we charge \$10 for each ENQUEUE operation and \$0 for each DEQUEUE operation. Imagine that this \$10 is “attached” to the element that was ENQUEUED: \$2 pays for pushing the element into  $S_1$ , \$5 for migrating the element from  $S_1$  to  $S_2$  (1 PUSH + 1 POP), and \$3 for popping the element out of  $S_2$ . This \$10 per element is enough to pay for all operations since each element is pushed, migrated and popped at most once during any sequence of operations. Hence, the amortized cost per operation for any sequence is *at most* 10 units of time (maybe less depending on the sequence).