Analog measuring instruments

Classical Electrical Measurements

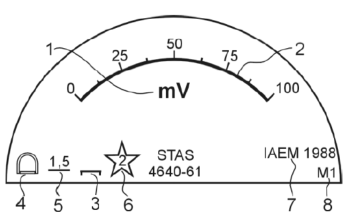
- **Indicating analog meters**
- **Analog Meters-Classifications and Symbols.**
- **Types of analog instruments.**
  - Moving Coil Instruments
  - Moving Iron Instruments
  - Electrodynamic Instruments
- **Voltmeters. Ammeters, Wattmeters, Ohmmeters**
- **Measurements with Bridges**
  - Wheatstone Bridge. Principle. Applications
  - The Balanced and the Unbalanced DC Bridges
  - Stress, Strain, and Strain Gages
  - Types of AC bridges

Indicating analog meters

- A pointer is moving along a graduated scale
- Continuous varying readout
- Easy to track changes in meter reading
- Analog multimeters are sometimes referred to as "volt-ohm-meters", abbreviated **VOM**.
- They are simple, reliable and they have low price
- Indicating instruments:
  - Electromechanical type
  - Electronic type

Analog meters. Classifications and symbols

Analog meter panel

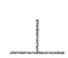


Analogical instrument scale, with conventional symbols

1. The parameter to be measured (voltage, current…)

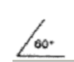
2.The scale (linear, square, logarithmic)

3. Scale position



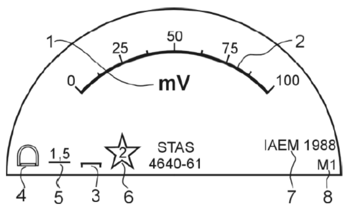
vertical

horizontal



inclined

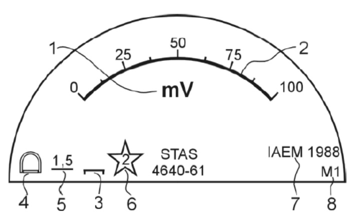
Analog meter panel



Analogical instrument scale, with conventional symbols

4. Conventional symbol of the instrument

	Permanent-magnet moving-coil instrument		Vibrating-reed instrument
	Permanent-magnet ratiometer		(*) Non-insulated thermocouple (thermal converter)
	Moving permanent-magnet instrument		(*) Insulated thermocouple (thermal converter)
	Moving permanent-magnet ratiometer		(*) Electronic device in a measuring circuit
	Moving-iron instrument		(*) Rectifier
	Polarized moving-iron instrument		Shunt
	Moving-iron ratiometer	R	Series resistor
	Ironless electrodynamic instrument		Electric screen
	Iron-cored electrodynamic (ferro-dynamic) instrument		Magnetic screen
	Ironless electro-dynamic ratiometer	ast	Astatic instrument
	Iron-cored electrodynamic (ferro-dynamic) ratiometer		Earth (ground) terminal (general symbol)
	Induction instrument		Zero (span) adjuster



Analogical instrument scale, with conventional symbols

5. Class of accuracy and the nature of measured quantity

1.5	2	1.5
dc	ac	dc and ac

6. Testing voltage for electric isulation –kV (Instrument safety)

☆	Test voltage 500 V
☆ 2	Test voltage above 500 V (e.g. 2 kV)
☆ 0	Apparatus not subjected to a voltage test

Analog meters principles

x- measured quantity (electrical energy)
a- pointer deflection (mechanical energy)

$$\boxed{x} \xrightarrow{force} F \xrightarrow{torque} M_d \xrightarrow[M_r]{balance} M_d + M_r = 0 \ \boxed{\rightarrow \alpha}$$

M_d – **Deflecting Torque**

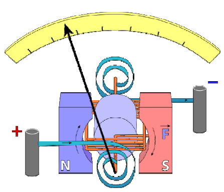
$$M_d = f(x)$$
$$M_d = f(x, \alpha)$$

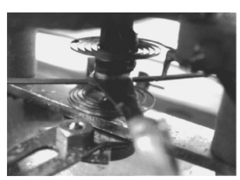
M_r – **Controlling (restoring) Torque**

Spring control
Gravity control

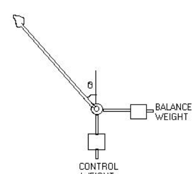
Spring control- is twisted in opposite direction

$$M_r = -D\alpha$$
$$D - specific \quad restoring \quad torque$$





Gravity control



When the moving system **is in motion** there are also:

M_{da} – **Damping Torque** $\quad M_{da} = -F\dfrac{d\alpha}{dt}$

•F- is the specific damping torque
•The pointer comes to its final position without overshooting

M_f – **Friction Torque** $\quad M_f = k_f G^n$

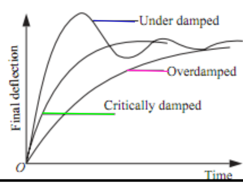
k_f-friction coefficient
G- movement weight

M_J – Inertia Torque $\quad M_J = -J\dfrac{d^2\alpha}{dt^2}$

J-moment of inertia

The general equation of the moving mechanism in dynamic conditions:

$$M_d + M_r \pm M_f + M_J + M_{da} = 0 \qquad J\frac{d^2\alpha}{dt^2} + F\frac{d\alpha}{dt} + D\alpha \pm M_f = f(x,\alpha)$$



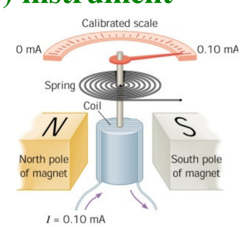
The balance condition will be:

$$\boxed{M_d + M_r = 0}$$

Types of analog instruments

Permanent-magnet moving-coil (PMMC) instrument

$M_d = k_d \cdot I$ DC-direct current $\quad K_d$-dynamic constant

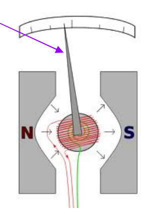


$k_d = NBS$
- N-number of wire
- B-induction
- S-coil area

$M_r = -D\alpha$

$M_d + M_r = 0 \Leftrightarrow k_d I - D\alpha = 0$

α-angle of rotation



The permanent deflection will be: $\boxed{\alpha_p = \dfrac{k_d}{D}\cdot I}$

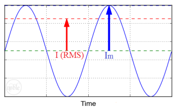
The sensitivity of this instrument: $\boxed{S_I = \dfrac{d\alpha}{dI} = \dfrac{k_d}{D}}$

Finally: $\boxed{\alpha_p = S_I \cdot I}$ $\longrightarrow$ This instrument type has a *linear* scale

These instruments work only in d.c !!!

Moving coil instruments with rectifiers

Considering a PMMC instrument connected to an AC (alternative current) power supply :

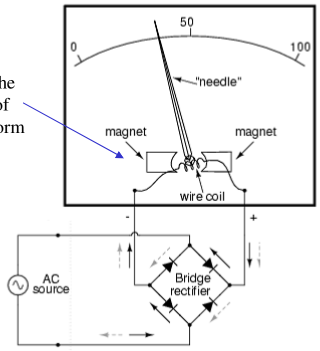


RMS (root mean square)

$$I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt}$$

$$i = I_m \sin \omega t = I\sqrt{2}\sin \omega t$$

meter displays the **average value** of the input waveform



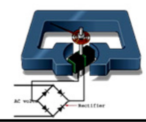
The instantaneous torque is:

$$m_d = k_d i$$

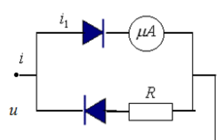
The average (mean) torque for a full period will be ***zero***:

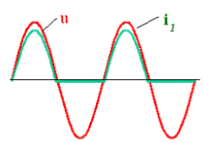
$$M_{d\,average} = \frac{1}{T}\int_0^T m_d dt = \frac{1}{T}k_d\int_0^T I_m \sin \omega t\, dt = 0$$

The solution is the ***rectifier*** instrument.



Moving-coil instrument with half-wave rectifier (1)





The ***average rectified value (arv)*** of the current that flows through the instrument coil is:

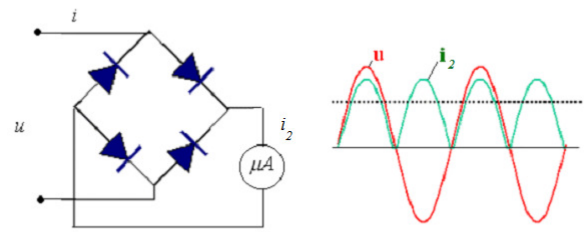
$$I_{arv1} = \frac{1}{T}\int_0^T i_1\, dt \qquad \text{where} \qquad i_1 = \begin{cases} \sqrt{2}I \sin \omega t, \cdots t \in \left[0, \frac{\pi}{2}\right] \\ 0 \qquad\qquad \ldots t \in \left[\frac{\pi}{2}, \pi\right] \end{cases}$$

Half wave rectified (1)

$$I_{arv1} = \frac{\sqrt{2}}{\pi} I = \frac{1}{2{,}22} I$$

The permanent deflection will be: $\alpha_p = S_I I_{arv1} = \frac{S_I}{2{,}22} I$

Moving-coil instrument with full-wave rectifier



The ***average rectified value (arv)*** of the current that flows through the instrument coil is:

$$I_{arv2} = \frac{1}{T}\int_0^T i_2\,dt = \frac{1}{T}\int_0^T |i|\,dt = \frac{1}{T}\int_0^T \sqrt{2}I|\sin\omega t|\,dt = 2\frac{\sqrt{2}}{\pi}I = \frac{1}{1{,}11}I$$

Full wave rectified (2)

$$I_{arv2} = \frac{1}{1{,}11}I$$

The permanent deflection will be: $\alpha_p = \frac{S_I}{1{,}11}I$

Meters are often calibrated to directly display **r.m.s.** of sinusoidal waves !!!

The form factor of sinusoidal current is:

$$k_f = \frac{rms\ value}{arv\ value}$$

$$k_{f\sin} = \frac{I_{rms}}{I_{avr2}} = \frac{I}{\frac{1}{1.11}I} = 1.11 \Rightarrow k_{f\sin} = 1.11$$

The instruments are often calibrated in rms (effective values) I of sinusoidal wave ($k_{f\,sin}$), so we read:

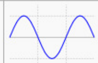
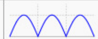
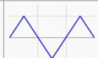
$$\mathrm{I_{read}} = \mathrm{I_{k_f\,sin=1.11}} \longrightarrow \text{r.m.s for 1.11}$$

If we measure another waveform (another form factor k'_f) the result must be adjusted in with a correction factor:

$$correction\ \ factor = \frac{k'_f}{1.11}$$

$$I' = \frac{k'_f}{1.11}\cdot I_{read}$$

Form Factors-Different waveforms

Waveform	Image	RMS	ARV	Form Factor
Sine wave		$\frac{a}{\sqrt{2}}$	$a\frac{2}{\pi}$	$\frac{\pi}{2\sqrt{2}} \approx 1.11072073$
Half-wave rectified sine		$\frac{a}{2}$	$\frac{a}{\pi}$	$\frac{\pi}{2} \approx 1.5707963$
Full-wave rectified sine		$\frac{a}{\sqrt{2}}$	$a\frac{2}{\pi}$	$\frac{\pi}{2\sqrt{2}}$
Square wave, constant value		a	a	$\frac{a}{a} = 1$
Pulse wave $D = \frac{\tau}{T}$ duty cycle		$a\sqrt{D}$	aD	$\frac{1}{\sqrt{D}} = \sqrt{\frac{T}{\tau}}$
Triangle wave		$\frac{a}{\sqrt{3}}$	$\frac{a}{2}$	$\frac{2}{\sqrt{3}} \approx 1.15470054$
Sawtooth wave		$\frac{a}{\sqrt{3}}$	$\frac{a}{2}$	$\frac{2}{\sqrt{3}}$
Gaussian white noise $U(-1,1)$		$\frac{1}{\sqrt{3}}$	$\frac{1}{2}$	$\frac{2}{\sqrt{3}}$

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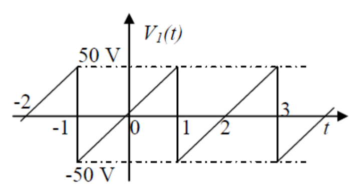
Application: Measuring non-sinusoidal waveforms

The voltage waveform shown in the figure is applied to an AC voltmeter (PMMC instrument with full wave rectifier). The instrument is calibrated to measure voltages with sinusoidal waveform.

a) Find the form factor of the measured waveform

b) Find the voltage that is read on the instrument and find the correction factor



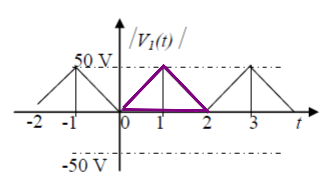
$$\left.\begin{aligned} v_1(t) &= At + B \\ -50 &= -A + B \\ 50 &= A + B \end{aligned}\right\} \Rightarrow A = 50,\ B = 0 \Rightarrow v_1(t) = 50t$$

a) $k_{f_{saw-tooth}} = \frac{V_{rms}}{V_{avr\,2}}$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v_1^2(t)dt} = \sqrt{\frac{1}{2}\int_{-1}^{1}(50t)^2 dt} = \sqrt{\frac{1}{2}\cdot 2500\left.\frac{t^3}{3}\right|_{-1}^{1}} = \sqrt{\frac{1}{2}\cdot 2500\frac{1+1}{3}} = \frac{50}{\sqrt{3}}[V]$$

$$V_{arv2} = \frac{1}{T}\int_0^T |v_1(t)|dt = \frac{1}{2}\frac{2\cdot 50}{2} = 25[V]$$

Aria of triangle



$$k_{f_{saw-tooth}} = \frac{\frac{50}{\sqrt{3}}}{25} = \frac{2}{\sqrt{3}} = 1.154$$

b) $V_{read} = V_{avr\,2}\cdot k_{f\,\sin} = 25\cdot 1.11 = 27.75[V]$

$$correction\ factor = \frac{k'_f}{1.11} = \frac{1.154}{1.11} = 1.04$$

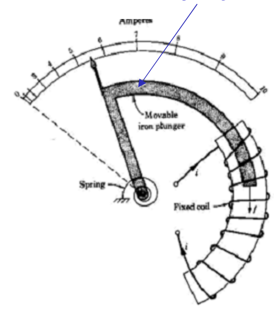
Moving-iron instrument (in ac and dc)

The iron is magnetized by the coil carrying the operating current.

They can be of two types:

- **With attraction**: *the soft iron vane is drawn into the field*
- **With repulsion:** *double iron instrument*

They measure a.c. signals at frequencies up to 125 Hz.



Attraction type

The deflecting torque $M_d = \frac{1}{2}\frac{dL}{d\alpha}I^2$

$\frac{dL}{d\alpha} \longrightarrow$ is the rate of change of the inductance of coil with the rotation of moving iron

$I \longrightarrow$ is the effective value of ac through the coil

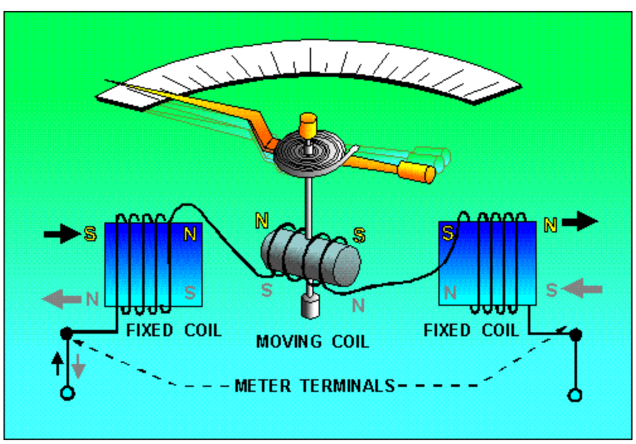
$$M_d + M_r = 0$$

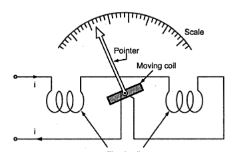
$$\frac{1}{2}\frac{dL}{d\alpha}I^2 - D\alpha = 0 \qquad \boxed{\alpha_p = \frac{1}{2D}\frac{dL}{d\alpha}I^2} \longrightarrow \text{Nonlinear scale}$$

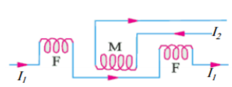
These instruments are used in industrial applications (c=1; 2; 2,5)

Electrodynamic instruments

The most accurate indicating instruments



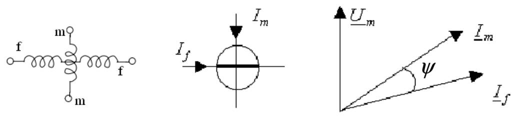




They measure both d.c. signals and a.c. signals up to a frequency of 2 kHz

They are transfer-type instruments.

A transfer instrument is one that may be calibrated with a d.c. source and then used without modification to measure a.c.



The deflecting torque

$$M_d = \frac{dM}{d\alpha} I_f I_m \cos\psi \quad (AC)$$

$$M_d = \frac{dM}{d\alpha} I_f I_m \quad (DC)$$

M is the mutual inductance between the fixed coil and moving coil.

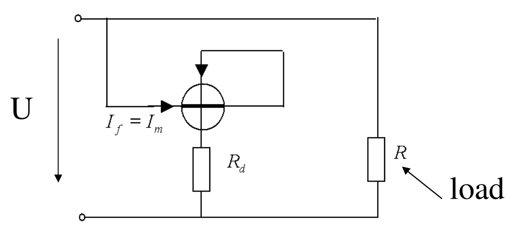
If the restoring torque is $M_r = -D\alpha$ than the permanent deflection of these kind of instruments will be:

$$\alpha_p = \frac{1}{D}\frac{dM}{d\alpha} I_f I_m \cos\psi \quad (AC)$$

$$\alpha_p = \frac{1}{D}\frac{dM}{d\alpha} I_f I_m \quad (DC)$$

These kind of instruments have a high accuracy (c=0,05). They are used like standard instruments. In ac they can be used until 500-1000Hz. They are sensitive to the influence of external magnetic fields, so they have an astatic construction.

Electrodynamic voltmeter



$$I_m = I_f = \frac{U}{R_V}$$

The total resistance R_V of the measuring circuit

$$R_V = R_m + R_f + R_d$$

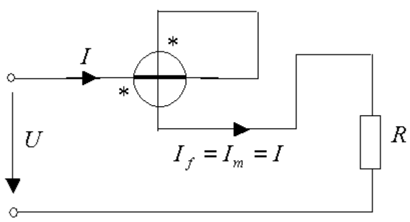
- R_m is the resistance of moving coil
- R_f is the resistance of fixed coil
- R_d is the dropping resistor

The permanent deflection will be

In d.c $\longrightarrow$ $\alpha_p = \frac{1}{D} I_m I_f \frac{dM}{d\alpha} = \frac{1}{D}\frac{U^2}{R_V^2}\frac{dM}{d\alpha} = f(U^2)$ In a.c $\longrightarrow$ $\alpha_p = f'(U^2)$

Electrodynamic ammeter

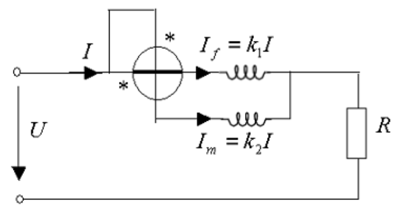
For I< 0.5A



In d.c

$$\alpha_p = \frac{1}{D} I_m I_f \frac{dM}{d\alpha} = \frac{1}{D} I^2 \frac{dM}{d\alpha} = f(I^2)$$

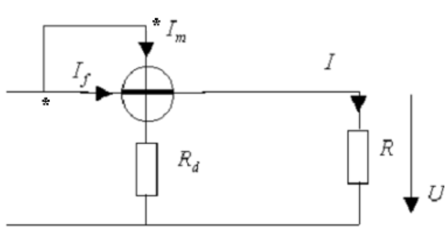
For I> 0.5A



In d.c

$$\alpha_p = \frac{1}{D} I_m I_f \frac{dM}{d\alpha} = \frac{k_1 k_2}{D} I^2 \frac{dM}{d\alpha} = f(I^2)$$

Electrodynamic wattmeter



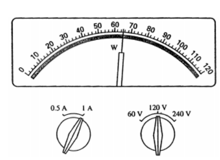
$$I_f = I$$

$$I_m = \frac{U}{R_m + R_d}$$

$$\frac{dM}{d\alpha} = k$$ (Through a proper design)

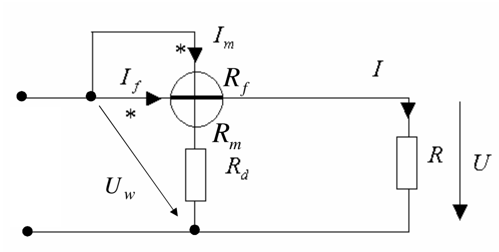
In d.c $\longrightarrow$ $\alpha_p = \dfrac{1}{D} k \dfrac{U}{R_m + R_d} I = K'P$

In a.c $\longrightarrow$ $\alpha_p = \dfrac{1}{D} k \dfrac{U}{R_m + R_d} I \cos\psi = K'P$



The wattmeter constant $\longrightarrow$ $\boxed{C_W = \dfrac{U_n I_n}{\alpha_{max}}}$

Wattmeter connection



Upstream

$$P_R = UI$$

$$U = U_w - R_f I_f$$

$$I = I_f$$

$$P_R = (U_w - R_f I_f) I_f = P_w - R_f I_f^2$$

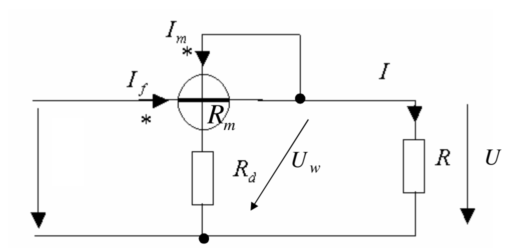
$$P_R = P_w - R_f I^2$$

$$P_R = P_w - R_f I^2$$

$$P_R = P_w - R_f \left(\frac{U}{R}\right)^2$$

$P_R \approx P_w$ → For medium and high resistance values (R)

Wattmeter connection



Downstream

$$P_R = UI$$

$$U = U_w$$

$$I = I_f - I_m = I_f - \frac{U_w}{R_d + R_m}$$

$$P_R = U_w \left(I_f - \frac{U_w}{R_d + R_m}\right)$$

$$P_R = U_w I_f - U_w \frac{U_w}{R_d + R_m}$$

$$P_R = P_w - \frac{U^2}{R_m + R_d}$$

$$P_R = P_w - \frac{(RI)^2}{R_m + R_d}$$

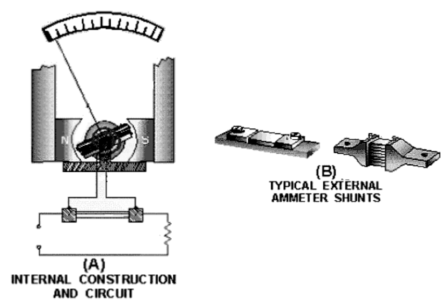
$P_R \approx P_w$ → For low resistance values (R)

Voltmeters, ammeters, ohmmeters

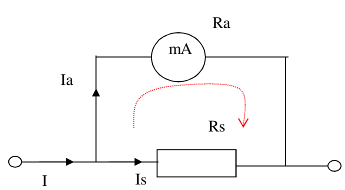
Voltmeter….voltage measurement…parallel connection
Ammeter…current measurement…series connection

Ammeter shunt

A shunt is a **low-resistance conductor connected in parallel (shunt) with the meter terminals**. It is used to carry the majority of the load current.



Internal shunts
External shunts



$$R_a I_a = R_s I_s \quad \Rightarrow R_s = \frac{R_a I_a}{I_s}$$

$$I = I_a + I_s, \quad \Rightarrow I_s = I - I_a$$

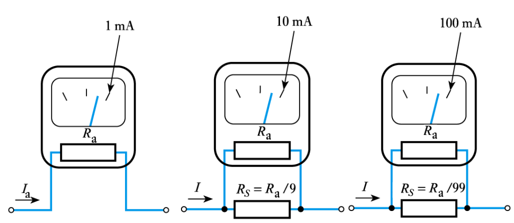
$$R_s = \frac{I_a R_a}{I - I_a} \quad \Rightarrow R_s = \frac{R_a}{\frac{I}{I_a} - 1}$$

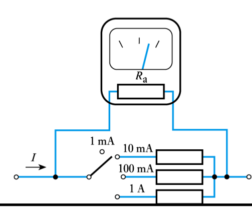
The multiplying power”of the shunt:

$$n = \frac{I}{I_a}$$

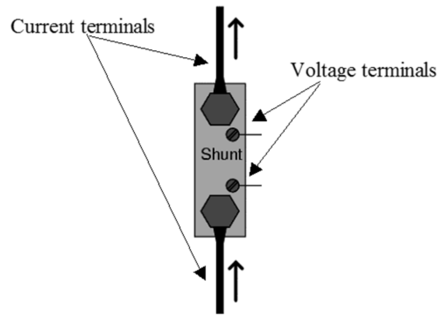
The value of the shunt will be:

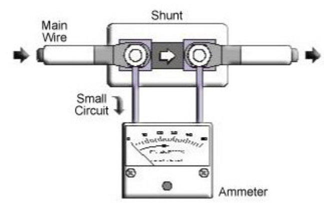
$$\boxed{R_s = \frac{R_a}{n-1}}$$





Ammeter shunt



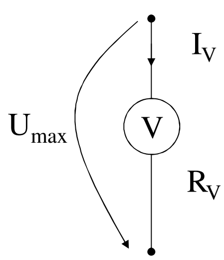


•The shunt resistor's resistance: mΩ or μΩ→

•Resistance this low is comparable to wire connection resistance, which means voltage measured across such a shunt must be done so in such a way as to avoid detecting voltage dropped across wire connections →

•Shunts are usually equipped with four connection terminals

Voltmeter sensitivity



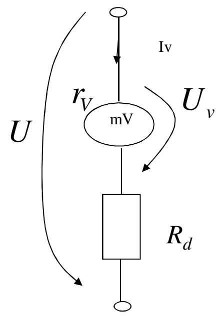
$$S_V = \frac{1}{I_V} = \frac{1}{full - scale\ current}$$

$$S_V = \frac{R_V}{U_{\max}} \qquad [S_V] = \frac{\Omega}{V}$$

$$R_V = S_V \cdot U_{\max}$$

Specific resistance On the scale

Voltmeter dropping resistor

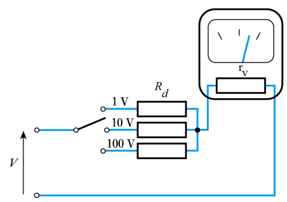


$$U = R_d I_v + U_v$$

$$U = R_d \frac{U_v}{r_V} + U_v$$

$$U = U_v\left(\frac{R_d}{r_V} + 1\right) \quad \Rightarrow R_d = r_V\left(\frac{U}{U_v} - 1\right)$$

Multiplying power $\boxed{m = \frac{U}{U_v}}$

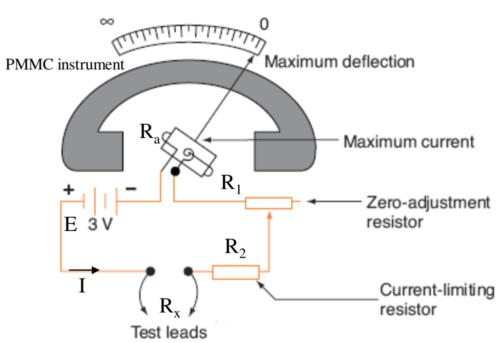


$$R_d = r_V(m-1)$$

Analog ohmmeter

An **ohmmeter** is normally part of a **volt-ohm-milliammeter (VOM)**.
Ohmmeters do not usually exist as individual instruments.

Series ohmmeter: for measuring low values of R_x (10^2-$10^5\Omega$)



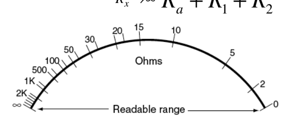
$$I = \frac{E}{R_a + R_1 + R_2 + R_x}$$

Short circuit

$$If \quad R_x = 0 \Rightarrow I_{\max} = \frac{E}{R_a + R_1 + R_2}$$

Open break

$$If \quad R_x = \infty \Rightarrow I_{\min} = \lim_{R_x \to \infty} \frac{E}{R_a + R_1 + R_2} = 0$$



Inverse and non linear scale!!!

Parallel ohmmeter (Shunt type ohmmeter): for measuring **very low** values of R_x (10^{-1}-$10^2\Omega$)

The megohmmeter (MΩ)(Megger): for measuring **very high** resistance →to test the insulation found in power transmission systems, electrical machinery, transformers and so on.

Application :Series ohmmeter

A series ohmmeter is made up of a E=1.5 V battery, a low-current PMMC instrument and $R_a+R_1+R_2=15k\Omega$.
(a) Determine the current in the instrument when $R_x=0$.
(b) Determine how the resistance scale should be marked at 0.5 FS, 0.25 FS, and 0.75 FS. (FS-Full Scale)

(a) $$I=\frac{E}{R_a+R_1+R_2+R_x}=\frac{1.5}{15\cdot 10^3}=100\mu A$$

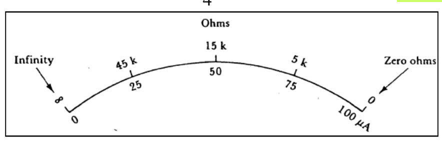
(b) At 0.5FS $\longrightarrow$ $I_{0.5FS}=\frac{100\mu A}{2}=50\mu A$ $\longrightarrow$ $\frac{E}{R_a+R_1+R_2+R_{x1}}=I_{0.5FS}$

$$R_a+R_1+R_2+R_{x1}=\frac{E}{I_{0.5FS}}=\frac{1.5}{50\cdot 10^{-6}}=30k\Omega$$

$$15k\Omega+R_{x1}=30k\Omega \Rightarrow R_{x1}=15k\Omega$$

At 0.25FS $\longrightarrow$ $I_{0.25FS}=\frac{100\mu A}{4}=25\mu A$ $\longrightarrow$ $R_{x2}=45k\Omega$

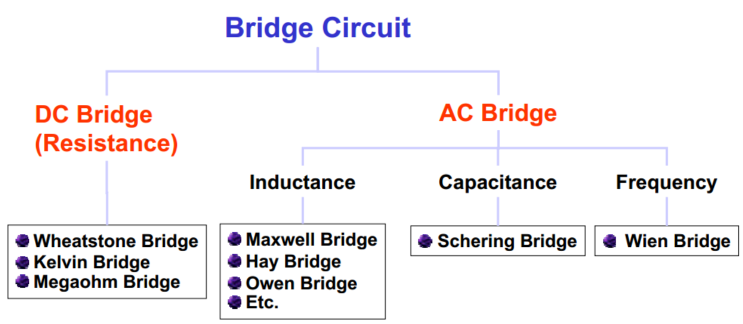
At 0.75FS $\longrightarrow$ $I_{0.75FS}=3\frac{100\mu A}{4}=75\mu A$ $\longrightarrow$ $R_{x3}=5k\Omega$



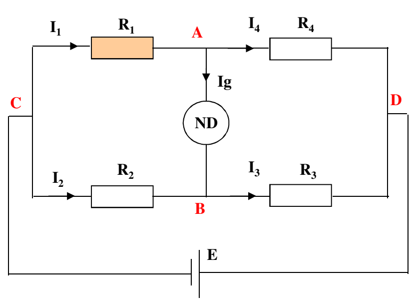
Measurements with Bridges
DC&AC Bridges

Measurements with Bridges

- Bridges use the null measurement method (comparison principle).
- A known (standard) value is adjusted until it is equal to the unknown value.



The balanced DC Wheatstone Bridge



- $R_x = R_1$; R_x-the unknown resistor (range of $1–10^6\Omega$)
- R_4-a calibrated variable resistance
- R_2 and R_3-fixed known resistors
- ND -Null Detector-Galvanometer (PMMC type sensitive ammeter)

Balance condition:

$$U_{AB} = 0 \quad (I_g = 0)$$

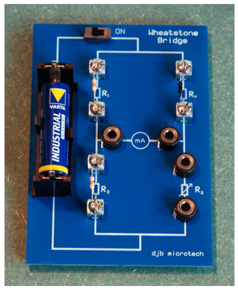
No potential difference across the galvanometer (no current through the galvanometer)

$$U_{AB} = 0 \quad \Leftrightarrow \quad U_{AC} = U_{BC} \quad \Leftrightarrow \quad R_1 \cdot I_1 = R_2 \cdot I_2 \quad (a)$$

$$U_{AB} = 0 \quad \Leftrightarrow \quad U_{AD} = U_{BD} \quad \Leftrightarrow \quad R_4 \cdot I_4 = R_3 \cdot I_3 \quad (b)$$

But: $\quad I_g = 0 \quad \Leftrightarrow \quad I_1 = I_4 \quad I_2 = I_3$

It results: $\quad (a)/(b) \Leftrightarrow \dfrac{R_1}{R_4} = \dfrac{R_2}{R_3} \quad \Rightarrow \quad R_1 \cdot R_3 = R_2 \cdot R_4$



$$R_1 \cdot R_3 = R_2 \cdot R_4$$

The unknown resistor will be:

$$R_x = R_1 = \frac{R_2}{R_3} R_4$$

$\frac{R_2}{R_3} \longrightarrow$ is the *measure ratio* and it gives the order of R_x

$R_4 \longrightarrow$ is the *adjustable resistance* (fine adjustments)

Application: Wheatstone Bridge

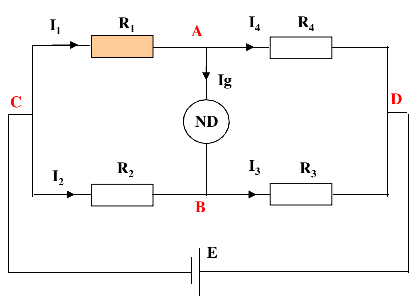
Consider using a Wheatstone bridge having R_2 = 2000 Ω and R_3 = 200 Ω to measure a resistance, R_1 , of a temperature sensor. Suppose the resistance of the temperature sensor, $R_1[\Omega]$, is related to the temperature T[°C], by the equation

$$R_1 = 1500 + 25T$$

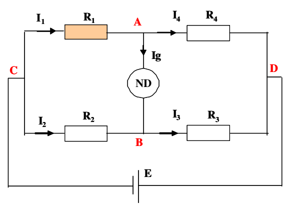
The bridge is balanced by adjusting R_4 until R_4 = 250 Ω. What is the value of the temperature?

$$R_1 = \frac{R_2}{R_3} R_4 = \frac{2000 \cdot 250}{200} = 2500\Omega$$

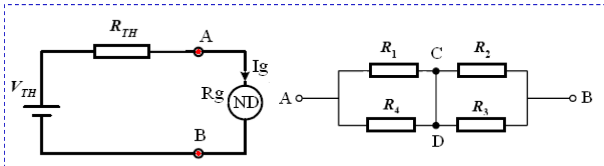
$$T = \frac{R_1 - 1500}{25} = \frac{2500 - 1500}{25} = 40^\circ C$$



The Wheatstone bridge sensitivity



Thévenin Equivalent Circuit



$$I_g = \frac{V_{TH}}{R_{TH} + R_g} \qquad R_{TH} = R_1 \| R_4 + R_2 \| R_3$$

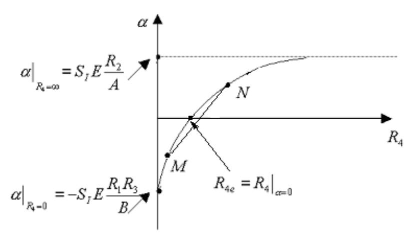
The *permanent deflection* of the ND (galvanometer):

$$\alpha_p = S_I I_g = S_I E \frac{R_2 R_4 - R_1 R_3}{A R_4 + B} \quad where\ A = f_1(R_1, R_2, R_3, R_g);\ B = f_2(R_1, R_2, R_3, R_g)$$

The absolute sensitivity: $\boxed{S = \lim_{\Delta R_4 \to 0} \frac{\Delta\alpha}{\Delta R_4} = S_I E \frac{R_2}{A R_4 + B}}$

The Wheatstone bridge sensitivity

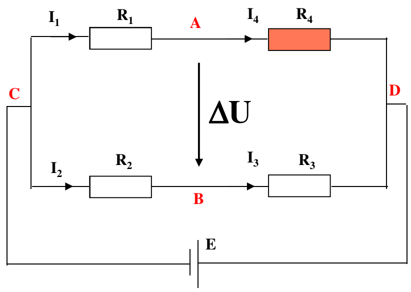
$$\boxed{S = \lim_{\Delta R_4 \to 0} \frac{\Delta\alpha}{\Delta R_4} = S_I E \frac{R_2}{A R_4 + B}}$$



- Bridge sensitivity increase with S_I and with E. The value of **E is limited** because the currents in the bridge branches are limited.
- Bridge sensitivity tends to zero when $R_4 = \infty$. $\Rightarrow$
- **Wheatstone bridge** is used to measure **medium values of resistances.**

- **Kelvin Bridge is used to measure small values** of resistances (0.00001 to 1 Ω)
- **Megaohm Bridge is used to measure high values** of resistances (10^6 to 10^{12} Ω)

The unbalanced Wheatstone bridge

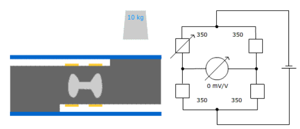


$R_4 \rightarrow$ sensor
- Temperature (thermistor)
- Force (Load cell-strain gauge)
- Pressure (strain gauge)
- Light (photoresistor)
- •piezoresistive sensors
- •Hall sensors

Initially $\Delta U=0$ ($R_1R_3=R_2R_4$) $\longrightarrow$ Balanced bridge

$R_4 \rightarrow R_4+\Delta R$ $\longrightarrow$ $\Delta U \neq 0$ $\longrightarrow$ Unbalanced bridge

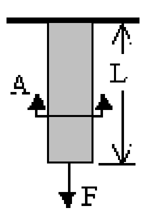
The amount of ΔU is the amount of parameter being measured.
ΔU must be amplified!!!



Load cell

Stress, Strain and strain gages

We consider a wire or cylinder, fixed at the top, and hanging down. A force F is applied.



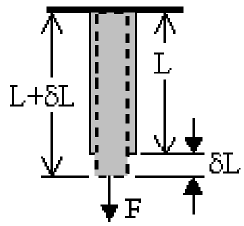
A- original cross sectional area

L-original wire length

The material will experience a stress, called ***axial stress***.

$$\sigma_a = \frac{F}{A} \qquad [\sigma_a] = \frac{N}{m^2} \longrightarrow \text{pressure}$$

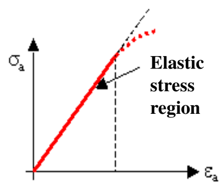
When F increases then L increases and A decreases.



Axial strain (deformation)

$$\varepsilon_a = \frac{\delta L}{L}$$ (nondimensional)

The Hooke's law: For elastic materials→ stress is proportional to strain.



$$\sigma_a = E\varepsilon_a$$

E→Young's modulus (modulus of elasticity)
For a given material is a constant.

The electrical resistance R of a wire: $R = \frac{\rho L}{A}$

We apply logarithm: $\ln R = \ln\rho + \ln L - \ln A$

By differentiating: $\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} - \frac{dA}{A}$

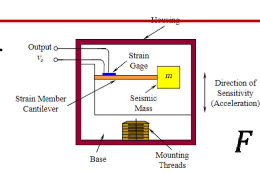
It is demonstrated that:

$$\frac{dR}{R} = S\frac{dL}{L}$$

- The resistance of the wire increases with deformation
- S-strain gauge factor (sensitivity)
- Usually S=2 (2-6 metals, 40-200 semiconductors)
- The strain and stress can be measured

Displacement, acceleration, pressure, temperature, liquid level, stress, force or torque can be determined using ***strain measurements***.

- A cantilever beam fixed to the housing of the instrument.
- A mass is fixed to the free end of the cantilever beam.
- Two bounded strain gauges are mounted on the cantilever beam
- Damping is provided by a viscous fluid filled inside the housing.



$$F = ma$$

A strain gauge accelerometer

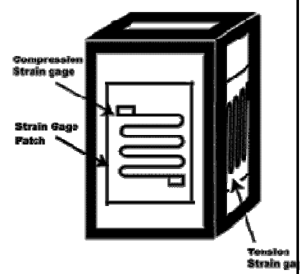
Strain gauges

•Strain gauges are measuring elements that convert force, pressure, tension, etc., into an electrical signal.

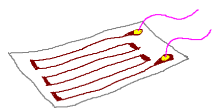
•A strain gauge is a resistive elastic sensor whose resistance is a function of applied strain (unit deformation).

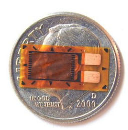
•A Wheatstone bridge converts this change in resistance to an absolute voltage.

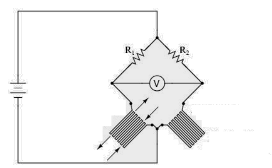
•Most strain gauges are smaller than a postage stamp.



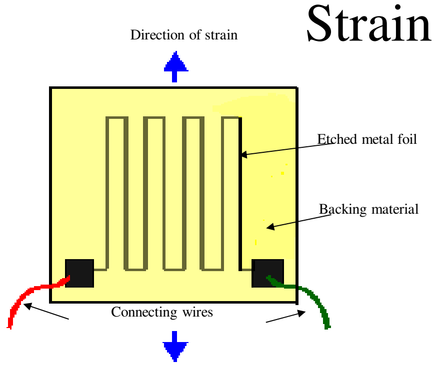
Tension & Compression







Strain gauges



Typically R=120Ω or 350 Ω

gravat

•A ***strain gauge*** consists of a small diameter wire (actually an etched ***metal foil***), which is attached to a ***backing material*** (usually plastic).

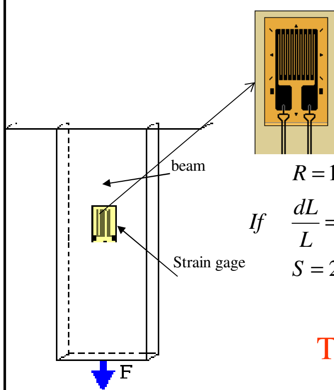
•The wire is looped back and forth several times to create an ***effectively longer wire***. The longer the wire, the larger the resistance, and the larger the change in resistance with strain.

How are strain gauges used?

- Sustained beam →axial strain
- The strain must be measured.
- A strain gauge is glued to the surface of the beam.

$$\frac{dR}{R} = S\frac{dL}{L}$$

- S is the gauge factor
- R is the nominal resistance of the gauge
- dR is the change in resistance due to the strain.



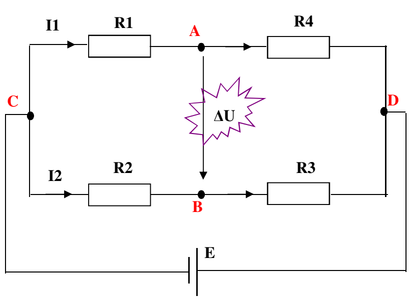
$$If\ \left.\begin{array}{l} R = 120\,\Omega \\ \dfrac{dL}{L} = 10^{-6}\ldots10^{-3} \\ S = 2 \end{array}\right\} \Rightarrow dR = RS\frac{dL}{L} = 0{,}00024\Omega\ldots0{,}24\Omega$$

VERY SMALL

The ohmmeter can not measure it!

SOLUTION: **The Unbalanced** Wheatstone Bridge

The unbalanced Wheatstone bridge



$$\Delta U = V_A - V_B = V_A - V_D - (V_B - V_D) = U_{AD} - U_{BD}$$

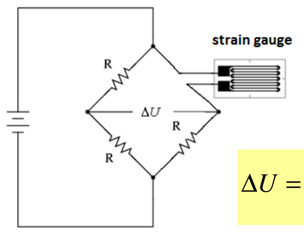
$$U_{AD} = E\frac{R_4}{R_1 + R_4}$$

$$U_{BD} = E\frac{R_3}{R_2 + R_3}$$

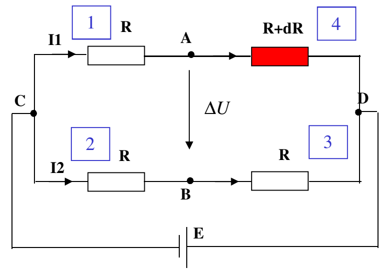
$$\Delta U = E\left(\frac{R_4}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right) = E\frac{R_2R_4 + \cancel{R_3R_4} - R_1R_3 - \cancel{R_4R_3}}{(R_1 + R_4)(R_2 + R_3)}$$

$$\boxed{\Delta U = E\frac{R_2R_4 - R_1R_3}{(R_1 + R_4)(R_2 + R_3)}}$$

The unbalanced Bridge with a single strain gauge (Quarter Bridge)



$$\Delta U = E\frac{R_2R_4 - R_1R_3}{(R_1+R_4)(R_2+R_3)}$$



Considering the bridge *with equal arms* then $R_1 = R_2 = R_3 = R$

Linear response

$$\Delta U = E\frac{R(R+dR) - RR}{(R+R+dR)(R+R)} = E\frac{RdR}{(2R+dR)2R}$$

$$\Delta U = E\frac{dR}{(2R+\underbrace{dR}_{\approx 0})2} \approx E\frac{dR}{4R} \Rightarrow \frac{dR}{R} = 4\frac{\Delta U}{E}$$

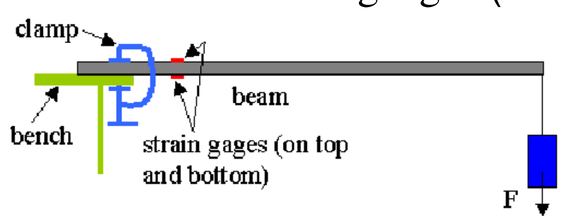
Bridge sensitivity

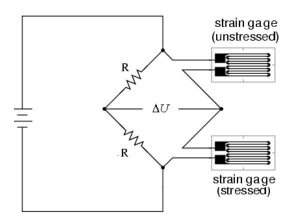
$$S_r = \frac{\Delta U / E}{dR / R} = \frac{1}{4}$$

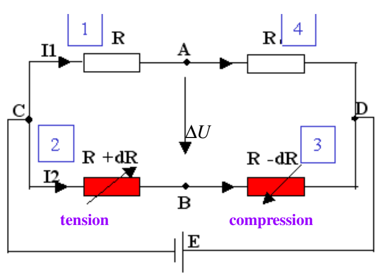
$$\frac{dR}{R} = S\frac{dL}{L} \Leftrightarrow \frac{dL}{L} = \frac{1}{S}\frac{dR}{R}$$

$$\frac{dL}{L} = \frac{1}{S}4\frac{\Delta U}{E}$$

The Unbalanced Bridge with two strain gauges (half bridge)







$$\Delta U = E\frac{R_2R_4 - R_1R_3}{(R_1+R_4)(R_2+R_3)}$$

$$\Delta U = E\frac{(R+dR)R - R(R-dR)}{2R2R} = E\frac{2RdR}{2R2R} = \frac{E}{2R}dR$$

If sensor 1 and sensor 2 are exposed to the pressure but they are exposed to the same temperature variation, is the output changing?

Half bridge

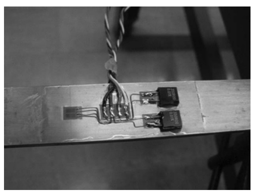
Bridge sensitivity

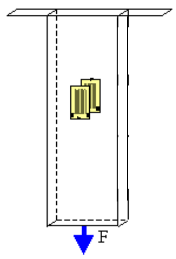
$$S_r = \frac{\Delta U / E}{dR/R} = \frac{1}{2}$$

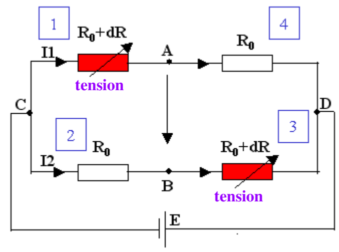
$$\frac{dL}{L} = \frac{1}{S} 2 \frac{\Delta U}{E}$$

Linear response

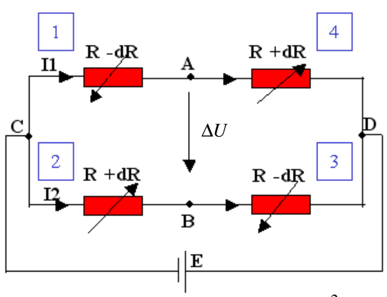
The same result is obtained in the next situation:

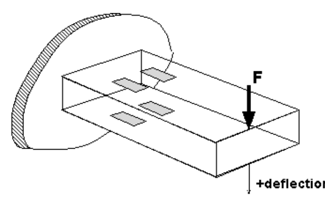






The Unbalanced Bridge with four strain gauges (full bridge)





$$\Delta U = E \frac{R_2 R_4 - R_1 R_3}{(R_1 + R_4)(R_2 + R_3)}$$

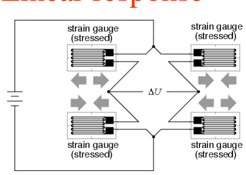
$$\Delta U = E \frac{(R + dR)^2 - (R - dR)^2}{2R2R} = E \frac{2R2dR}{2R2R} = \frac{E}{R} dR$$

Bridge sensitivity

$$S_r = \frac{\Delta U / E}{dR/R} = 1$$

$$\frac{dL}{L} = \frac{1}{S} \frac{\Delta U}{E}$$

Linear response



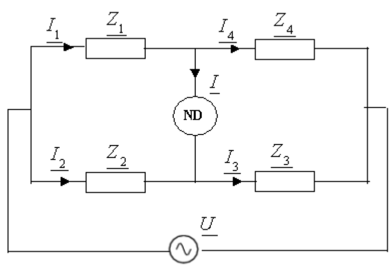
Two force sensors suitable for a bathroom scale, each containing a pair of strain gauges that will flex oppositely under load

AC Bridges (Impedance Bridges)

They are used to measure: inductance L, capacitance C, storage factor (quality factor), loss factor, frequency of audio signals

They can be used also a oscillator circuits

- signal generators
- to determine an unknown circuit parameter by measuring the oscillating frequency



$$\underline{Z}_1 = Z_1 e^{j\varphi 1}$$
$$\underline{Z}_2 = Z_2 e^{j\varphi 2}$$
$$\underline{Z}_3 = Z_3 e^{j\varphi 3}$$
$$\underline{Z}_4 = Z_4 e^{j\varphi 4}$$

ND-null detector-headphone; vibration galvanometer

The ac source is an oscillator of f=40…125Hz

The balance condition $\underline{I} = 0$ leads to the balance equations:

$$\begin{cases} \underline{Z}_1 \cdot \underline{I}_1 = \underline{Z}_2 \cdot \underline{I}_2 \\ \underline{Z}_4 \cdot \underline{I}_4 = \underline{Z}_3 \cdot \underline{I}_3 \\ \underline{I}_1 = \underline{I}_4; \quad \underline{I}_2 = \underline{I}_3; \end{cases} \Leftrightarrow \quad \frac{\underline{Z}_1}{\underline{Z}_4} = \frac{\underline{Z}_2}{\underline{Z}_3} \quad \Leftrightarrow \quad \boxed{\underline{Z}_1 \cdot \underline{Z}_3 = \underline{Z}_2 \cdot \underline{Z}_4}$$

$$\boxed{Z_1 Z_3 e^{j(\varphi_1+\varphi_3)} = Z_2 Z_4 e^{j(\varphi_2+\varphi_4)}}$$ (polar form)

The balance condition will be:

$$\begin{cases} Z_1 \cdot Z_3 = Z_2 \cdot Z_4 & \text{magnitude condition} \\ \varphi_1 + \varphi_3 = \varphi_2 + \varphi_4 & \text{phase condition} \end{cases}$$

$$\boxed{(R_1 + jX_1)\cdot(R_3 + jX_3) = (R_2 + jX_2)\cdot(R_4 + jX_4) \Rightarrow}$$ (Cartesian form)

The balance condition will be:

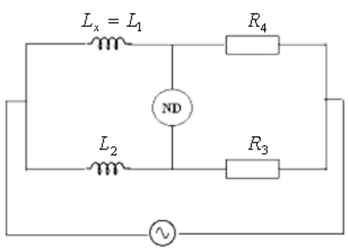
$$\begin{cases} R_1R_3 - X_1X_3 = R_2R_4 - X_2X_4 \\ R_1X_3 + R_3X_1 = R_2X_4 + R_4X_2 \end{cases}$$

Remarks

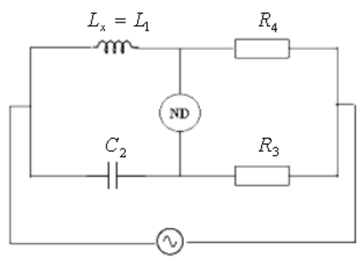
$$Z_1 Z_3 e^{j(\varphi_1+\varphi_3)} = Z_2 Z_4 e^{j(\varphi_2+\varphi_4)}$$

If $\underline{Z}_3 = R_3$; $\underline{Z}_4 = R_4 \Rightarrow \varphi_3 = \varphi_4 = 0 \Rightarrow \varphi_1 = \varphi_2 \Rightarrow$

$\Rightarrow$ the others arms should contain either L_1, L_2 or C_1, C_2



The balance is possible



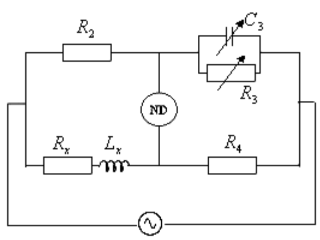
The balance is impossible

A huge number of various AC bridge circuits were designed and developed: *Maxwell, Maxwell-Wien, Wien, Schering, Hay, Owen, Anderson, de Sauty.*

Examples of ac bridges

The Maxwell-Wien bridge

For unknown inductors: R_x, L_x, (low Q-values inductors)



$$R_2 R_4 = (R_x + j\omega L_x)\frac{R_3 \dfrac{1}{j\omega C_3}}{R_3 + \dfrac{1}{j\omega C_3}}$$

$$R_2 R_4 = (R_x + j\omega L_x)\frac{R_3}{1 + j\omega R_3 C_3}$$

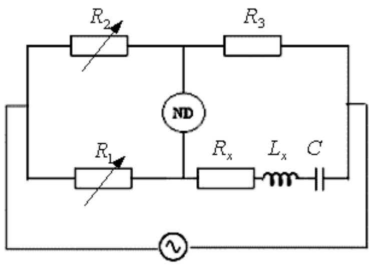
$$R_2 R_4 (1 + j\omega R_3 C_3) = R_3 (R_x + j\omega L_x)$$

$$R_2 R_4 + j\omega R_2 R_3 R_4 C_3 = R_3 R_x + j\omega L_x R_3$$

It results: $R_x = \dfrac{R_2 R_4}{R_3}$ and $L_x = R_2 R_4 C_3$

The quality factor (Q value) of the inductor will be: $Q = \dfrac{\omega L_x}{R_x} = \omega \dfrac{R_2 R_4 C_3}{R_2 R_4} R_3 = \omega C_3 R_3$

The resonance bridge (for measure R, L, Q)

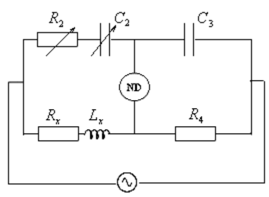


$$R_1R_3 = R_2(R_x + j\omega L_x + \frac{1}{j\omega C})$$

$$(R_1R_3 - R_2R_x)\,j\omega C = 1 - \omega^2 L_x C$$

$$R_x = \frac{R_1R_3}{R_2} \qquad L_x = \frac{1}{\omega^2 C}$$

The Owen bridge (for measure R, L-high values)



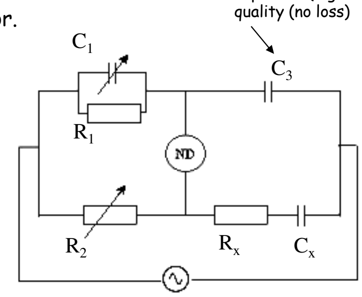
$$R_x = \frac{R_4C_3}{C_2} \qquad L_x = R_2R_4C_3$$

Application: Schering Bridge (for measurement of capacitors and their insulation)

A Schering bridge has the following components values: C_1=0.1µF, R_1=2kΩ, R_2=5kΩ,C_3=0.25 µF, f=2kHz.

Determine the unknown capacitance and dissipation factor.

C_3-standard capacitor (high quality (no loss)



$$\frac{\frac{1}{j\omega C_1}R_1}{\frac{1}{j\omega C_1}+R_1}\left(R_x + \frac{1}{j\omega C_x}\right) = R_2\frac{1}{j\omega C_3}$$

$$\frac{R_1}{1+R_1 j\omega C_1}\left(\frac{1+j\omega R_x C_x}{j\omega C_x}\right) = \frac{R_2}{j\omega C_3}$$

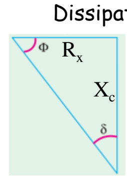
$$\frac{R_1(1+j\omega R_x C_x)}{(1+R_1 j\omega C_1)C_x} = \frac{R_2}{C_3}$$

$$R_1C_3(1+j\omega R_x C_x) = R_2C_x(1+R_1 j\omega C_1)$$

$$R_1C_3 = R_2C_x \Rightarrow C_x = \frac{R_1C_3}{R_2} = \frac{2K\Omega}{5k\Omega}\cdot 0.25\mu F = 0.1\mu F$$

$$R_xC_x = \omega R_1C_1 \Rightarrow R_x = \frac{R_1C_1}{C_x} = \frac{R_1C_1}{\frac{R_1C_3}{R_2}} = \frac{R_2C_1}{C_3} = \frac{5\cdot 10^3\cdot 0.1}{0.25} = 2k\Omega$$

Dissipation factor.

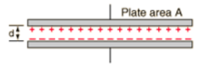


$$D = tg\delta = \frac{R_x}{\frac{1}{\omega C_x}} = \omega R_x C_x =$$

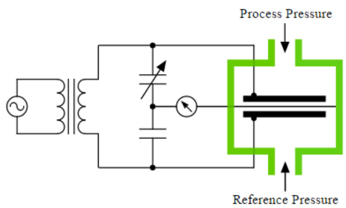
$$= 2\pi\cdot 2\cdot 10^3\cdot 2\cdot 10^3\cdot 0.1\cdot 10^{-6} =$$

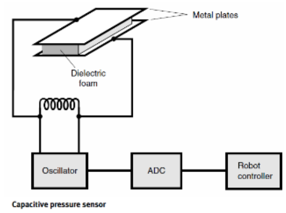
$$= 2.513$$

Application
Capacitive Pressure Sensors



$$C = \frac{\varepsilon A}{d}$$





- The capacitive pressure sensors use a thin diaphragm, usually metal or metal-coated quartz, as one plate of a capacitor.
- The diaphragm is exposed to the ***process pressure on one side*** and to a ***reference pressure on the other.*** Changes in pressure cause it to deflect and change the capacitance, **that is detected by a bridge circuit.**

A capacitive pressure sensor consists of two metal plates separated by a layer of nonconductive (dielectric) foam.
The resulting variable capacitor is connected in parallel with an inductor; the inductance/capacitance (LC) circuit **determines the frequency of an oscillator.**

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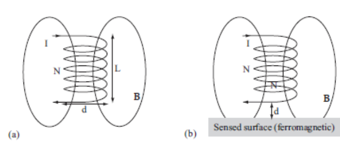
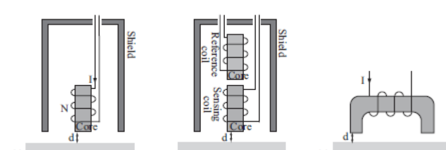
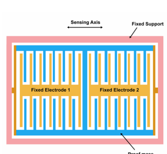
Application
Inductive proximity sensors

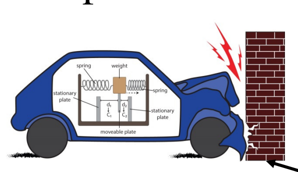


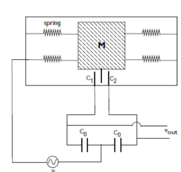


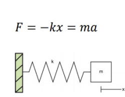
AC bridge- **measuring the inductance L-** a measure of the proximity to the surface being sensed.

Application
MEMS capacitive accelerometers









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https://www.youtube.com/watch?v=T_iXLNkkjFo
https://www.youtube.com/watch?time_continue=7&v=RLQGZl0lpjQ

Airbag deployment