Problem Set #5 CSC236 Fall 2018

Mengning Yang, Chenxu Liu, Licheng Xu

December 6, 2018

We declare that this assignment is solely our own work, and is in accordance with the University of Toronto Code of Behaviour on Academic Matters.

This submission has been prepared using LaTeX.

Problem 1.

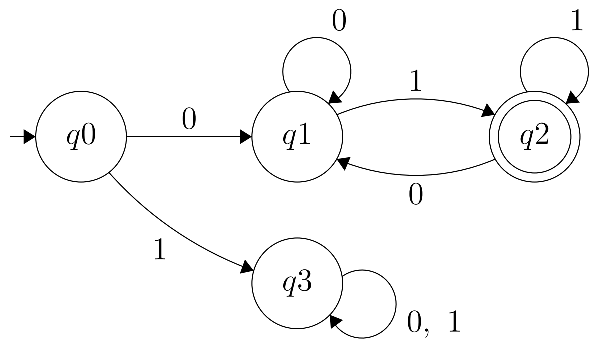
(WARMUP - THIS PROBLEM WILL NOT BE MARKED).

For the following language, give an NFA that matches the language. Give informal justification for why your NFA is correct.

$L = \{w \in \{0,1\}^* \mid w \text{ starts with 01 but does not end with 01}\}$

Problem 2.

(12 MARKS) Consider the following DFA for the language $L = \{w \in \{0,1\}^* \mid w$ begins with a 0 and ends with a 1$\}$, and answer the questions below. For each question, you should **only use the techniques that we covered in this course**.



1. (5 marks) Prove that the DFA is correct (it matches exactly the language described above). Before beginning the proof, clearly state each of your **state invariants**.

2. (2 Marks) Prove that this DFA has the minimal number of states (4 states).

3. (5 Marks) Using the **state elimination** method, convert the above DFA to its equivalent regular expression. You must show all your steps in **separate** figures.

Answers:

1. state invariants:
   q_0 p(w):initial state, accepts empty strings
   q_1 p(w):begins with 0,ends with 0
   q_2 p(w):begins with 0, ends with 1
   q_3 p(w):begins with 1

   Base case:
   Initial state is q_0, and ϵ is an empty string, so ϵ satisfies the invariant of q_0.
   Base case holds.

   Induction step:
   We have seven transitions to check.

I: Assume some string w satisfies the invariant of q_0
if 0 transition is added which will give us w0 = 0, which satisfies the state invariant of q_1
if 1 transition is added which will give us w1= 1, which satisfies the state invariant of q_3

II: Assume some string w satisfies the invariant of q_1, then it must begin with 0.
if transition 1 is added, it will give us $w1 = 01$, which satisfies the state invariant of q_2
if transition 0 is added, then $w0 = 00$, satisfies the state invariant of q_1

III: Assume some string w satisfies the state invariant of q_2, then it must begin with 0 and ends with 1
if transition 0 is added, w0 now begins with 0 and ends with 0, so it goes back to q_1, and satisfies the state invariant of q_1
if transition 1 is added, w1 still begins with 0 and ends with 1, so it stays at q_2 and satisfies the state invariant of q_2

IV: Assume some string w satisfies the state invariant of q_3, then it must begin with 1
if transition 1 and transition 0 is added, it will still give us a string that begins with 1, which satisfies the state invariant of q_3

Lastly, show that the state invariant of the accepting state exactly describe the languages that we want the DFA to accept. There is one accepting state which is q_2. If the string begins with 0 and ends with 1, it will satisfy the state invariant of q_2.

2. Suppose for contradiction that we can find a correct DFA that has 3 states, then let
$w_0 = \epsilon$
$w_1 = 1$
$w_2 = 0$
$w_3 = 01$
For one of the pairs of strings, the supposed 3-state DFA is forced into the same state for both strings (by the Pigeonhole principle), and $w_i x$ and $w_j x$

must be both accepted or both rejected, for any string x. We will show, for each possible pair, that this is NOT true.

pair 1:
$w_0 = \epsilon$ and $w_1 = 1$, $x = 01$:
$w_0 x = 01$, accepted
$w_1 x = 101$, rejected
contradiction

pair 2:
$w_0 = \epsilon$ and $w_2 = 0$, $x = 1$:
$w_0 x = 1$, rejected
$w_2 x = 01$, accepted
contradiction

pair 3:
$w_0 = \epsilon$ and $w_3 = 01$:
no need to choose an x
w_0 is rejected
w_3 is accepted
contradiction

pair 4:
$w_1 = 1$ and $w_2 = 0$, $x = 1$:
$w_1 x = 11$, rejected
$w_2 x = 01$, accepted
contradiction

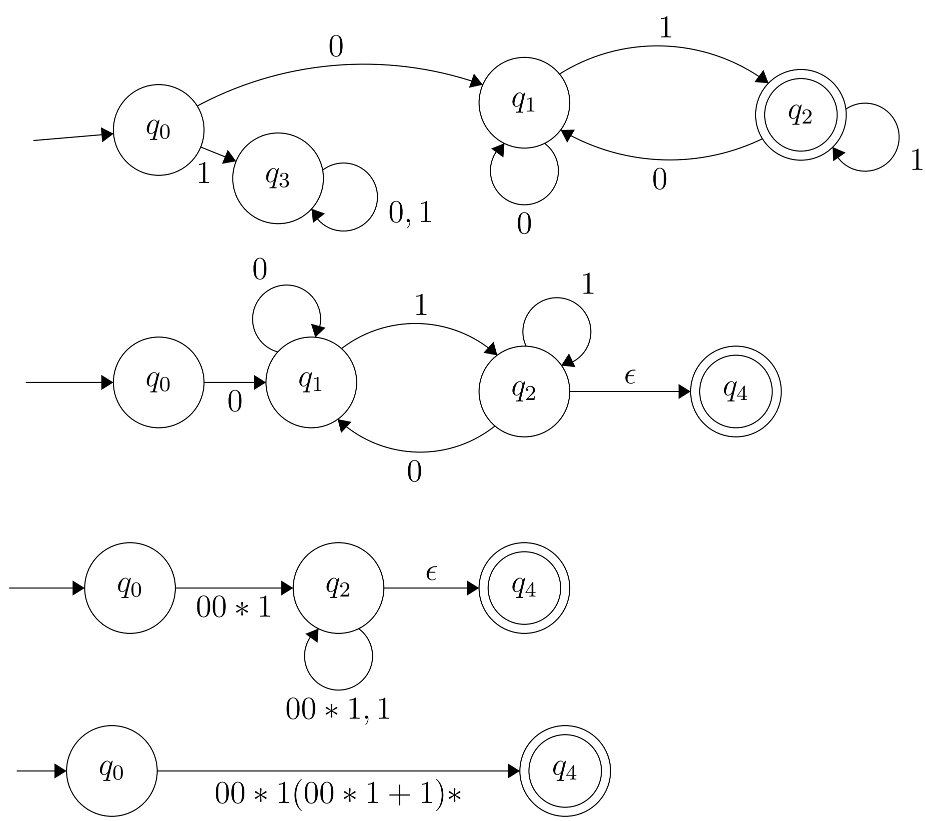
pair 5:
$w_1 = 1$ and $w_3 = 01$:
no need to choose an x
w_1 is rejected
w_3 is accepted
contradiction

pair 6:
$w_2 = 0$ and $w_3 = 01$:

no need to choose an x
w_2 is rejected
w_3 is accepted
contradiction

$\therefore$ None of these pairs is both accepted nor rejected.
$\therefore$ There are at least 4 states.

3.



Problem 3.

(4 Marks) Use the subset construction algorithm from lecture to produce an equivalent DFA from the following NFA. Please show your work so that we know how each state is generated. Name your DFA states so that the link to the NFA states is clear; i.e. you should have DFA states that look like $\{q_0, q_1\}$.

Old State	Symbol	New State
q_0	0	q_2
q_0	0	q_3
q_0	1	q_1
q_0	1	q_4
q_1	0	q_2
q_1	1	q_1
q_2	0	q_1
q_2	1	q_2
q_3	0	q_3
q_3	1	q_4
q_4	0	q_4
q_4	1	q_5
q_5	0	q_5

The start state of this NFA is q_0; the accepting (final) states are q_0, q_1, and q_5.

Answer:
$\{q_0\}$–ϵ–$\{q_0\}$(initial state of the DFA)
$\{q_0\}$–0–$\{q_2,q_3\}$(new state!)
$\{q_0\}$–1–$\{q_1,q_4\}$(new state!)
$\{q_2,q_3\}$–0–$\{q_1,q_3\}$(new state!)
$\{q_2,q_3\}$–1–$\{q_2,q_4\}$(new state!)
$\{q_1,q_4\}$–0–$\{q_2,q_4\}$
$\{q_1,q_4\}$–1–$\{q_1,q_5\}$(new state!)
$\{q_1,q_3\}$–0–$\{q_2,q_3\}$
$\{q_1,q_3\}$–1–$\{q_1,q_4\}$
$\{q_2,q_4\}$–0–$\{q_1,q_4\}$
$\{q_2,q_4\}$–1–$\{q_2,q_5\}$(new state!)
$\{q_1,q_5\}$–0–$\{q_2,q_5\}$
$\{q_1,q_5\}$–1–$\{q_1\}$(new state!)
$\{q_2,q_5\}$–0–$\{q_1,q_5\}$
$\{q_2,q_5\}$–1–$\{q_2\}$(new state!)

$\{q_1\}$–0–$\{q_2\}$
$\{q_1\}$–1–$\{q_1\}$
$\{q_2\}$–0–$\{q_1\}$
$\{q_2\}$–1–$\{q_2\}$