3. a)

x	0	1
S_1	$S_2/0$	$S_3/0$
S_2	$S_1/0$	$S_4/1$
S_3	$S_5/0$	$S_6/1$
S_4	$S_2/1$	$S_3/1$
S_5	$S_6/1$	$S_4/1$
S_6	$S_4/0$	$S_6/0$

b)

x	0	1
S_1	$S_2/0$	$S_3/0$
S_2	$S_4/0$	$S_4/1$
S_3	$S_5/0$	$S_6/1$
S_4	$S_2/1$	$S_2/1$
S_5	$S_6/1$	$S_4/1$
S_6	$S_4/0$	$S_6/0$

a) We build the forwards table

state	z=0	z=1
S_1	S_2S_3	-
S_2	S_1	S_4
S_3	S_5	S_6
S_4	-	S_2S_3
S_5	-	S_4S_6
S_6	S_4S_6	-
S_2S_3	S_1S_5	S_4S_6
S_4S_6	S_4S_6	S_2S_3
S_1S_5	S_2S_3	S_4S_6

There are no duplicate states, so the automaton is lossless.

To determine the input sequence we also need the backwards table

z=0	z=1	state
S_2	-	S_1
S_1	S_4	S_2
S_1	S_4	S_3
S_6	S_2S_5	S_4
S_3	-	S_5
S_6	S_3S_5	S_6
S_1S_3	S_4	S_2S_5
S_1S_3	S_4	S_3S_5
S_1S_2	S_4	S_1S_3
S_1S_2	S_4	S_1S_2

$S_2 \rightarrow S_3$

Output seq = 110101

~~Input seq =~~

output	1		1		0		1		0		1	
initial state ↓												Final state
S_2	→	S_4	→	S_2 S_3	→	S_1 S_5	→	S_4 S_6	→	S_4 S_6	→	S_2 S_3 = S_3
$S_2 \in$ S_2 S_4 S_5	←	S_3 S_4 S_5	←	S_1 S_5 S_6	←	S_3 S_4 S_5	←	S_2 S_6	←	S_1 S_4	←	S_3
S_2	→	S_4	→	S_3	→	S_5	→	S_6	→	S_4	→	S_3 - the lines merged

=) Input sequence is: 110001

b) Forwards table:

State	z=0	z=1
S_1	S_2S_3	-
S_2	S_1	S_4
S_3	S_5	S_6
S_4	-	S_2S_2
S_5	-	S_4S_6
S_6	S_4S_6	-
	...	

→ We found duplicates, so the automaton has losses and there is no point in continuing the problem

table could have continued but we have already found a duplicate