cpGRUPA	30 413	NUME	SANDOR	PRENUME	DOROTEEa

Subiect nr	3

Semnatura: Doroteea

A. $\int_0^\infty \frac{\sin^2 x}{x} dx = \int_0^\infty \sin x \cdot \frac{\sin x}{x} dx$

$x \in \mathbb{R}$ ~~Let~~ $f: \mathbb{R} \to \mathbb{R}$ be $f(x) = \sin x$

$\cancel{f(x+\pi) = f(\pi - x)}$

$\sin^2 x = \frac{1 - \cos 2x}{2} \Rightarrow \sin^2 x = \frac{\cos(0 \cdot x) - \cos(2x)}{2} =$

$= \frac{\cos(0 \cdot x)}{2} - \frac{\cos(2 \cdot x)}{2} =$

$\sin^2 x = f(0) - f(2 \cdot x)$

First Frullani Formula: $\int_0^\infty f(ax)$

B. $D = (x,y) \in \mathbb{R}^2$ st $\begin{cases} x, y > 0 \\ x \leq y^3 \leq 2x \;\Big|\sqrt[3]{\ } \\ x^3 y^2 \leq 3 \end{cases}$

$\Rightarrow \sqrt[3]{x} \leq y \leq \sqrt[3]{2x}$

$x^3 y^2 \leq 3 \Leftrightarrow x^3 \leq \frac{3}{y^2} \;\Big|\sqrt[3]{\ } \Leftrightarrow \quad x \leq \sqrt[3]{\frac{3}{y^2}}$

$$\iint_D x^2 y \, dx dy = \int_{\sqrt[3]{x}}^{\sqrt[3]{2x}} \int_0^{\sqrt[3]{\frac{3}{y^2}}} x^2 y \, dx dy = \int_{\sqrt[3]{x}}^{\sqrt[3]{2x}} y \frac{x^3}{3} \Big|_0^{\sqrt[3]{\frac{3}{y^2}}} dy =$$

$$= \int_{\sqrt[3]{x}}^{\sqrt[3]{2x}} \frac{\cancel{y}}{\cancel{3}} \cdot \frac{\cancel{3}}{y^{\cancel{2}}} dy = \ln y \Big|_{\sqrt[3]{x}}^{\sqrt[3]{2x}} = \frac{1}{3} \ln \frac{2x}{x} = \frac{1}{3} \ln 2.$$