Homework 2 Solutions

Ex 1, Page 78

Yes, this is an instance of Modus Ponens

Ex 2, Page 78

Yes, this is an instance of Modus Tollens

Ex 5, Page 78

Let H: Randy works hard
D: Randy is a dull boy
J: Randy gets the job

$$\begin{array}{l} H \\ H \rightarrow D \\ D \rightarrow \neg J \\ \hline \neg J \end{array} \quad ?$$

This argument is valid since we can conclude D by modus ponens and $\neg J$ by modus ponens.

Alternatively, through hypological sylogism, we have that $H \rightarrow \neg J$, and since we have H, through modus ponens, we have $\neg J$.

Ex 6. Page 78

Let:

R :	It rains	
F :	It is foggy	
S :	Sailing race will be held	
L :	Life saving demo will be held	
T :	Trophy will be awarded	

Show that:

$(\neg R \vee \neg F) \rightarrow (S \wedge L)$

$S \rightarrow T$

$\neg T$

R

1. $(\neg R \vee \neg F) \rightarrow S$ (Simplification)
2. $\neg S$ (Modus Tollens)
3. $\neg(\neg R \vee \neg F)$ (Modus Tollens)
4. $R \wedge F$ (De Morgan)
5. R (Simplification)

Ex 1, Page 91.

Must show that the sum of two odd integers is even.

Let $a = 2n+1$
$b = 2k+1$

$$
\begin{aligned}
a+b &= 2n+1+2k+1 \\
&= 2n+2k+2 \\
&= 2(n+k+1)
\end{aligned}
$$

$\therefore$ $a+b$ is even

Ex 2, Page 91

Must show that the sum of two even integers is even.

Let $a = 2n$
$b = 2k$

$$
\begin{aligned}
a+b &= 2n+2k \\
&= 2(n+k)
\end{aligned}
$$

$\therefore$ $a+b$ is even

Ex 3, page 91

Must show that the square of an even number is even

let $a = 2n$

$$a^2 = (2n)^2$$
$$= 4n^2$$
$$= 2(2n^2)$$

$\therefore$ a^2 is an even number

Ex 6, Page 91

Must show that the product of two odd numbers is odd.

let $a = 2n+1$
$b = 2k+1$

$$ab = (2n+1)(2k+1)$$
$$= 4nk + 2n + 2k + 1$$
$$= 2(2nk + n + k) + 1$$

$\therefore$ ab is odd.

Ex 9, page 91

Must show that the sum of a rational number and an irrational number is irrational

Let's assume that the sum of a rational and an irrational number is rational. We therefore have:

$I + R = P$, where I is irrational, R is rational, and P is rational

$\therefore I + \frac{r}{s} = \frac{p}{q}$

$I = \frac{p}{q} - \frac{r}{s}$

$= \frac{ps - qr}{qs}$

$\therefore I$ is rational.

This is a contradiction, therefore our assumption is invalid

$\therefore P$ is irrational

Ex 11, Page 91

Must prove or disprove that the product of two irrational numbers is irrational.

Proof: We know that $\sqrt{2}$ is an irrational number.

So $\sqrt{2} \cdot \sqrt{2}$ is a product of two irrational numbers, which is itself a rational number

Therefore by counter example; the product of two irrational numbers is not always irrational, so we have disproved the claim.

Ex 17a), Page 91

Must show if n^3+5 is odd then n is even, where $n \in \mathbb{Z}$.

Proof by contraposition: We have to prove that $ODD(n^3+5) \rightarrow EVEN(n)$. Taking the contrapositive:

$ODD(n) \rightarrow EVEN(n^3+5)$

Let $n = 2k+1$

$$
\begin{aligned}
n^3+5 &= (2k+1)^3 + 5 \\
&= 8k^3 + 12k^2 + 6k + 1 + 5 \\
&= 8k^3 + 12k^2 + 6k + 6 \\
&= 2(4k^3 + 6k^2 + 3k + 3)
\end{aligned}
$$

$\therefore$ n^3+5 is even, $\therefore$ $ODD(n^3+5) \rightarrow EVEN(n)$

Ex 17b, Page 91

Must show that if n^3+5 is odd, then n is even:

Proof by contradiction:

Assume n is odd, So let $n = 2k+1$

$$\begin{aligned} n^3+5 &= (2k+1)^3+5 \\ &= 8k^3+12k^2+6k+1+5 \\ &= 8k^3+12k^2+6k+6 \\ &= 2(4k^3+6k^2+3k+3) \end{aligned}$$

$\therefore$ n^3+5 is even, this is a contradiction since we know that n^3+5 is odd.

$\therefore$ Our assumption that n is odd is invalid

$\therefore$ n is even.

Ex 18a, page 91

Must show that if $3n+2$ is even then n is even, where $n \in \mathbb{Z}$.

Proof by contraposition.

We must show that $\text{EVEN}(3n+2) \rightarrow \text{EVEN}(n)$

Take the contrapositive:

$$\text{ODD}(n) \rightarrow \text{ODD}(3n+2)$$

Let $n = 2k+1$

$$\begin{aligned}
3n+2 &= 3(2k+1)+2 \\
&= 6k+3+2 \\
&= 6k+5 \\
&= 6k+4+1 \\
&= 2(3k+2)+1
\end{aligned}$$

$\therefore$ $3n+2$ is odd

$\therefore$ $\text{EVEN}(3n+2) \rightarrow \text{EVEN}(n)$

Ex 18b, Page 91

Must show that if $3n+2$ is even, then n is even.

Proof by contradiction:

Assume n is odd, so let $n = 2k+1$

$$
\begin{aligned}
3n+2 &= 3(2k+1)+2 \\
&= 6k+3+2 \\
&= 6k+5 \\
&= 6k+4+1 \\
&= 2(3k+2)+1
\end{aligned}
$$

$\therefore$ $3n+2$ is odd. This is a contradiction. So our assumption that n is odd is invalid.

$\therefore$ n is even.