cpGRUPA =	30 413	NUME =	SANDOR	PRENUME =	DOROTEEa

Subiect nr	1

Semnatura: ______________________

A) If $A : C[a,b] \to \mathbb{R}$ is a PLF, $A(1)=1$, and the functions f and $g \in C[a,b]$ are synch. monotonic (have the same monotony on each subset of $[a,b]$), then $A(fg) \geq A(f) \cdot A(g)$

$$(f(x) - f(y)) \cdot (g(x) - g(y)) \geq 0, \forall x,$$

$$\Rightarrow (f(x) - f)(g(x) - g) \geq 0, \forall x \in [a,b], \forall x \in [a,b]$$

$$\Rightarrow f(x) \cdot g(x) \cdot 1 - f(x) \cdot g - g(x) \cdot f + f \cdot g \geq 0, \forall x \in [$$

$$\Rightarrow f(x) \cdot g(x) \cdot A(1) - f(x) \cdot A(g) - g(x) \cdot A(f) + A(\cancel{f}g)$$

$$\Leftrightarrow f \cdot g - f A(g) - g A(f) + A(f \cdot g) \cancel{\geq 0} \cdot 1 \geq 0$$

$$\Rightarrow A(f \cdot g) - A(f) \cdot A(g) - A(f) \cdot A(g) + A(f g) \geq 0$$

$$\Rightarrow A(f \cdot g) \geq A(f) \cdot A(g)$$

$$\int_a^b f(x)g(x)\,dx \geq \frac{1}{b-a}\int_a^b f(x)\,dx \cdot \int_a^b g(x)\,dx$$

We apply Chebyshey: $A(f) = \frac{1}{b-a}\int_a^b f(x)\,dx$

3) $\lim\limits_{x\to\infty} \frac{1}{x} \int_0^x \frac{dy}{5+4\cos y}$

$$\int_0^x \frac{dy}{5+4\cos} =$$

$u = tg\frac{y}{2}$ / arctg

$arctg\, u = \frac{y}{2} \Rightarrow 2\, arctg\, u = y \Rightarrow \frac{2}{1+u^2}\, du = dy$

I $y = 0 \Rightarrow u = 0$

II $y = x \Rightarrow u = tg\frac{x}{2}$

$$\cos y = \frac{1-\left(tg\frac{y}{2}\right)^2}{1+tg\left(\frac{y}{2}\right)^2} = \frac{1-u^2}{1+u^2}$$

$$\int_0^u \frac{1}{5+4\cdot\frac{1-u^2}{1+u^2}} \cdot 2 \cdot \frac{1}{1+u^2}\, du$$