grupa	30 413	NUME	SANDOR	PRENUME	DOROTEEa

Subiect nr 2 Semnatura: Dorotee

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B. $\Gamma\left(\frac{4}{3}\right)\Gamma\left(\frac{2}{3}\right) = \Gamma\left(1+\frac{1}{3}\right)\Gamma\left(1-\frac{1}{3}\right) = \frac{1}{3}\Gamma\left(\frac{1}{3}\right)\Gamma\left(1-\frac{1}{3}\right)$

$= \frac{1}{3}\cdot\frac{\pi}{\sin\frac{\pi}{3}}$, $\frac{1}{3}\in(0,1)$

$= \frac{1}{3}\cdot\frac{\pi}{\frac{\sqrt{3}}{2}} = \frac{2\pi}{3\sqrt{3}} = \frac{2\pi\sqrt{3}}{3\cdot(\sqrt{3})^2} = \frac{2\sqrt{3}\cdot\pi}{9}$

A. $I = \int_0^\infty \frac{dx}{(x^2+1)^{m+1}}$

$B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1}\,dx$

we subst. $x = \frac{u}{1+u} \Rightarrow B(a,b) = \int_0^\infty \frac{u^{a-1}}{(1+u)^{a+b}}\,du$

$u = x^2 \Rightarrow x = \sqrt{u} \Rightarrow dx = \frac{1}{2}\cdot\frac{1}{\sqrt{u}}$

$I = \int_0^\infty \frac{1}{(u+1)^{m+1}}\cdot\frac{1}{2}\cdot u^{-\frac{1}{2}}\,du = \frac{1}{2}\int_0^\infty \frac{u^{-\frac{1}{2}}}{(u+1)^{m+1}}\,du$

$\begin{cases} a-1 = -\frac{1}{2} \Rightarrow a = \frac{1}{2} \\ a+b = m+1 \Rightarrow b = m+\frac{1}{2} \end{cases} \Rightarrow I = \frac{1}{2}B\left(\frac{1}{2}, m+\frac{1}{2}\right)$