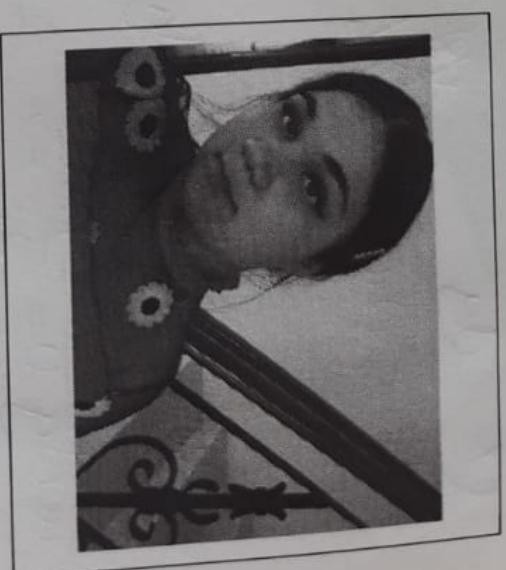


GRUPA =	30 413	NUME =	SANDOR	PRENUME =	DOROTEEA
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Subject nr	1
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Semnatura: _____



A) If $A: \mathcal{C}(\mathcal{A}) \rightarrow \mathbb{R}$ is a FZF, $A(1)=1$, and the functions f and $g \in \mathcal{C}(\mathcal{A})$ are synchron. monotonic (have the same monotony on each subset of \mathcal{A}), then $A(fg) \geq A(f) \cdot A(g)$

$$\Rightarrow (f(x) - f(y)) \cdot (g(x) - g(y)) \geq 0, \forall x, y \in [a, b]$$

$$\Rightarrow f(x) \cdot g(x) - 1 - f(x) \cdot g - g(x) \cdot f + f \cdot g \geq 0, \forall x \in D$$

$$\Rightarrow f(x) \cdot g(x) - A(1) - f(x) \cdot A(g) - g(x) \cdot A(f) + A(f \cdot g)$$

$$\Rightarrow f \cdot g - f A(g) - g A(f) + A(f \cdot g) \geq 0 \cdot 1 \geq 0$$

$$\Rightarrow A(f \cdot g) - A(f) \cdot A(g) - A(f) \cdot A(g) + A(f \cdot g) \geq 0$$

$$\Rightarrow A(f \cdot g) = A(f) \cdot A(g)$$

$$\int_a^b f(x)g(x) dx \geq \frac{1}{b-a} \int_a^b f(x) dx \cdot \int_a^b g(x) dx$$

We apply Chebyshev: $A(fg) = \frac{1}{b-a} \int_a^b f(x)g(x) dx$

$$2) \lim_{x \rightarrow \infty} \frac{1}{x} \int_0^x \frac{dy}{5+4 \cos y}$$

$$\int_0^x \frac{dy}{5+4 \cos} = \int_0^x \frac{1}{6}$$

$$u = \log \frac{y}{2} \quad | \quad \text{arctg}$$

$$\text{arctg } u = \frac{y}{2} \Rightarrow 2 \text{ arctg } u = y \Rightarrow$$

$$\frac{2}{1+u^2} du = dy$$

$$I \quad y=0 \Rightarrow u=0$$

$$II \quad y=x \Rightarrow u = \log \frac{x}{2}$$

$$\cos y = \frac{1 - \left(\log \frac{x}{2} \right)^2}{1 + \left(\log \left(\frac{x}{2} \right) \right)^2} = \frac{1-u^2}{1+u^2}$$

$$\int_0^x \frac{1}{5+4 \cdot \frac{1-u^2}{1+u^2}} \cdot 2 \cdot \frac{1}{1+u^2} du$$