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Subject nr 3

Semnatura: Doroteea



$$A. \int_0^{\infty} \frac{\sin^2 x}{x} dx = \int_0^{\infty} \sin x \cdot \frac{\sin x}{x} dx$$

$x \in \mathbb{R}$ Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be $f(x) = \sin x$

$$f(x+\pi) = f(\pi-x)$$

$$\begin{aligned} \sin^2 x &= \frac{1 - \cos 2x}{2} \Rightarrow \sin^2 x = \frac{\cos(0 \cdot x) - \cos(2x)}{2} = \\ &= \frac{\cos(0 \cdot x)}{2} - \frac{\cos(2 \cdot x)}{2} = \end{aligned}$$

$$\sin^2 x = f(0) - f(2 \cdot x)$$

First Trapezium Formula: $\int_0^{\infty} f(ax)$

B. $D = \{(x, y) \in \mathbb{R}^2 \mid \text{et}$

$$\left. \begin{array}{l} x, y > 0 \\ x \leq y^3 \leq 2x \\ x^3 y^2 \leq 3 \end{array} \right\}$$

$$\Rightarrow \sqrt[3]{x} \leq y \leq \sqrt[3]{2x}$$

$$x^3 y^2 \leq 3 \Leftrightarrow x \leq \frac{3}{y^2} \sqrt[3]{\frac{3}{y^2}} \Leftrightarrow x \leq \frac{3}{y^2} \sqrt[3]{\frac{3}{y^2}}$$

$$\iint_D x^2 y \, dx \, dy = \int_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} \int_0^{\sqrt[3]{2x}} x^2 y \, dx \, dy = \int_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} \left[\frac{x^3 y}{3} \right]_0^{\sqrt[3]{2x}} dy = \int_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} \frac{x^3 \sqrt[3]{2x}}{3} dy = \frac{x^3 \sqrt[3]{2x}}{3} \left[y \right]_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} = \frac{x^3 \sqrt[3]{2x}}{3} \cdot \sqrt[3]{2x} = \frac{x^3 \sqrt[3]{2x}^2}{3}$$

$$= \int_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} \frac{x^3 \sqrt[3]{2x}^2}{3} dy = \frac{2x}{3} \int_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} dy = \frac{2x}{3} \left[y \right]_{\sqrt[3]{2x}}^{\sqrt[3]{2x}} = \frac{2x}{3} \cdot \sqrt[3]{2x} = \frac{2x \sqrt[3]{2x}}{3}$$