University of Toronto
CSC343 Fall 2014

In-class Exercises: Functional Dependencies

Suppose we have a relation R with attributes $ABCD$

1. **What an FD means.** Suppose the functional dependency $BC \rightarrow D$ holds in R. Create an instance of R that violates this FD.

   Solution:

   In order to violate this FD, we need two tuples with the same value for B and the same value for C (both!), yet different values for D.

A	B	C	D
1	3	6	4
2	3	6	5

2. **Equivalent sets of FDs.**

   (a) Are the sets $A \rightarrow BC$ and $A \rightarrow B, A \rightarrow C$ equivalent? If yes, explain why. If no, construct an instance of R that satisfies one set of FDs but not the other.

   Solution:

   These are equivalent — there is no instance of the relation that satisfies one but not the other. This can be proven, as follows:

   - Assume that $A \rightarrow BC$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \rightarrow B$, and $A \rightarrow C$.
   - Assume that $A \rightarrow B$, and $A \rightarrow C$.
     - Under this assumption, $A^+ = ABC$.
     - Therefore $A \rightarrow BC$.
   - Therefore each set of FDs follows from the other. They are equivalent.

   (b) Are the sets $PQ \rightarrow R$ and $P \rightarrow Q, P \rightarrow R$ equivalent? If yes, explain why. If no, construct an instance of R that satisfies one set of FDs but not the other.

   Solution:

   These are *not* equivalent, as demonstrated by this instance that satisfies $PQ \rightarrow R$ but not $P \rightarrow Q, P \rightarrow R$:

P	Q	R
1	2	4
3	2	5

   In fact we can always "split the RHS" of an FD.

3. **Keys and FDs.**

   (a) We claimed that if a set of attributes K functionally determines all attributes, K must be a superkey (*i.e.*, no two tuples can agree on all attributes in K). Do you believe this? Suppose these FDs hold in R: $A \rightarrow BC$, $C \rightarrow D$. Does A functionally determine all attributes of R? Can two tuples agree on A?

   Solution:

   This is left as an exercise to explore on your own, in order to build your intuition.

(b) We also said that if K is a superkey (*i.e.*, no two tuples can agree on all attributes in K) K must functionally determine all attributes. Do you believe this? Suppose A is a superkey of R Does A functionally determine all attributes of R?

Solution:

Again, this is left as an intuition-building exercise.

4. **Does an FD follow from a set of FDs?** Suppose we have a relation on attributes $ABCDEF$ with these FDs:

$$AC \to F, CEF \to B, C \to D, DC \to A$$

(a) Does it follow that $C \to F$?

(b) Does it follow that $ACD \to B$?

Solution:

We use the closure test to check whether an FD follow from a set of FDs.
$C^+ = CDAF$. Therefore, $C \to F$ does follow.
$ACD^+ = ACDF$. Therefore, $ACD \to B$ does not follow.

5. **Projecting a set of FDs onto a subset of the attributes.** Suppose we have a relation on attributes $ABCDE$ with these FDs:

$$A \to C, \quad C \to E, \quad E \to BD$$

(a) Project the FDs onto attributes ABC.

Solution:

To project onto a set of attributes, we systematically consider every possible LHS of an FD that might hold on those attributes.

- $A^+ = ACEBD$, therefore $A \to BC$. (It also functionally determines DE, but these are not in our set of attributes. And it functionally determines itself, but we don't need to write down dependencies that are tautologies.)
- $B^+ = B$. This yields no FDs for our set of attributes.
- $C^+ = CEBD$, therfore $C \to B$.
- We don't need to consider any supersets of A. A already determines all of our attributes ABC, so supersets of A will be only yield FDs that already follow from $A \to BC$.
- The only remaining subset of the attributes ABC to consider is BC. $BC^+ = BCED$. This yields no FDs for our set of attributes.
- So the projection of the FDs onto ABC is: $\{A \to BC, \quad C \to B\}$.

(b) Project the FDs onto attributes ADE.

Solution:

- $A^+ = ACEBD$, therefore $A \to DE$.
- $D^+ = D$. This yields no non-trivial FDs..
- $E^+ = EBD$, therfore $E \to D$.
- Again, we don't need to consider any supersets of A, since A determines all the attributes ADE also.
- The only remaining subset of the attributes ABC to consider is DE. $DE^+ = DEB$. This yields no FDs for our set of attributes.
- So the projection of the FDs onto ADE is: $\{A \to DE, \quad E \to D\}$.