Electronic Measurements and Sensors

SINGLE PHASE AC CIRCUITS MEASUREMETS

1.1. Theoretical Approach

Meters can be classified not only by the parameter that they measure, but also by the type of display or used readout. Meter displays fall into two categories: analog and digital.

Analog meters generally use some sort of electromechanical mechanism to cause a small arm to move, depending on the voltage or current applied to the meter. A graduated measurement scale is imprinted behind the mechanism such that the moving arm points to the value of the meter reading. The important thing is that analog meters provide a *continuously varying readout*, without any discrete jumps in meter reading. The mechanism is not strictly necessary to be a mechanical one. It can be implemented using other technologies, as long the result is a continuous “analog” type of display.

The display mechanism of a meter is often referred to as a *movement*, borrowing from its mechanical nature to *move* a pointer along a scale so that a measured value may be read. Most mechanical movements are based on the principle of electromagnetism: that electric current through a conductor produces a magnetic field perpendicular to the axis of electron flow. The greater the electric current, the stronger the magnetic field produced. If the magnetic field formed by the conductor is allowed to interact with another magnetic field, a physical force will be generated between the two sources of fields. If one of these sources is free to move with respect to the other, it will do so as current is conducted through the wire, the motion (usually against the resistance of a spring) being proportional to strength of current.

On the analog meter panel (Fig.1.1) there are symbols that help us to identify the instrument:

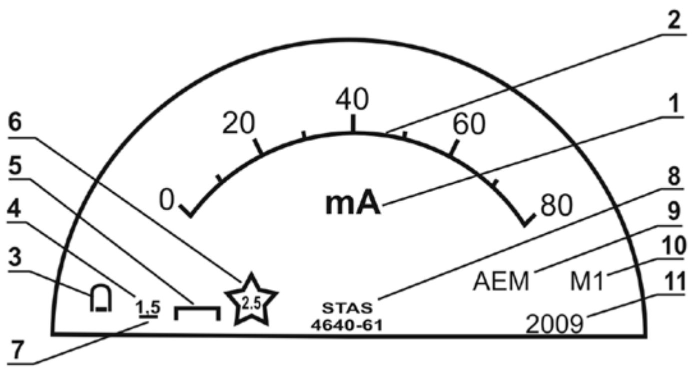


Fig.1.1 *Example of an Analog Meter Panel*

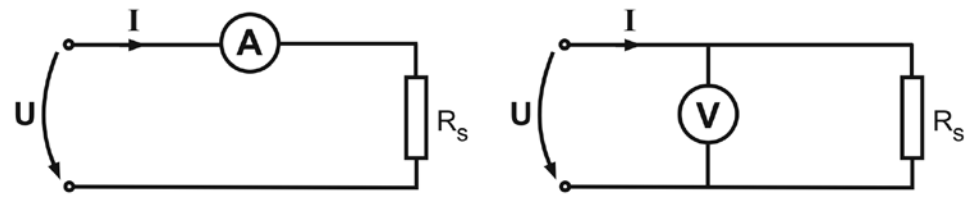
1. the parameter to be measured (voltage, current…);
2. the scale type (linear, square, logarithmic);
3. the general symbol of the instrument type;
4. the accuracy class (class index);
5. working position;
6. maximum insulating voltage (kV);
7. the usage current;
8. the standardization norm of manufacturing;
9. the manufacturer;
10. the meter’s identification code;
11. the year of manufacturing.

The most used analog measurement devices are:

- Ammeters*:* they allow the direct measurement of the electric current; are characterized by a serial connection, (Fig.1.2.a) and they have low internal resistance.

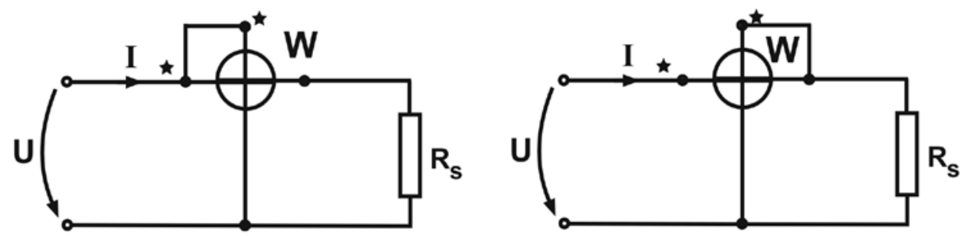
- Voltmeters: they allow the direct measurement of the electric voltage; are characterized by a parallel connection, (Fig.1.2.b); they have high internal resistance.

- Wattmeters: They allow the direct measurement of the active power; they can have an upstream (Fig.1.3.a) or a downstream connection (Fig.1.3.b). The wattmeter derivation is proportional with the active power - $U \cdot I \cdot \cos\varphi$ - and is a function of the phase shift.



a) b)

Fig.1.2 *Ammeter (a) and Voltmeter (b) Circuit Connection*



a) b)

Fig.1.3 *Wattmeter upstream (a) and downstream (b) circuit connection*

Considering a sinusoidal AC circuit and a load impedance Z (Fig.1.4), this is characterized by the relations:

$$Z = R + jX = Z(\cos\varphi_z + j\sin\varphi_z)$$
$$Z = \frac{U_z}{I_z} \qquad (1.1)$$
$$\varphi = arctg\frac{X}{R}$$

It’s possible to measure the power consumed on this impedance using a wattmeter connected in upstream circuit (the beginning of the voltage coil connected to the beginning of the current coil) or in downstream circuit (the beginning of the voltage coil connected to the end of the current coil).

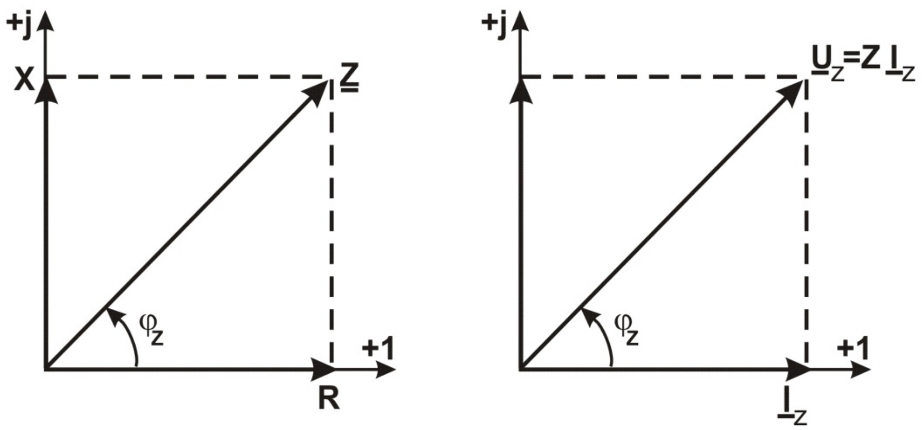


Fig.1.4 *The Phasors Diagram*

In the case of the **upstream circuit** (Fig.1.5), the active power indicated by the wattmeter is:

$$P = UI\cos\varphi \qquad (1.2)$$

and is different from the real power consumed on the load, by the consume on the current coil. If we consider the current impedance to be a pure resistance having the value R_{aw}, the real consumed power on the load is:

$$P_z = P - R_{aw}I^2 \qquad \Delta P_{aw} = R_{aw}I^2 \qquad (1.3)$$

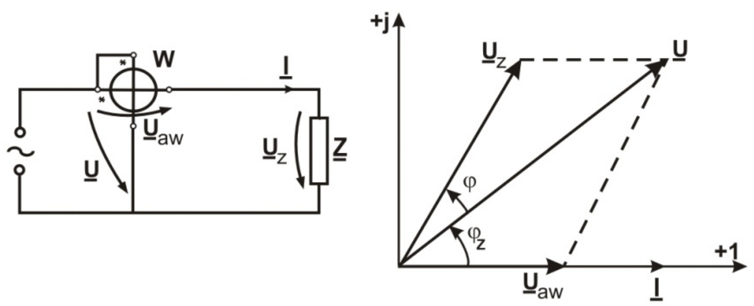


Fig.1.5 *Upstream AC Circuit and the Correspondent Phasors Diagram*

From the phasors diagram it can be seen that in this case:

$$U_Z^2 = U^2 + U_{aw}^2 - 2UU_{aw}\cos\varphi$$
$$U_{aw} = R_{aw}I \qquad (1.4)$$

The modulus of the load impedance is:

$$Z = \frac{U_Z}{I} = \frac{\sqrt{U^2 - R_{aw}(2P - R_{aw}I^2)}}{I} \qquad (1.5)$$

and the power factor:

$$\cos\varphi_Z = \frac{P - R_{aw}I^2}{I\sqrt{U^2 - R_{aw}(2P - R_{aw}I^2)}} \qquad (1.6)$$

From the last two relations one can determine the real and the imaginary components of the load impedance:

$$R_Z = Z\cos\varphi_Z \qquad (1.7)$$

$$X_Z = Z\sin\varphi_Z \qquad (1.8)$$

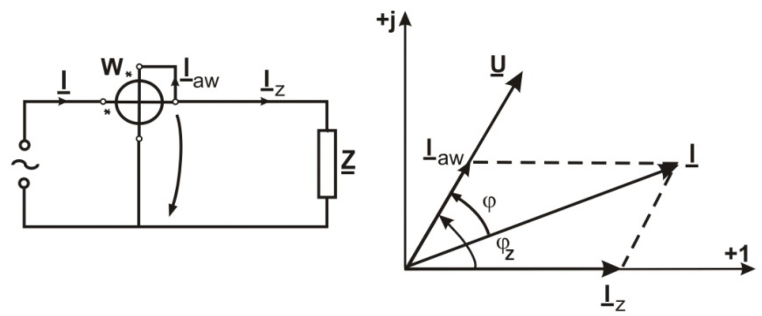


Fig.1.6 *Downstream AC Circuit and the Correspondent Phasors Diagram*

In the case of **the downstream circuit** (Fig.1.6), the active power indicated by the wattmeter is given by the relation 1.2 and is different from the real power consumed on the load, due to the consume on the voltage coil. Considering the voltage impedance to be a pure resistance having the value R_{vw}, the real power consumed on the load Z will be:

$$P_Z = P - \frac{U^2}{R_{vw}} \qquad \Delta P_{aw} = \frac{U^2}{R_{vw}} \qquad (1.9)$$

From the phasors diagram it can be seen that in this case:

$$I_{vw} = \frac{U}{R_{vw}} \qquad I_Z^2 = I^2 + I_{vw}^2 - 2\cdot I\cdot I_{vw}\cdot\cos\varphi \qquad (1.10)$$

Like in the previous case, the modulus of the load impedance can be computed with the relation:

$$Z = \frac{U}{I_Z} = \frac{U}{\sqrt{I^2 - \frac{1}{R_{vw}}(2P - \frac{U^2}{R_{vw}})}} \qquad (1.11)$$

and the power factor is:

$$\cos\varphi_Z = \frac{P - \frac{U^2}{R_{vw}}}{U\sqrt{I^2 - \frac{1}{R_{vw}}(2P - \frac{U^2}{R_{vw}})}} \qquad (1.12)$$

The real and the imaginary components of the load impedance can be determined in this case with the relations:

$$R_Z = Z\cos\varphi_Z \qquad (1.13)$$

$$X_Z = Z\sin\varphi_Z \qquad (1.14)$$

The *method errors* that appear at the measurement of the power, for the **upstream and downstream** connections, are:

$$\Delta P_{aw} = R_{aw} \cdot I^2 \qquad \Delta P_{aw} = \frac{U^2}{R_{vw}} \qquad (1.15)$$

These errors appear because of the consumption on the current and voltage coils of the device. The method errors are equal with:

$$Z^2 = R_{aw}R_{vw} \qquad (1.16)$$

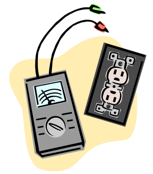
If $Z^2 > R_{aw}R_{vw}$, *then the upstream connection is recommended,* if $Z^2 < R_{aw}R_{vw}$, *then the downstream connection is recommended.*

1.2 Experimental Design and Procedure

1.2.1 Equipment

ATR 8 – adjustable autotransformer 0-250 V, 8 A;
K – encapsulated bipolar switch;
V – a.c. voltmeter , 0-150-300 V;
A – a.c. ammeter , 0-1-5 A;
W – a.c. wattmeter , 0-5 A, 0-150-300 V;
KS – load switch;
R - resistive load;
C - capacitive load;
L – inductive load, it can be an autotransformer ATR 8 connected between the terminals 0-220.

1.2.3 Description of the Experimental Procedure



Activity 1. Voltage, current and power measurement. Load: resistive

For the voltage, current and power measurements the circuits from Fig.1.7 will be used. Write the measuring devices characteristics by completing the Table 1.1 and Table 1.2

Before plug in, check the cursors positions such that the autotransformer should be on "0" and the loads on maximum resistance position. Set up the input voltage (using the "C" cursor of the autotransformer) corresponding with the wattmeter range, then adjust the load resistors R_{s1} and R_{s2} ,R_{s3} and modify the current values in the range of the measurement devices, reading minimum 5 values on their scale. After that we record the results in the Tables 1.3.

ATENTION: You must check, all time, the value of the current in the circuit (on ammeter scale), such that it doesn't exceeds the nominal values of the rheostat and of the wattmeter coil. It is not allowed to exceed the nominal current of the wattmeter under no circumstances.

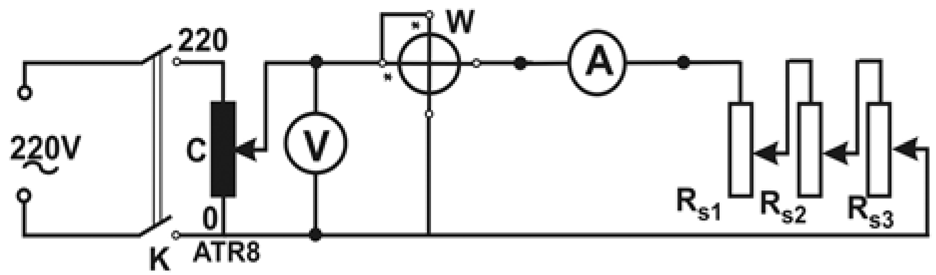
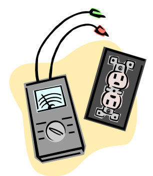


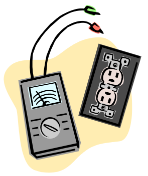
Fig.1.7 *The experimental circuit with resistive load*



Activity 2. Power measurement. Upstream connection

In the case of the measurements with the wattmeter in upstream connection, the circuit used is the one from Fig.1.8.

Put the "load switcher" LK on '0'position, adjust the load potentiometer on the position that corresponds to a value of the current of 1 A, and then measure several values with the KS on each of the positions 0,I,II,III, meaning for the resistive load, resistive-capacitive load, resistive-inductive load, respectively resistive-inductive-capacitive load. With the switch KS on '0' position, adjust again the load potentiometer in order to obtain minimum 5 values of currents in the ammeter range. The values measured are record in the Table 1.4.



Activity 3. Power measurement. Downstream connection

For the measurements with wattmeter connected in downstream circuit, use the circuit from Fig.1.9, using the same devices like in the previous case.

The only modifications are related to the position of the ammeter and of the voltmeter and to the connection of the wattmeter. The measured values must be recorded in the Table 1.5.

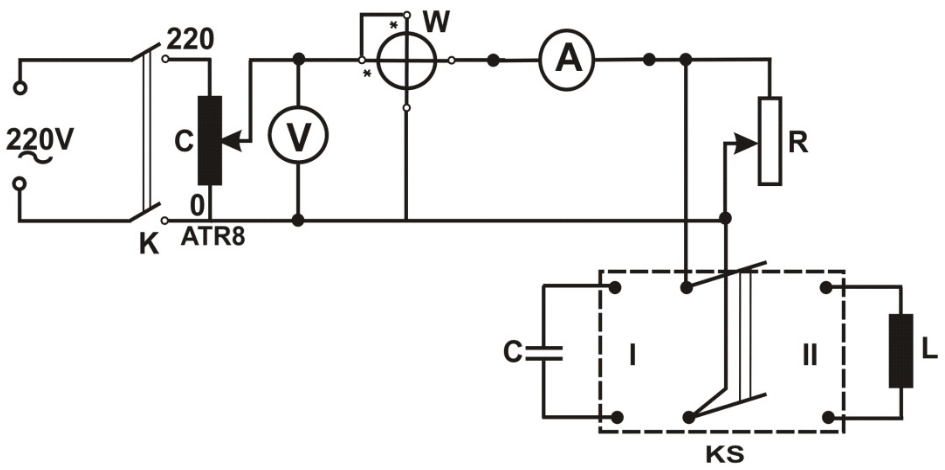


Fig.1.8 *The upstream AC experimental circuit with R,L or C load*

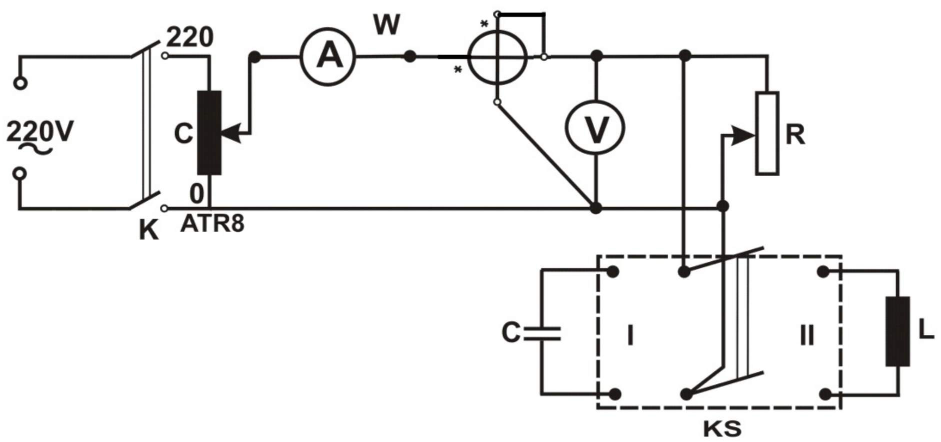


Fig.1.9 *The downstream AC experimental circuit with R,L or C load*

1.3 Experimental Results

After you've done the laboratory work, you must analyze the results and write some conclusions about how the measurements errors depend on the load impedance, for both of the wattmeter connections.

For ammeters and voltmeters, the constant of the instruments is determined as the ratio between the range and the maximum deflection:

$$C = \frac{X_N}{\alpha_{max}} \quad [X] / \text{div.} \qquad (1.17)$$

and the measured value is determined with the relation:

$$X_m = C \cdot \alpha \quad [X] \qquad (1.17')$$

For wattmeters the range is determined as the product between the voltage range and the current range, and the instrument constant is:

$$C_W = \frac{X_{WN}}{\alpha_{max}} = \frac{U_N \cdot I_N}{\alpha_{max}} \quad [\text{W/div}] \qquad (1.18)$$

The maximum absolute error for ammeters, voltmeters, wattmeters, will be determined according to the index of accuracy class "c" of the device with the relationship:

$$\varepsilon_{max} = \frac{c \cdot X_N}{100} \quad [X] \qquad (1.19)$$

The maximum relative error of the measurements can be calculated in the case of the ammeters, voltmeters and wattmeters with the relationship:

$$\varepsilon_{r\,max} = \frac{\varepsilon_{max}}{X_m} \cdot 100 = \frac{c \cdot X_N}{X_m} \quad [\%] \qquad (1.20)$$

1.4. Analysis and Conclusions

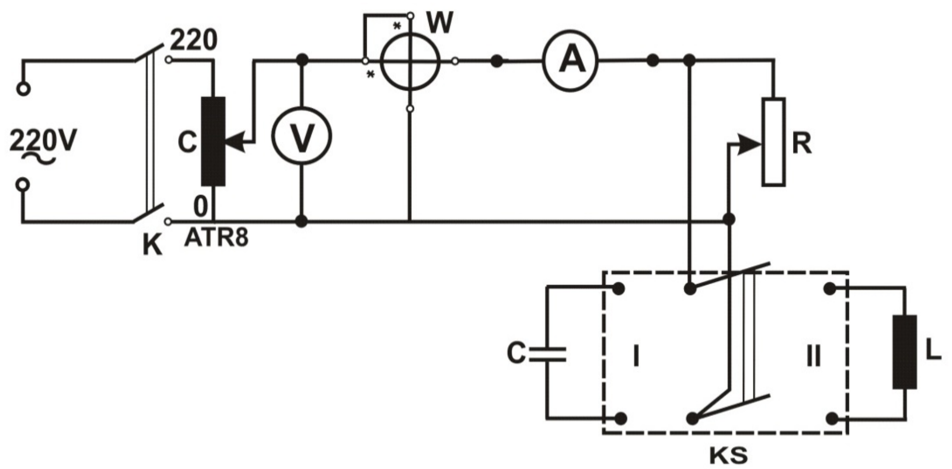
1. omplete the tables:

Nr.	Meter	Range X_N	Max. defl. α_{max}	Instr. Constant $C = \frac{X_N}{\alpha_{max}}$	Accuracy class c	Max. abs. error $\varepsilon_{max} = \frac{cX_N}{100}$
	type	[X]	[div.]	[X/div]	[%]	[X]

Range			R_a	R_v	R_{wi}	R_{wu}	R_{aw}	R_{vw}
ammeter	voltmeter	wattmeter						

R_a – ammeter internal resistance;
R_v – voltmeter internal resistance;
R_{wi} – wattmeter current coil resistance;
R_{wu} – wattmeter voltage coil resistance;
$R_{aw} = R_a + R_{wi}$;
$R_{vw} = R_v \,||\, R_{wu}$

2. Set up the **upstream** AC experimental circuit with R,L or C load



Complete the table:

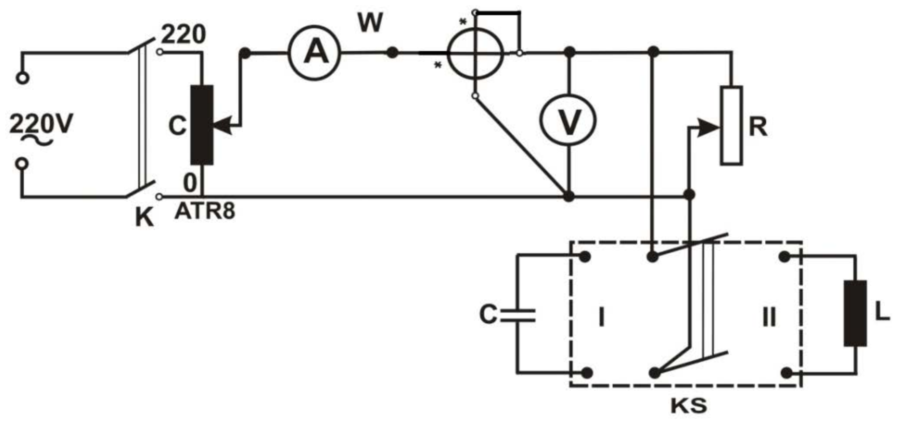
Load	U	I	P		ΔP_{aw}	ε_{rp}	Z	R_z	X_z	$\cos\varphi_z$	P_z	S_z	Q_z
					Rel (1.15)	$\frac{\Delta P_{aw}}{P}100$	Rel (1.5)	Rel (1.7)	Rel (1.8)	Rel. (1.6)	Rel (1.3)	UI	$\sqrt{S_z^2 - P_z^2}$
type	[V]	[A]	[div]	[W]	[W]	[%]	[Ω]	[Ω]	[Ω]		[VA]	[VAr]	[VA]

Write down the relations:

(1.15) ΔP_{aw} (1.5) (1.7) (1.8)

(1.6) (1.3)

1. Set up the **downstream** AC experimental circuit with R,L or C load



Complete the table:

Load	U	I	P		ΔP_{aw}	ε_{rp}	Z	R_z	X_z	cos φ_z	P_z	S_z	Q_z
					Rel (1.15)	$\frac{\Delta P_{aw}}{P}100$	Rel (1.11)	Rel (1.13)	Rel (1.14)	Rel. (1.12)	Rel (1.9)	UI	$\sqrt{S_z^2 - P_z^2}$
type	[V]	[A]	[div]	[W]	[W]	[%]	[Ω]	[Ω]	[Ω]		[VA]	[VAr]	[VA]

Write down the relations:

(1.15) **ΔP_{aw}** (1.11) (1.13) (1.14)

(1.12) (1.9) **P_z**

Electronic Measurements and Sensors

THE EXTENSION OF THE MEASUREMENT RANGE AT ANALOG INSTRUMENTS

2.1. Theoretical Approach

The range is the maximum value of the measurand that can be measured by a given meter. The range of voltmeters, ammeters, etc., is limited by the current carrying capacity of the control springs, high measuring parameters, etc. The permanent-magnet moving-coil instruments are used in d.c. for the currents measurements in a range of 100 μA …10mA, or for voltages measurements in a range of 1mV…100mV. For the measurement of large values, it is necessary to make use of a device which reduces the current and voltage of the instrument by a known proportion.

For the extension of the range at permanent-magnet moving-coil ammeters, shunts are used – these are low resistances connected in parallel with the meter coil. In Fig.2.1.a the shunt R_S is connected across the ammeter coil R_a on the terminals B_1, B_2. In Fig.2.1.b the real diagram of the terminal B_1 is presented (the diagram of the terminal B_2 is similar). As we can observe from the figure, there are three circuits connected on the same terminal: the measured current circuit I, the ammeter coil circuit I_a and the shunt circuit. Consequently R_1, R_2, and R_3 contact resistance appear. These resistances tend to zero, theoretically, but practically they have a finite value.

The equivalent electrical diagram of the terminal B_1 (and similarly to the terminal B_2) is a diagram in triangle (Fig.2.1.c), which for calculus considerations is transfigured into a star one. (Fig.2.1.d). If the real contact resistances are considered almost equals: $R_1 \approx R_2 \approx R_3 = R$, then the equivalent contact resistance will be:

$$r_{12} \approx r_{12}' \approx r_{13}' \approx r_{23} \approx r_{23}' = r = \frac{1}{3}R \qquad (2.1)$$

Theoretically the "multiplying power" of the shunt is:

$$n = \frac{I}{I_a} \qquad (2.2)$$

and the value of the shunt resistance is determined by the equation:

$$R_s = \frac{R_a}{n-1} \qquad (2.3)$$

In general the ammeter range extension means n>>1, and thus the shunt resistance means $R_s \ll R_a$.

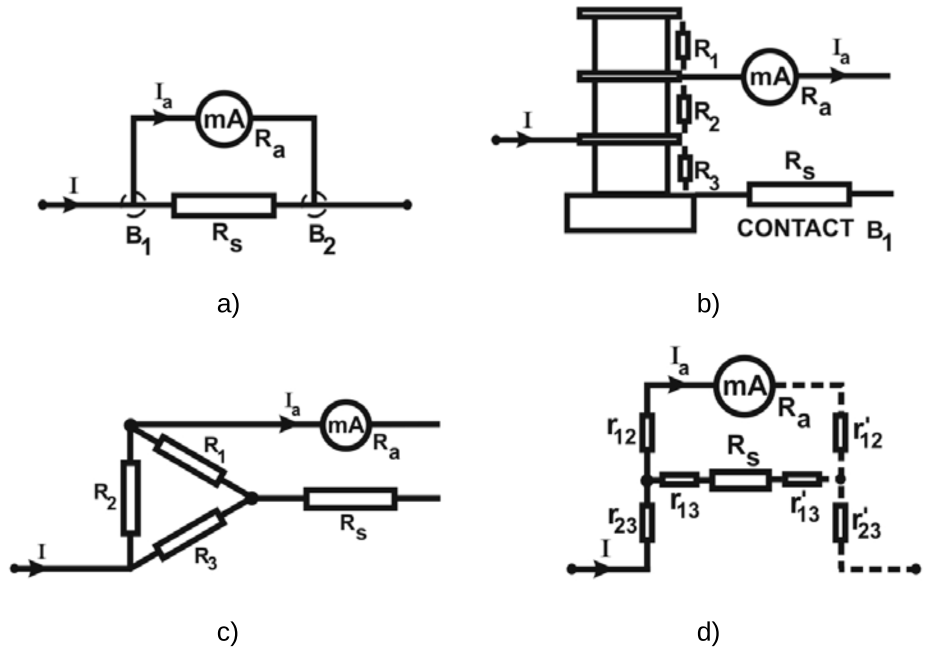


Fig.2.1 *The equivalent electric diagrams for the ammeters with simple shunt*

Practically, the real value shunt "multiplying power" is different from the theoretical value, because of the contact resistance r_{12}, r'_{12}, respectively r_{13}, r'_{13} that appear connected in series with the ammeter coil resistance respectively with the shunt resistance.

Thus, the real "multiplying power" of the shunt is:

$$n' = \frac{R_a'}{R_s'} + 1 = \frac{r_{12} + R_a + r_{12}'}{r_{13} + R_s + r_{13}'} + 1 = \frac{R_a + 2 \cdot r}{R_a + 2 \cdot r} + 1 \qquad (2.4)$$

The absolute error of the current measured with this extended measurement device is:

$$\varepsilon_a = I - I' = (n - n') \cdot I_a \qquad (2.5)$$

and the relative error will be:

$$\varepsilon_r = \frac{I - I'}{I'} = \left(\frac{n}{n'} - 1\right) \cdot 100\,[\%] \qquad (2.6)$$

For the reduction of these errors the shunts are constructed with terminals doubled at the both parts (2x2), like in the Fig.2.2.a. The equivalent electrical diagram of the ammeter with four connection terminals shunt is that from the Fig.2.2.b.

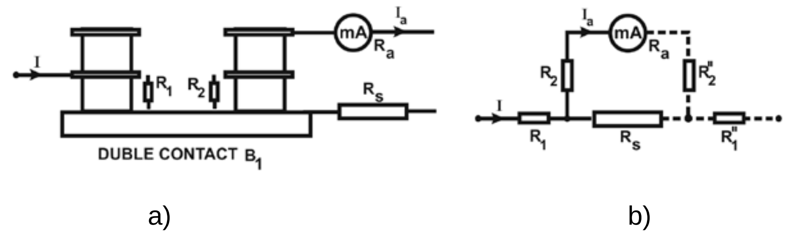


Fig.2.2 *Ammeter with four connection terminals shunt*

The essential difference from the previous case consists in the fact that the contact resistances appear connected in series in the main circuit and also in the ammeter coil circuit, but they do not affect anymore the shunt circuit.

Calculating the real ” multiplying power” of the shunt, we obtain:

$$n'' = \frac{R_a''}{R_s} + 1 = \frac{R_a + R_2 + R_2''}{R_s} + 1 = \frac{R_a + 2R}{R_s} + 1 \qquad (2.7)$$

The absolute error of the measured current, in the case of the utilization of a shunt with four connection terminals, will be:

$$\varepsilon_a'' = I - I'' = (n - n'')I_a \qquad (2.8)$$

and the relative error:

$$\varepsilon_r = \frac{I - I''}{I''} = (\frac{n}{n''} - 1) \cdot 100\,[\%] \qquad (2.9)$$

As the contact resistance are much lower than the resistance of the measurement device, $R << R_a$, the real extension rate of the current domain is practically equal with the theoretical rate.

In the case of permanent-magnet moving-coil voltmeters, the extension of the measurement range is done by putting an additional resistance (dropping resistance) in series with the device coil resistance, like in the Fig.2.3.a and Fig.2.3.b. The equivalent electrical diagram can be seen in the Fig.2.3.c.

Theoretically, the voltmeter “multiplier power” is:

$$m = \frac{U}{U_v} \qquad (2.10)$$

and the necessary value of the additional resistance (dropping resistance) is:

$$R_{ad} = R_v(m - 1) \qquad (2.11)$$

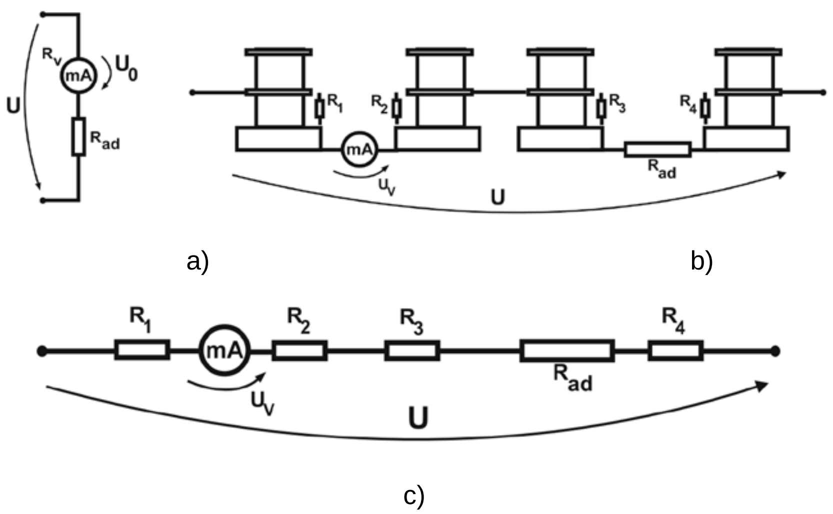


Fig.2.3 *The equivalent electric diagrams for the voltmeter with additional resistance*

Practically, the real extension value of the domain is determined by taking also into account the contact resistance that appears in series with the measurement device and with the additional resistance. With the assumption that $R_1 \approx R_2 \approx R_3 \approx R_4 \approx R$, the real voltmeter “multiplier power” is:

$$m' = \frac{R_{ad}'}{R_v} + 1 = \frac{R_{ad} + 4R}{R_v} + 1 \qquad (2.12)$$

The absolute error of the voltage measured with the extended device is determined with the relation:

$$\varepsilon_a = U - U' = (m - m') \cdot U_v \qquad (2.13)$$

and the relative error with:

$$\varepsilon_r = \frac{U - U'}{U'}100 = (\frac{m}{m'} - 1) \cdot 100 \qquad (2.14)$$

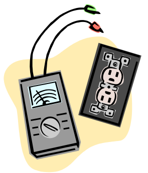
As the voltmeter “multiplying power” is m >>1, it results that $R_{ad} >> R_v$. Therefore in the case of the voltmeters the contact resistances can be neglected and they don’t introduce errors at the measurement of the voltages.

2.2 Experimental Design and Procedure

2.2.1 Equipment

- CVS – adjustable continuous voltage supply (0÷15÷30V; 0 ÷1A);
- mA – permanent-magnet moving-coil mili-ammeter, 60mV/10Ω
- Ae – standard d.c. ammeter, 0 ÷ 500mA; precision class c = 0,2;
- V – permanent-magnet moving-coil voltmeter; 0÷3V 500 Ω/V
- Ve – standard d.c. voltmeter, 0 ÷ 7,5 ÷15 ÷30 ÷75 V, accuracy class c =0,2;
- Rad – additional resistance box; 500 Ω/V; 3 ÷ 7,5 ÷ 15 ÷ 30 ÷ 75 ÷ … ÷750 V;
- Rs – multiple shunt, 60 mV/10Ω, 75 ÷100 ÷ mA
- Rp – protection rheostat for the limitation of the working current, 75 Ω, 3A;

2.2.3 Description of the Experimental Procedure



Activity 1. Current measurement. Ammeter shunt: two terminals

In order to study the permanent-magnet moving-coil ammeter extended in current with two connection terminals shunts the circuit from Fig.2.4 must be realized.

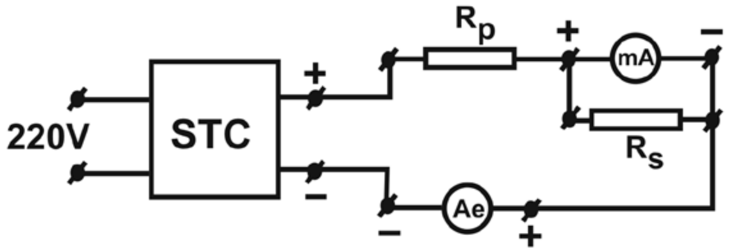
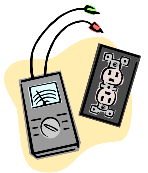


Fig.2.4 *Experimental circuit for the ammeters range extension using two connection terminal shunt*



Activity 2. Current measurement. Ammeter shunt: four terminals

In order to study the permanent-magnet moving-coil ammeter extended in current with four connection terminals shunts the circuit from Fig.2.5 must be realized.

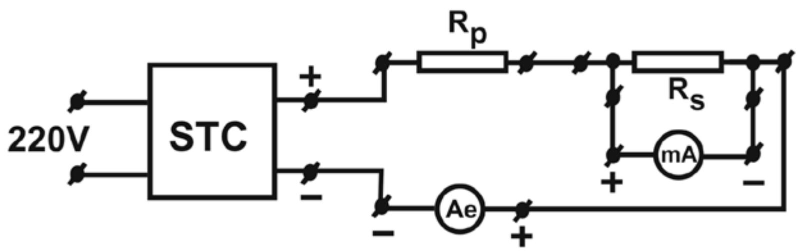
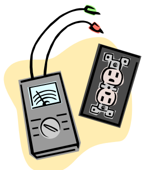


Fig.2.5 *Experimental circuit for the ammeters range extension using four connection terminal shunt*



Activity 3. Voltage measurement. Voltmeter range extension

In the case of the voltmeters, the extension of the measurement range is studied with the assembly diagram from the Fig.2.6

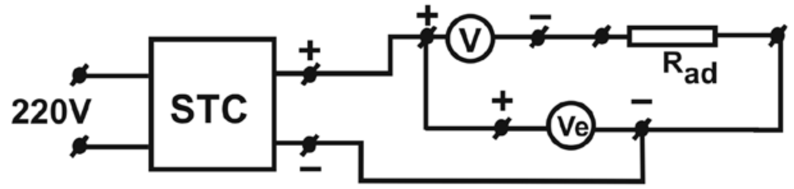


Fig.2.6 *Experimental circuit for the voltmeters range extension using additional resistances*

For each circuit case start the experiment by putting protection rheostat on its maximum value and the voltage supply on minimum.

In the circuit from Fig. 2.4 connect the shunt on each of the three domains (75 – 150 – 300 mA) and each time adjust the supply voltage, in a way that at least 10 values of the current on the scale of the extended device to be obtained. The measured values on the extended device and on the standard one are read and the results are introduced in Table 2.1.

Modify the diagram from Fig.2.5 according to the Fig.2.5 in order to study the shunt with double terminals. The modification is achieved by changing the shunt connection. Make the same measurements as in the case of two connection terminals and introduce the results in the Table 2.2.

To study the extension of the measurement domain at voltmeters the assembly diagram from the fig.2.6 must be done. With the additional resistance connected in turn on the domains 7,5 - 15 -30 - 75 V, the supply voltage of the source is adjusted in such a way that at least 10 values indicated in the scale of the extended device are obtained. The voltages on measured the extended device and on the standard one are read and the results are introduced in Table 2.3.

2.3 Experimental Results

For all the above presented situations, the constants of the device are:

$$C = \frac{X_n}{\alpha_t} \quad [x/div] \qquad (2.15)$$

and the indicated values:

$$x_i = C \cdot \alpha \quad [x] \qquad (2.16)$$

where:

- X_n – the nominal domain of the measurement device;
- α_t – the total deviation on the scale of the device;
- α - the deviation corresponding to the measured quantity.

In the case of the ammeters, the theoretical extension rate of the domain is:

$$n = \frac{I_{sN}}{I_{aN}} \quad (2.17)$$

where:

- I_{sN} – the value of the nominal current written on the shunt;
- I_{aN} – the value of the nominal current of the measurement device.

In the case of the voltmeters, the theoretical extension rate of the domain is :

$$m = \frac{U_{adN}}{U_{vN}} \quad (2.18)$$

where

- U_{adN} – the value of the nominal voltage written on the additional resistance;
- U_{vN} - the value of the nominal voltage of the used measurement device.

The value of the quantity measured by the extended device is determined with the equations:

$$\begin{aligned} I &= n \cdot I_a = n \cdot C \cdot \alpha \\ U &= m \cdot U_v = m \cdot C \cdot \alpha \end{aligned} \quad (2.19)$$

in which I_a, U_v are the values read from the proper measurement device, and the other notations have the meaning presented previously.

The measurement absolute error is determined with the equation:

$$\varepsilon_a = x - x_e \quad [X] \quad (2.20)$$

where x is the value read from the extended device and x_e is the value read from the standard device. The relative error is determined using the relation:

$$\varepsilon_r = \frac{x - x_a}{x_a} \cdot 100 \quad [\%] \quad (2.21)$$

For ammeters the real "multiplying power" of the domain according to the relation (2.5) and (2.8) is:

$$n'(n'') = n - \frac{\varepsilon_a}{I_a} \quad (2.22)$$

and for voltmeters, according to the relation (2.13):

$$m' = m - \frac{\varepsilon_a}{U_v} \quad (2.23)$$

where :

- *n, m* - the theoretical "multiplying power" of the domain;
- ε_a – the measurement absolute error;
- I_a, U_v – the values indicated by the used measurement device;

The medium value of the "multiplying power", in each case, is given by:

$$multiplying\ power_{med} = \frac{\sum_{i=1}^{k} multiplying\ power_i}{k} \quad (2.24)$$

where *k* – represents the number of the measurements done at the same theoretical "multiplying power" of the domain.

The contact resistances at the shunt terminals or the additional resistances are calculated with the relation:

- for ammeters with simple shunt:

$$R_c = \frac{3}{2} \cdot \frac{n - n'}{(n-1) \cdot (n'-2)} \cdot R_a \quad (2.25)$$

- for ammeters with shunt with double terminal:

$$R_c = \frac{1}{2} \cdot \frac{n'' - n}{n - 1} \cdot R_a \quad (2.26)$$

- for voltmeters with additional resistance:

$$R_c = \frac{m' - m}{4} \cdot R_v \quad (2.27)$$

The value of the medium contact resistance is calculated with the relation:

$$R_{c\,med} = \frac{\sum_{i=1}^{k} R_{ci}}{k} \quad (2.28)$$

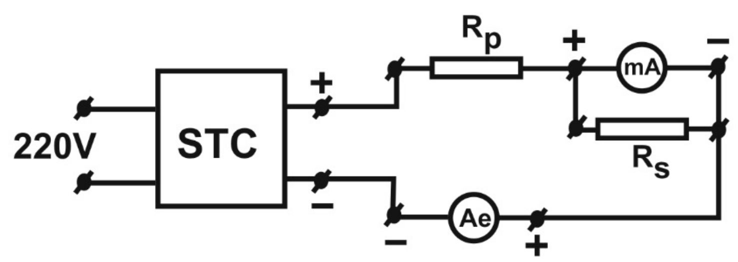
2.4. Analysis and Conclusions

The absolute and relative errors which appear due to the range extension of the measurement devices must be represented function of the measured quantity.

In the case of ammeters the both cases of the simple and the double shunt will be represented on the same graph.

1. **Question** . What is the influence of the shunt connection (with simple and double terminals) upon the measurement errors at the ammeters?
2. **Question** What is the influence of the connection of the additional resistance (of the contact resistance) upon the measurement errors at the voltmeters?

3. Set up the circuit for the ammeters range extension using **two connection terminal shunt**



Complete the table:

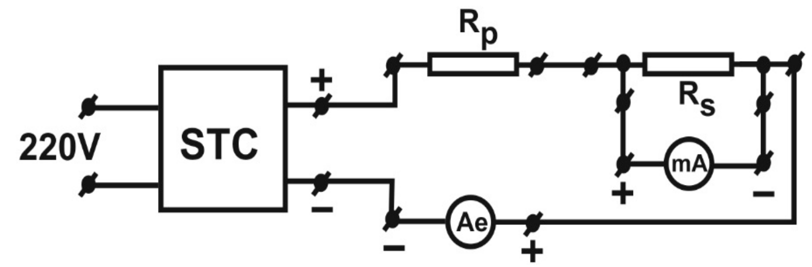
I_{SN}	n	The extended device			The standard device		ε_a *(rel 20)*	ε_r *(rel 21)*	n' *(rel 20)*	R_c
		I_a		I	I_e					
[mA]		[div]	[mA]	[mA]	[div]	[mA]	[mA]	[%]		[Ω]

Ra=…………………..

Write down the relations for:

ε_a ε_r ***n'*** R_c

1. Set up the circuit for the ammeters range extension using **four connection terminal shunt**



Complete the table:

I_{SN}	n	The extended device			The standard device		ε_a *(rel 20)*	ε_r *(rel 21)*	n' *(rel 20)*	R_c
		I_a		I	I_e					
[mA]		[div]	[mA]	[mA]	[div]	[mA]	[mA]	[%]		[Ω]

Ra=…………………..

Write down the relations for:

ε_a ε_r ***n'*** R_c

ANALOG AND DIGITAL MEASUREMENT DEVICES

3.1. Theoretical Approach

The digital measurement devices present the result of the measurement in digits. These devices are connected to the circuit depending on their function: ammeters in series, voltmeters in parallel, wattmeters with the current circuit connected in series with the load, and the voltage circuit connected across the line on which the measurement is done.

The nominal range represents a value that was chosen by the manufacturer and is between the minimum and maximum limits that can be measured, and for which the device keeps its characteristics.

The measured value is displayed on a certain number of digits, a digit representing an element of the display that allows visualizing the numbers from 0 to 9. A fraction of digit allows visualizing only some numbers between 0 and 9. Examples:

- on 1/2 digits can be displayed 0 and 1;
- on 2/3 digits can be displayed 0, 1 and 2;
- on 3/4 digits can be displayed 0, 1 ,2 and 3.

The number of points is the number of different results that can be represented on the display without taking into account the decimal comma. So, on a digit can be displayed 10 measurement points, and on a device with 3 ½ digits can be displayed 2000 different results (0000…1999), so it has 2000 points. The resolution of the device is the smallest measurable difference between 2 values and it's equal to the value of a point.

The measurement errors can be classified in three important categories:

- the error of the device;
- the reading error, which appears because of the modification of the last digit ±1, for example due to the perturbations;
- the error determined by the finite resolution of the device.

The digital devices have usually two inputs, one for DC and one for AC. On the DC input, the device displays the mean value of the input signal x(t),

$$X_A = \overline{X} = X_0 + \frac{1}{T}\int_0^T x_p(t)dt \qquad (3.1)$$

where: - X_A– the value displayed by the device;

- X_0– the DC component of the measured value;
- $X_p(t)$ – the periodic component (variable in time) of the measured value.

On the AC input, the displayed value can be the effective value of the measured value, calculated with the relation:

$$X_A = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \qquad (3.2)$$

or it can be the effective value calculated from the mean value and the form factor for the measured value, with the relation:

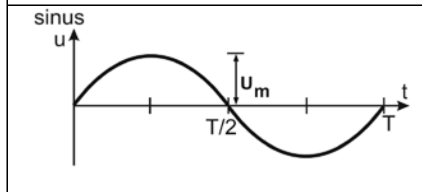
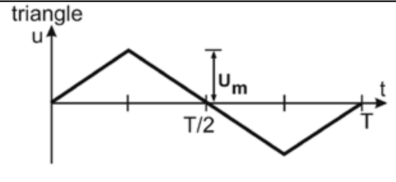
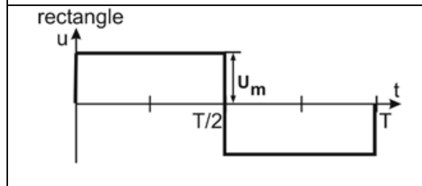
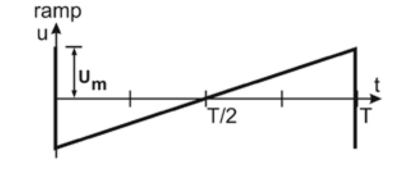
$$X_A = k\frac{1}{T}\int_0^T |x(t)|dt \qquad (3.3)$$

where: - $x(t)$ – the alternative signal that must be measured;

- k_f –the form factor.

In this last case, the devices display the effective value of the sinus corresponding to the mean value, and the device must be calibrated for the form factor of the sinus $k = \pi / 2\sqrt{2} = 1.11$. In table 3.1 some characteristic values of some waves are presented.

Table 3.1

Waveform	Effective value (U)	Mean value (Umean)	Crest factor. (k_c)	Form factor (k_f)	Distortion factor (k_d)
	$\frac{U_m}{\sqrt{2}}$	$\frac{2 \cdot U_m}{\pi}$	$\sqrt{2}$	$\frac{\pi}{2\sqrt{2}} = 1.11$	0
	$\frac{U_m}{\sqrt{3}}$	$\frac{U_m}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = 1.156$	0.12
	U_m	U_m	1	1	0.435
	$\frac{U_m}{\sqrt{3}}$	$\frac{U_m}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	0.626

The characteristic parameters of a variable quantity in time $x(t)$ with the period T are the followings:

-the maximum (peak) value:

$$X_{max} = X_m \qquad (3.4)$$

-the effective value:

$$X = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt} \qquad (3.5)$$

-the mean value:

$$X_{mean} = \frac{1}{T}\int_0^T x(t)dt \qquad (3.6)$$

-the crest (peak) factor:

$$k_c = \frac{X_m}{X} \qquad (3.7)$$

-the form factor:

$$k_f = \frac{X_m}{X_{mean}} \qquad (3.8)$$

3.2 Experimental Design and Procedure

3.2.1 Equipment

DC Power supply – 0 ÷ 15 ÷ 30 V ; 0 ÷ 1A;

A – Analog d.c. ammeter, 0 ÷ 500mA; accuracy class c = 0,2;

V – Analog d.c. voltmeter, 0 ÷ 7,5 ÷15 ÷30 ÷75 V, accuracy class c =0,2;

Rh – Protection rheostat for the limitation of the working current;

DM – Digital multimeter, type E0302 or HM8012;

FG – Functions generator, Tabor WW5061;

O – Oscilloscope.

3.2.3 Description of the Experimental Procedure

Activity 1. Working with MTX3250 Digital Multimeter (DMM)

The digital multimeter (DMM) MTX3250 main characteristics will be studied. According with the instrument characteristics for measuring DC voltages there are the next information:

VDC direct voltage On this position, users can measure a direct voltage value or the direct component of alternating voltage (you must keep to the range corresponding to the rms value, see Serial mode).

Range	Accuracy	Resolution	Input resistance	Admissible overload
500 mV	0.08% + 3D	0.01 mV	10 MΩ / (*)1 GΩ	1000 VDC or 700 VAC (1 min max.)
5 V	0.08% + 3D	0.1 mV	11 MΩ	
50 V	0.08% + 3D	1.0 mV	10 MΩ	
500 V	0.1% + 3D	10 mV	10 MΩ	
600 V	0.1% + 3D	0.1 V	10 MΩ	

The digital DC voltmeter is studied using circuit drawn in Fig.3.1. By adjusting the voltage generated by the power supply (rough adjustment) and by moving the cursor of the variable resistor *Rh* (smooth adjustment), at least 10 measured values will be chosen corresponding to the divisions of 3V domain of the analog voltmeter V. The values will be recorded in Table 3.2.

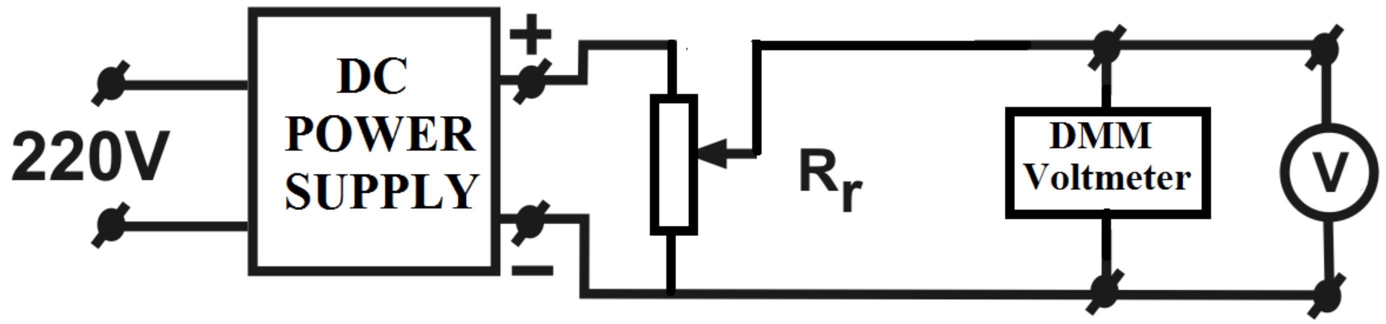


Fig.3.1 *The experimental circuit for digital DC voltmeter*

According with the instrument characteristics for measuring DC currents there are the next information:

Measuring current

ADC current In "DC" mode, users can measure a direct current value or an alternating current's direct component.

Rate	Accuracy	Resolution	Input resistance	Admissible overload
500 µA	0.2% + 5D	10 nA	about 350 Ω	20 Arms (30 s max.)
5 mA	0.2% + 3D	0.1 µA	about 35 Ω	
50 mA	0.2% + 3D	1.0 µA	about 5 Ω	
500 mA	0.2% + 5D	10.0 µA	about 0.5 Ω	
10 A	0.5% + 5D	1 mA	about 0.02 Ω	

For the study of the digital DC ammeter setup the circuit from the Fig.3.2. By adjusting the power supply and the resistor *Rh* we choose a minimum number of 10 measured values in the range of 20mA to 200mA, so that they correspond to some divisions of the analog ammeter A. The values shown by the digital ammeter are recorded in Table 3.3.

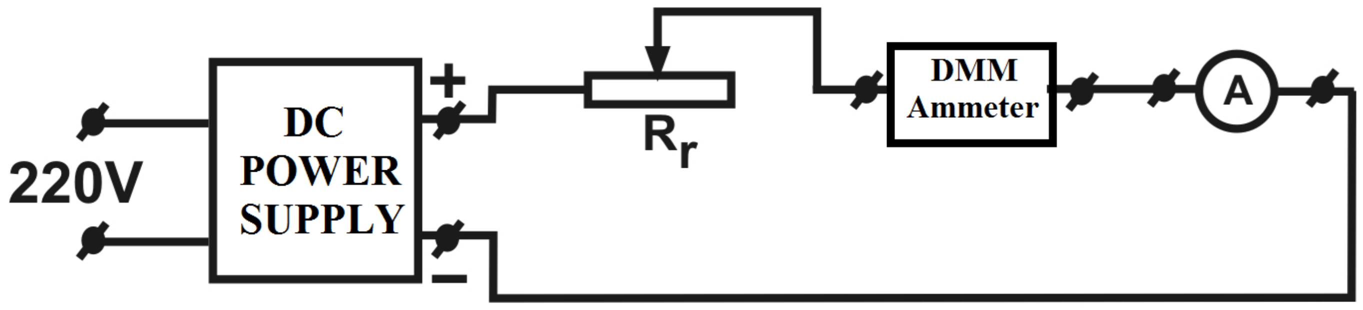


Fig.3.2 *The experimental circuit for digital ammeter*

For measuring the periodical signals, setup the circuit from the Fig.3.3 corresponding to the AC digital voltmeter. From the signal generator set the item to measure on the frequency of 1000 Hz, with the following waveforms: sinus, triangle, rectangle and ramp. For each waveform (which will be shown on the oscilloscope's screen), set a minimum of 5 voltage values indicated by the digital multimeter. The results are recorded in the Table 3.4.

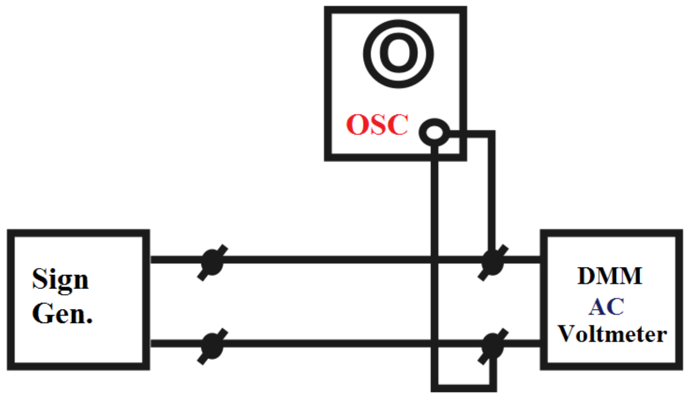
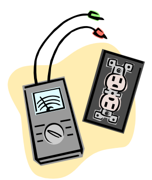


Fig.3.3 *The experimental circuit for measuring the periodical signals*



Activity 2. Working with HM 8012 Digital Multimeter

HM 8012 is a 4 3/4 programmable digital multimeter (DMM). The device is provided with Wdm8012 Windows software and allows us to measure different variables with the aid of the computer. All instrument functions can be controlled by a host computer through serial interface (RS 232) available in standard.

The Wdm8012 software is a virtual instrument allowing the command of the multimeter and reading of its configuration (Fig.3.4 and Fig.3.5).

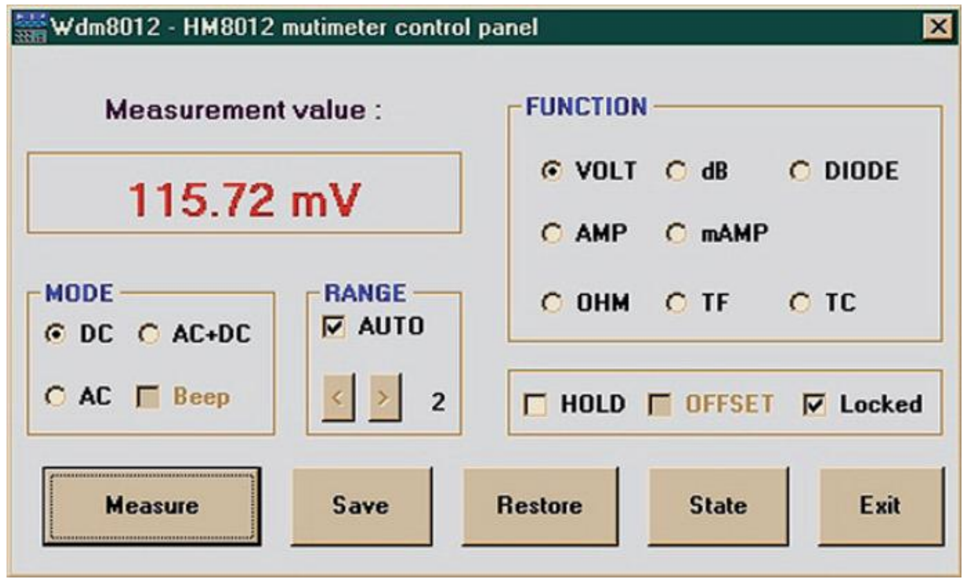


Fig.3.4 *The multimeter control panel*

When the configuration of the instrument is done, a set of measurements can be carried out and saved for future use. Furthermore, the software can show deviations of the values relative to two predetermined thresholds.

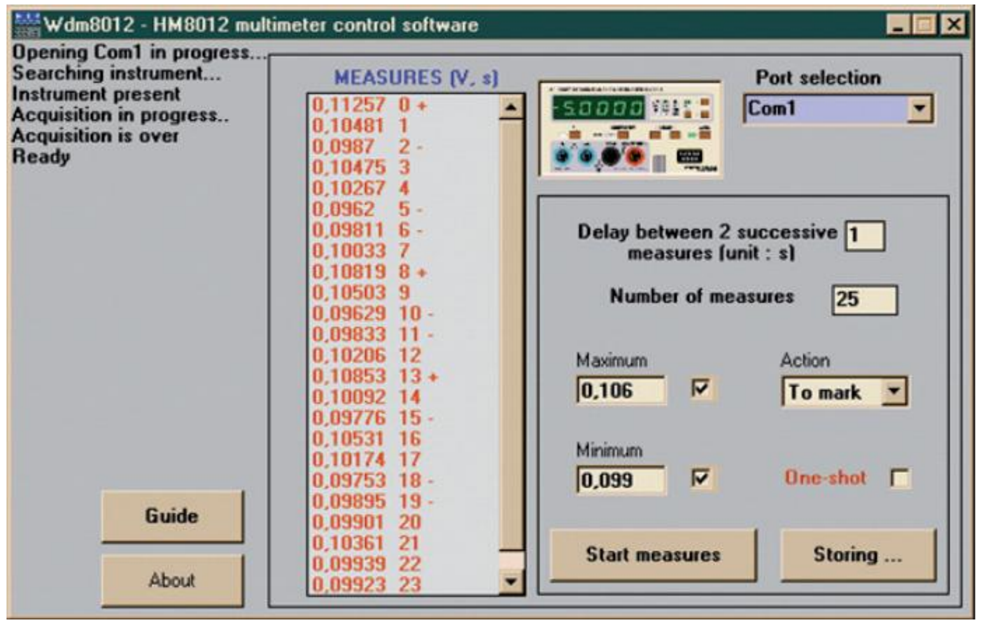


Fig.3.5 *The multimeter software panel*

3.3 Experimental Results

The maximum absolute measurement errors of digital multimeter in function of the specifications from the technical book are computed depending on the domain. If, for example the range of DC Voltmeter is 500mV and the voltmeter indication is 115.72mV then:

$$e_{aDMM} = 0.08\% \cdot 115.72\text{mV} + 3 \cdot 0.01\text{mV} = 9.28\text{mV} \quad (3.9)$$

where: e_{aDMM} – the maximum absolute measurement error of the digital voltmeter;

In general, the relative measurement error calculated using the device specifications is:

$$e_{rDMM} = \pm \frac{e_{aDMM}}{x_{iDMM}} 100 \quad (3.10)$$

The relative measurement error calculated using the analog instrument is:

$$e_{rAnalog} = \pm \frac{e_{aAnalog}}{x_{iAnalog}} \quad (3.11)$$

Record data in Tables 3.2 and Table 3.3, calculate errors according with relations (3.9), (3.10), and (3.11) and represent graphically these errors function of measured values.

In the case of periodical signals measurements, you must take into account that almost multimeters measure the effective value of the sinusoidal voltages by scaling the mean value. The characteristic values are computed with the relations:

$$U_{mean} = \frac{U_i}{k_{f\sin}} [\text{V}] \quad (3.12)$$

$$U = U_{mean} \cdot k_f = \frac{k_f}{k_{f\sin}} U_i [\text{V}] \quad (3.13)$$

$$\mathrm{U_m} = \mathrm{U} \cdot \mathrm{k_v} = \frac{\mathrm{k_f} \cdot \mathrm{k_c}}{\mathrm{k_{f\sin}}} \mathrm{U_i} [\text{V}] \quad (3.14)$$

U_i represents the indicated voltage value which is read directly from the measurement device. All of these values will be recorded in Table 3.4.

With the aid of the HM 8012 digital multimeter a set of 50 resistances will be measured and the experimental results will be analyzed by using statistical methods.

Experimental data will be written in Table 3.5.

The array of measured values will be represented as a histogram and the frequency polygon will be made.

For the measured values the length of the grouping interval shall be determined, using the *Sturges* formula:

$$d = \frac{\mathrm{X_{m\,max}} - \mathrm{X_{m\,min}}}{1 + 3.22 \log n} \quad (3.15)$$

where: - $X_{m\,max}$ - the maximum value of the measured resistance (from Table 3.5);

- $X_{m\,min}$ - the minimum value of the measured resistance (from Table 3.5);

- n - the number of measurements.

The d value will be rounded up to a unit or to a tenth of Ω (according to the value of the measured resistance).

Based on the grouping interval length value (d) the grouping intervals and the central value (the middle of every interval) will be established. These obtained data will be written in Table 3.6.

The number of data corresponding to every grouping interval -the absolute frequency (n_i)-will be determined.

The relative frequency (f_i) that represents the ratio between the absolute frequency and the total number of measurements (n) will be also determined:

$$f_i = \frac{n_i}{n} \quad (3.16)$$

The histogram must be drawn and it's consists of rectangle shapes having their basis equal to the grouping interval and their heights equal to the frequency (absolute or relative).

The frequency polygon will be made, and it's obtained by connecting the superior middles of the histogram with segments.

After these steps it is necessary to compute some static indicators:

a. Arithmetic mean of measured sizes:

$$\overline{X} = \frac{X_{m1} + X_{m2} + \ldots + X_{mn}}{n} = \frac{1}{n} \sum_{i-1}^{n} X_{mi} \quad (3.17)$$

b. Geometrical mean:

$$G = \sqrt[n]{X_{m1} \cdot X_{m2} \cdot \ldots \cdot X_{mn}} = \sqrt[n]{\prod_{i-1}^{n} X_{mi}} \quad (3.18)$$

c. Quadratic mean:

$$X_p = \sqrt{\frac{1}{n}(X_{m1}^2 + X_{m2}^2 + \ldots + X_{mn}^2)} \quad (3.19)$$

and the variation of the static indicators:

a. The variation domain of the studied size:

$$\mathrm{W} = X_{m\max} - X_{m\min} \quad (3.20)$$

b. Standard deviation:

$$S = \sqrt{\frac{\sum_{i-1}^{n}(X_{mi} - \overline{X})^2}{n-1}} \quad (3.21)$$

c. Variance:

$$S^2 = \frac{\sum_{i-1}^{n}(X_{mi} - \overline{X})^2}{n-1} \quad (3.22)$$

d. Probable error of the average measured value:

$$\Delta\overline{X} = 0.6745 \frac{S}{\sqrt{n}} \quad (3.23)$$

Based on the calculus already made, the value of the measured size will be written:

$$X = \overline{X} \pm \Delta X \quad (3.24)$$

The Gauss normal distribution curve (the probability density function) of the obtained measurement results, will be calculated and built.

The Gaussian probability density function of the results (sample of sizes), considered as purely random variables, will be written as it follows:

$$f(X_m) = \frac{1}{S\sqrt{2\pi}} e^{-\frac{(X_m - \overline{X})^2}{2S^2}} \qquad (3.25)$$

for - ∞< X_m < +∞ where $f(X_m)$ represents the relative frequency of the samples of the measured values X_m. For the curve construction, $f(X_m)$ will be calculated for the following values of deviation (S = 0; ±0.5; ±1; ±1.5; ±2; ±2.5; ±3)-see Fig. 3.6.

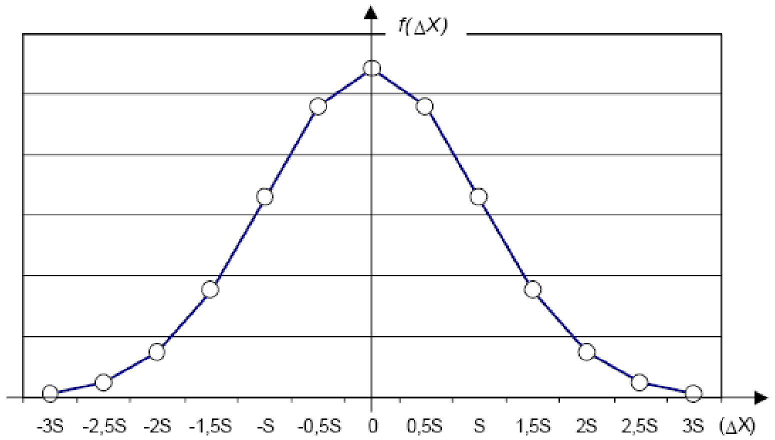


Fig.3.6. *Gauss normal distribution curve (probability density function) of the measurement results for a sample of sizes (example)*

3.4. Analysis and Conclusions

After finishing the work, the results will be analyzed and will be formed conclusions referring to the:

- absolute and relative errors of the digital multimeters;
- possibility of measuring periodical signals other than sinusoidal type.

3. **Assignment.** Make the statistical analysis of the indicated parameters using Excel and compare the results with those made by hand

1. Complete the tables:

Nr.	Meter	Range X_N	Max. defl. α_{max}	Instr. Constant $C = \frac{X_N}{\alpha_{max}}$	Accuracy class c	Max. abs. error $\varepsilon_{max} = \frac{cX_N}{100}$
	type	[X]	[div.]	[X/div]	[%]	[X]

Range (X_N)			R_a	R_v	R_{wi}	R_{wu}	R_{aw}	R_{vw}
ammeter	voltmeter	wattmeter						

R_a – ammeter internal resistance;

R_v – voltmeter internal resistance;

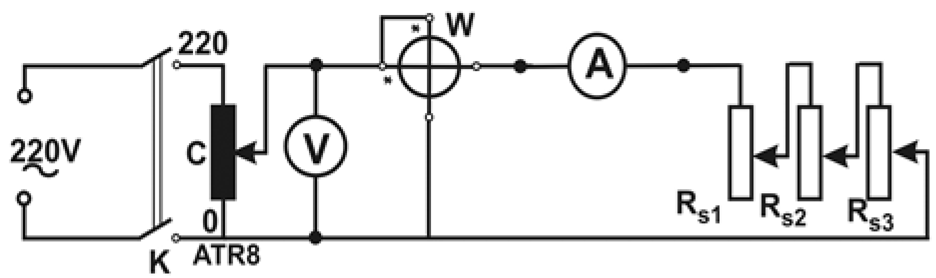
R_{wi} – wattmeter current coil resistance;

R_{wu} – wattmeter voltage coil resistance;

$R_{aw} = R_a + R_{wi}$;

$R_{vw} = R_v \,||\, R_{wu}$

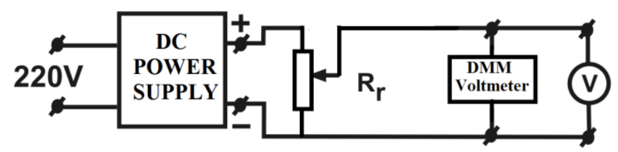
1. Set up the experimental circuit **with resistive load**



Complete the table:

Ammeter				Voltmeter				Wattmeter	
Deflection α	Measured value I	True value $I \pm \varepsilon_{max}$	Relative error ε_r $\frac{\varepsilon_{max}}{I}100$	Deflection α	Measured value U	True value $U \pm \varepsilon_{max}$	Relative error ε_r $\frac{\varepsilon_{max}}{U}100$	Measured value P	
[div]	[A]	[A]	[%]	[div]	[A]	[A]	[%]	[div]	[W]

Set up the experimental circuit for digital DC voltmeter



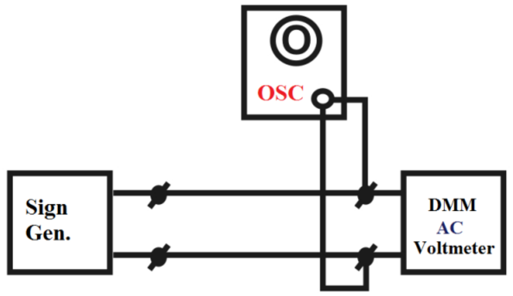
Complete the table:

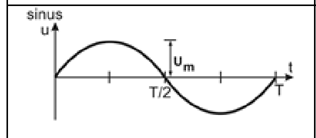
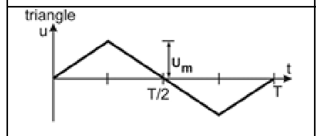
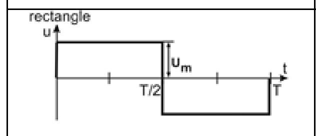
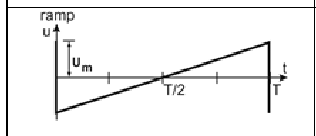
DMM/Digital Voltmeter	Analog Voltmeter	DMM U_i	Analog U_i		DMM e_{max}	Analog e_{max}	DMM e_r[%] $\frac{e_{max}}{U_i}100$	Analog e_r[%] $\frac{e_{max}}{U_i}100$	DMM U_m $U_i \pm e_{max}$
Range U_N/ Resolution	Range/ U_N Resolution	[V]	[div]	[V]	[V]	[V]	[%]	[%]	[V]

Write down the relations for:

DMM BK Precision	$e_{max} = 0.012\% \cdot U_i + 5\text{digit}$ 1digit=……………….	**Analog** $c =$ U_N=	$e_{max} = \dfrac{c \cdot U_N}{100}$	

1. Set up the experimental circuit for measuring the periodical signals:



Waveform	Effective value (U)	Mean value (U_{mean})	Crest factor. (k_c)	Form factor (k_f)	Distortion factor (k_d)
	$\frac{U_m}{\sqrt{2}}$	$\frac{2 \cdot U_m}{\pi}$	$\sqrt{2}$	$\frac{\pi}{2\sqrt{2}} = 1.11$	0
	$\frac{U_m}{\sqrt{3}}$	$\frac{U_m}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}} = 1.156$	0.12
	U_m	U_m	1	1	0.435
	$\frac{U_m}{\sqrt{3}}$	$\frac{U_m}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	0.626

Complete the table:

Waveform	**Pick coeff. (k_v) (table)**	**Shape coeff. (k_f) (table)**	**Indicated value (U_i)**	**Mean value** $U_{mean} = \frac{U_i}{k_{f\,sin}}$	**RMS value** $U = \frac{k_f}{k_{f\,sin}} U_i$	**Max. value** $U_m = \frac{k_f \cdot k_c}{k_{f\,sin}} U_i$	**Osc. pk to pk ($U_{v\text{-}v}$)**
			[V]	**[V]**	**[V]**	**[V]**	**[V]**

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ELECTRONIC TEST INSTRUMENTS

5 1/2 digits Dual Display Multimeter

Model 5492

Data Sheet

Features

- Selectable 120,000 / 40,000 / 4,000 Count
- Dual Display
- True RMS (AC, AC+DC), 40Hz to 30KHz Measurement Bandwidth
- 2 or 4 wire selectable for Resistance Measurements
- MIN/MAX
- Selectable measurement rates
- Data Hold
- RS 232 interface
- GPIB version available (model 5492GPIB)

Specifications

DC Voltage
    Maximum Range: 1000V
    Best Accuracy: 0.012% + 5 dgts.*
    Best Resolution: 10mV*
AC Voltage (True RMS) Freq. 50Hz to 5KHz
    Maximum Range: 750V
    Best Accuracy: 1% + 40 dgts.*
    Best Resolution: 10mV*
DBm (600W Ref.)
    Range: -31 to 59*
    Resolution: 0.01dB
    Best Accuracy: 0.8dB*
DC Current
    Maximum Range: 12A
    Best Accuracy: 0.1 + 3 dgts.*
    Best Resolution: 1mA*
AC Current (True RMS, AC Coupled) Freq. 50Hz to 2KHz
    Maximum Range: 12A
    Best Accuracy: 0.5 + 12 dgts.*
    Best Resolution: 1mA*
Resistance
    Maximum Range: 300MW
    Best Accuracy: 0.06 + 3 dgts.*
    Best Resolution: 10mW*
Frequency
    Maximum Range: 120KHz
    Best Accuracy: 0.005 + 2 dgts.
    Best Resolution: 10mHz

* = Medium Measurement Rate

THE WHEATSTONE BRIDGE

This laboratory will present a null measurement method based on a DC Wheatstone bridge for measuring resistances in a range of 1Ω-10MΩ. The laboratory also provides the methods for computing the measurement accuracy, the bridge sensibility and the bridge convergence.

4.1. Theoretical Approach

The Wheatstone bridge is used for measuring resistance and it has the advantage of being highly accurate and very sensitive. The bridge scheme is shown in Figure 4.1. The balance condition (for zero deviation of the null instrument, α=0) will be:

$$R_x R_3 = R_2 R_4, \quad or \qquad R_x = R_4 \frac{R_2}{R_3} \tag{4.1}$$

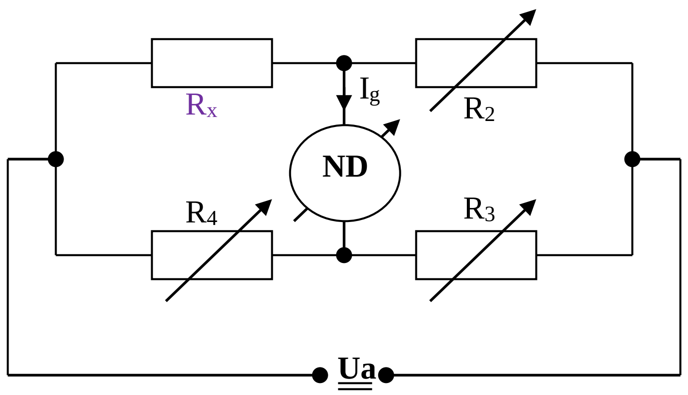


Fig.4.1. *The Wheatstone bridge principle*

In relation (4.1), R_X is the resistance to be measured. The ratio R_2/R_3 represents the “measure ratio” and it gives the order of R_X. Normally it must be modified at the beginning of the measurements. The adjustable resistor R_4 is used for fine tuning in order to obtain the balance, so the null Detector ND will indicate zero.

The permanent deflection of the ND (usually an galvanometer) is given by:

$$\alpha = S_I I_g = S_I U_a \frac{R_2 R_4 - R_x R_3}{R_4 A + B} = k \frac{R_2 R_4 - R_x R_3}{R_4 A + B} \tag{4.2}$$

In (4.2), S_I represents the ND sensitivity (galvanometer sensitivity) and A and B are some parameters expressed function of R_1, R_2., R_3, and R_g. The bridge balance characteristic, α=α(R_4), is shown in Figure 4.2.

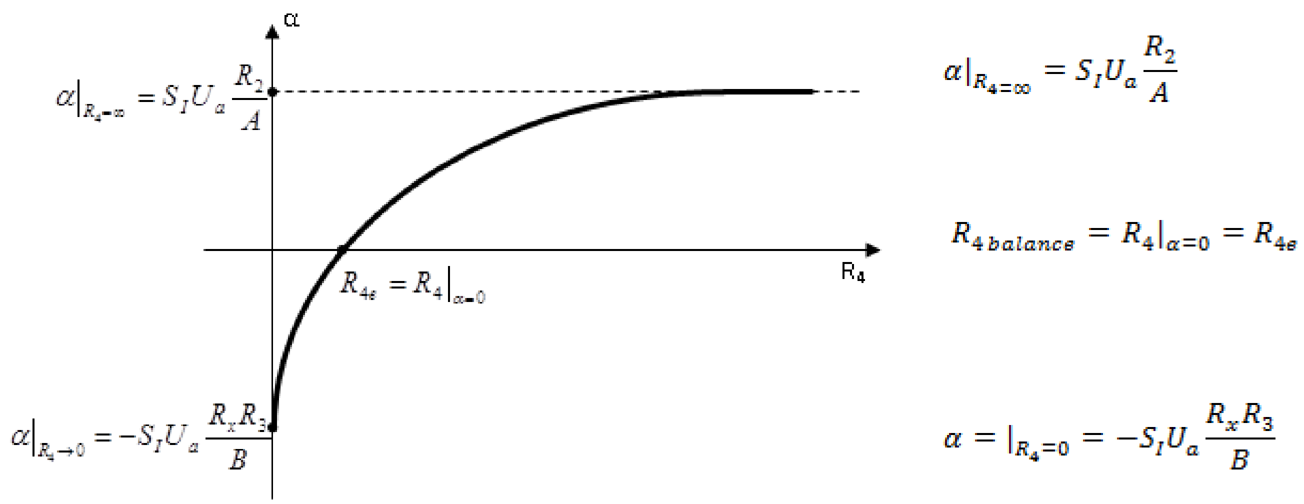


Fig.4.2. *The bridge balance characteristic*

If the fine tuning of the resistance R_4 will not give a zero deflection of ND (α=0), but two nonzero deflections will result (αl left, αr right)for two successive values of the balance resistance (R_{4l} and R_{4r}), the value of the R4 resistance (Figure 3) will be calculated with the interpolation relation:

$$R_{4i} = R_{4s} + (R_d - R_{4s}) \frac{\alpha_s}{\alpha_s + \alpha_d} \tag{4.3}$$

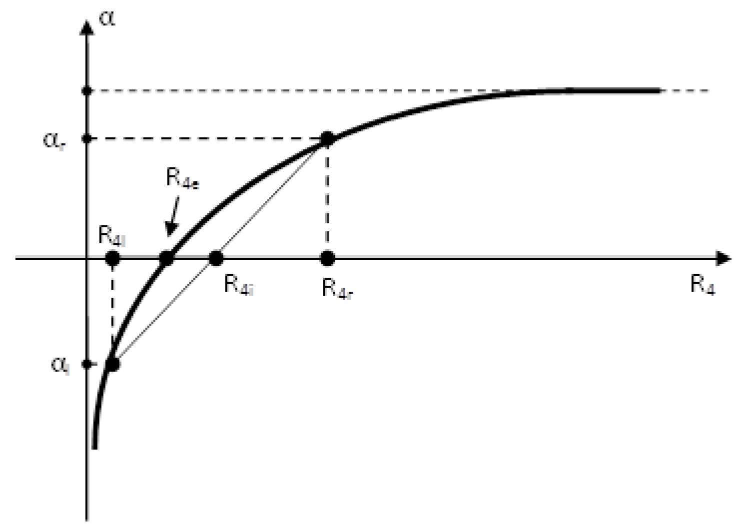


Fig.4.3 *The balance R4 interpolated value computation*

The Wheatstone bridge sensitivity in report with the balance resistance R4 is:

$$S = \lim_{\Delta\alpha \to 0} \frac{\Delta\alpha}{\frac{\Delta R_{4e}}{R_{4e}}} = \frac{d\alpha}{\frac{dR_{4e}}{R_{4e}}} \tag{4.4}$$

The relation (4.4) is valid in the vicinity of R_4 values, where the curve α=f(R_4) can be considered linear.

To give the value of the unknown resistor and the measurement accuracy, the following will be taken into consideration:

- The relative error of measured resistance is given by:

$$e_{rRx} = (e_{rk} + e_{rR4}) + e_{rd} = (e_{r2} + e_{r3} + e_{rR4}) + e_{rd} = e_{rc} + e_{rd} \qquad (4.5)$$

In (4.5) e_{rk} is the relative error given by the measure ratio R2/R3, e_{rR4} is the relative error given by R_4 and e_{rd} is the realtive error coresponding to the interpolated value of R_4 (rel 4.6), e_{rc} is the constructive relative error of the Wheatstone bridge.

$$e_{rd} = \frac{R_{4d} - R_{4s}}{R_{4i}} \frac{\Delta\alpha}{\alpha_d + \alpha_s} \qquad (4.6)$$

In (4.6) $\Delta\alpha$ is the a fraction from ND resolution (usually a half of one division or digit).
The absolute error of the measured unknown resistance:

$$e_a = \frac{e_r R_{mas}}{100} \qquad (4.7)$$

In (4.7) R_{RxN} is the nominal value of the unknown resistance. The interval in which the probabil value of the unknown resistance is found:

$$R_X = R_{mas} \pm e_a \qquad (4.8)$$

Another element that is very important when talking about a bridge (DC or AC type) is the its convergence. In the Wheatstone bridge the problem is solved by computing the variation of the logarithmic decrement of the ND indication, which reflects balance process (number of intermediary balances-n, the balance rapidity). The logarithmic decrement is defined as being the ratio between two consecutive ND indications in the balance process (Figure 4.4):

$$\delta = ln\frac{\alpha_2}{\alpha_1} = 2{,}303 lg\frac{\alpha_2}{\alpha_1} \qquad (4.9)$$

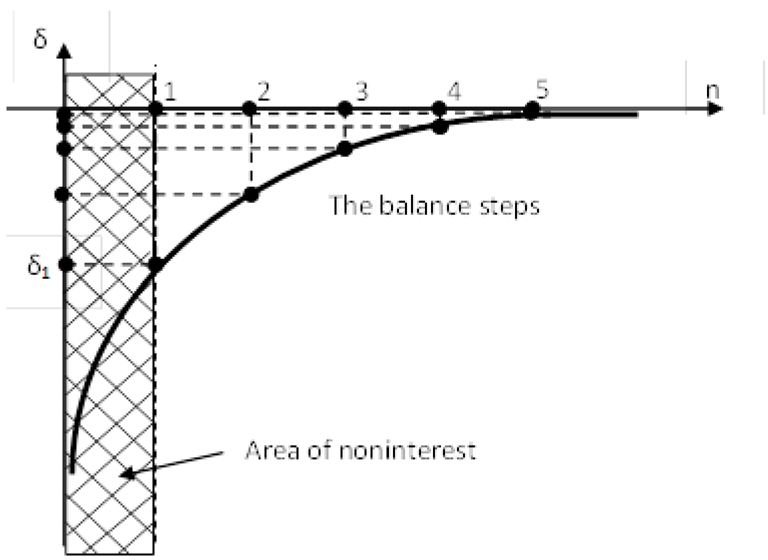


Fig.4.4*The balance R4 interpolated value computation*

The bridge is in balance when the curve δ=f(n) becomes asymptotic at the abscises axis. The value of δ_{max} ($\delta_{max}=ln\alpha_n/\alpha_1$) is depending on the sensitivity and range of the ND instrument. In general, the ratio α_n/α_1 will be not smaller than 10^{-3}, $10^{-4}$and is limited by the ranges and sensitivities usually null detectors (multimeters, galvanometers, nano and pico ammeters or micro voltmeter). Considering the above, the value of δ_{max} will lie in a range of (-6.909…. -9.212).

4.2 Experimental Design and Procedure

4.2.1 Equipment

DC Power supply – 0 ÷ 15 ÷ 30 V ; 0 ÷ 1A;

- *Wheatstone bridge, which contains tree adjustable resistances R2, R3, R4, with a precision class of c=0,05;*
- *MTX3250 Digital Multimeter (DMM)*
- *BK Precision 9121 Single Output Programmable DC Power Supply , adjustable from 0-60V, 5A*
- *Rx – 10 unknown resistances.*

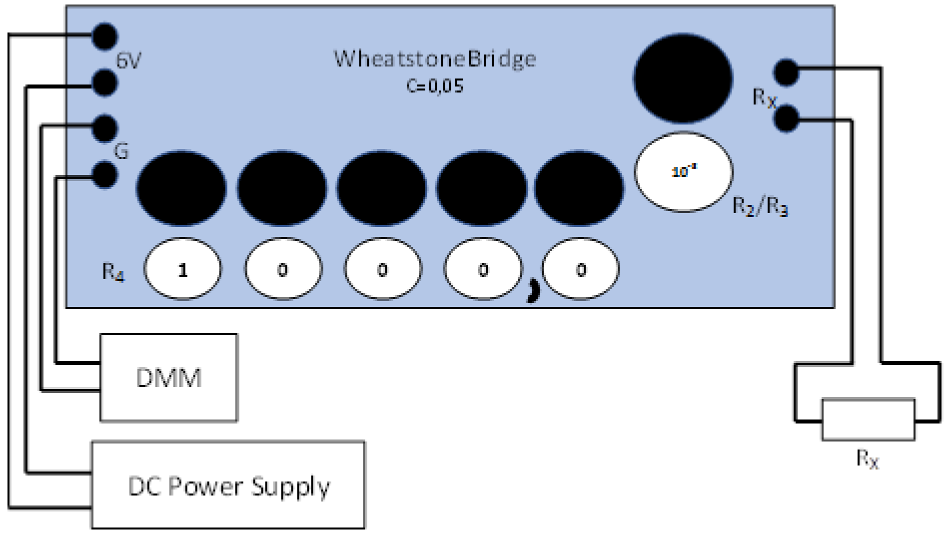


Fig.4.5 *The experimental Wheatstone bridge setup*

4.2.2 Description of the Experimental Procedure

Activity 1. Determining the unknown resistances value and the measurement error

For the resistance to be measured the next steps will be followed:

a) The ND will be set to its smallest sensitivity value (mV). The bridge will be supplied being careful not to have a zero-resistance arm. The most significant decade R4 will be set to R=1x10000 value. Then the ratio R2/R3 will be modified until the deviation (in negative values) on the ND will be the closest possible to the zero value. The value of α1 will be written down in Table 4.1.

b) The fine adjustments of the bridge will be done by modifying R4. First, the adjustments start with most significant decades of R4 and continue one by one to the least significant one. (x1000, x100, x10, x1, x0.1). For every decade, the measured values of ND will be written down, there for it results a series of values α1…αn for the n steps of balancing. If the least significant decades won't modify significantly the measured value they won't be taken into consideration.

c) In order to calculate the value of Rx and the measurement errors the bridge must be balanced. The table 2 will be completed. The value of R4e will be used in the case of a perfect balance (α =0) or R4i if the interpolation is used. The obtained value is Rmas. If there are more than one measurement for the same resistance the obtained value will be the mean value $R_{med\ mas}$.

Table 4.1

Measured resistance	The ND (null detector) α deviation after the adjustment of:					
	R_2/R_3 [mV]	**The first** decade (the most significant) α1[mV]	**The second** decade α2 [mV]	**The third** decade α3[mV]	**The forth** decade α4[mV]	**The fifth** decade α5[mV]
R_1						
R_2						
R_3						
R_4						
R_5						
R_6						

Tabelul 4.2

Meas resist. R_{xN}	**Measured values**						**Computed values**						
	R_2/R_3	**R_{4e}**	**α_l** left	**R_{4l}** left	**α_r** right	**R_{4r}** right	**R_{4i}**	**R_{mas}**	**e_{rc}**	**e_{rd}**	**e_{rRx}**	**e_a**	**R_X**
	[Ω/Ω]	[Ω]	[div]	[Ω]	[div]	[Ω]	[Ω]	[Ω]	[%]	[%]	[%]	[%]	[Ω]
R_1													
R_2													
R_3													
R_4													
R_5													
R_6													

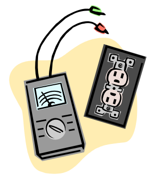
Notations:

R_{RxN} - the nominal value of the unknown resistance
R_{4e} – the value of R_4 at bridge balance where $\alpha_s = \alpha_d = 0$;
α_l, α_r – the smallest possible deviation to the left and right;
R_{4l},R_{4r} – the values of R_4 proper to α_l, α_r deflection;
R_{mas} – value of the measured resistance;
e_{rRx} – relative error for determining of the resistance;
e_{rd} – the relative error corresponding to the interpolated value of R4;
e_a – known absolute error of the measured resistance;
R_x – probable interval of values for the measured resistance.

Use the relations for:

e_{rc} constructive rel. error	e_{rc}=	e_{rRx} relative error of Rx	$e_{rRx} = e_{rc} + e_{rd}$	e_a abs. error	$e_a = \frac{e_r R_{mas}}{100}$	$R_X = R_{mas} \pm e_a$

The α=f(R_4) will be represented on a graphic according to Figure 4.2.



Activity 2.. Determining the sensitivity of the bridge

There are two possible situations:

-Total balance if α=0, results R_{4e};

-Balance through interpolation, α_l, α_r, gives R_{4i};

-

a. Total balance

The characteristic of α=f(R_4) shown in Figure 4.2, can be written in the near vicinity of the balance resistance according to Figure 4.6.

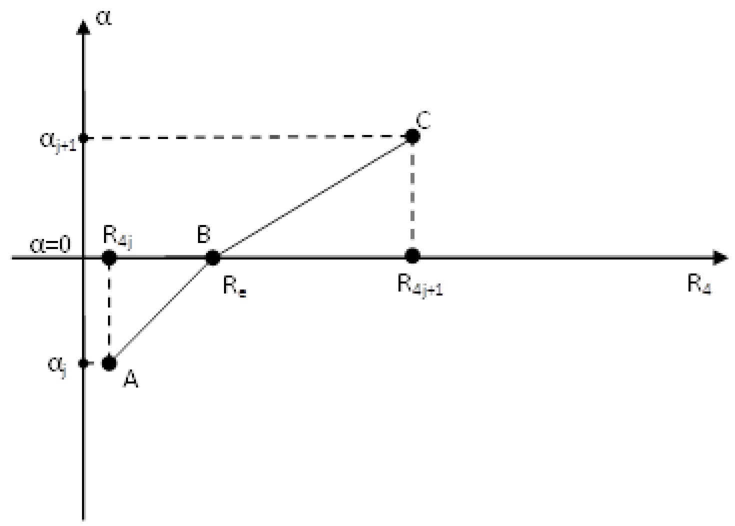


Fig. 4.6

In the near balance vicinity of the resistance R_4 two sensibilities will be defined, according to the two slopes of the AB, respectively to BC segments. Determining the two sensitivities will be done according to the relation (4.4), with indications to the left (s) and to the right (d) relative to the balance condition:

$$S_l = \frac{d\alpha}{\frac{dR_{4e}}{R_{4e}}} = \frac{\alpha_j}{\frac{R_{4e}-R_{4j}}{R_{4e}}}; \qquad S_r = \frac{d\alpha}{\frac{dR_{4e}}{R_{4e}}} = \frac{\alpha_{j+1}}{\frac{R_{4j+1}-R_{4e}}{R_{4e}}} \qquad (4.10)$$

Practically it will result the next procedure:

-If the balance α=0 is reached than the value will be R_{4e}

- A variation is given by lowering the resistance of R_{4e} (from one of the least significant decades) and the deviation α_j will be noted corresponding to R_{4j} . Then the sensitivity S_l is calculated with equation (4.10)
- A variation is given by increasing the resistance R_{4e} and the deviation $\alpha_{j+}1$ will be noted corresponding to $R_{j+1.}$ Then the sensitivity S_r is calculated with equation (4.10)

The above discussed items are available for a specific value of R_2/R_3 of the bridge. This value is usually chosen in a manner that as many as possible balance resistance decades can be used on the bridge. To observe the influence of the R_2/R_3 towards the bridge sensitivity, after measuring each resistance two ratios of R_2/R_3 will be chosen in a way so for one of them all the decades can be used and for the second one, one or two of them will be left out. The calculations for the sensitivities will be done as above mentioned, for the two values of ratio and the curves will be as the one from Figure 4.6, the two graphics will be drawn on the same graph for them to be compared.

b. Balance through interpolation

The characteristic from 4.3 described in the second section allows the calculation of the sensitivity:

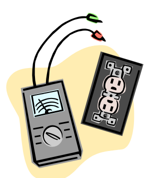
$$S = \frac{d\alpha}{\frac{dR_e}{R_e}} = \frac{\alpha_r - \alpha_l}{\frac{R_{4r} - R_{4l}}{R_{4i}}} \tag{4.11}$$

Same as at the total balance, two ratios of R_2/R_3 will be chosen for comparing the values obtained.

NOTE: The sensibilities S_l, S_r, S, will be determined separately for each resistance. The Table 4.3 should be completed.

Table 4.3

Meas resist. R_{xN}	R_2/R_3	α_s (α_j)	$R_{4l}(R_{4j+1})$	α_d (α_{j+1})	$R_{4r}(R_{4j+1})$	S_s	S_d	S
R_1								
R_2								
R_3								
R_4								
R_5								
R_6								



Activity 3. Determining the logarithmic decrement

With the values from ***Activity 1*** and using relation (4.9) the logarithmic decrement values will be calculated for each unknown resistance. The Table 4.4 will be completed and the graphic δ=f(n) should be drawn, where n=1..6.

Tabelul 4.4

Meas resist. R_{xN}	The logarithmic decrement values				
	$\delta 1 = \ln \alpha 2/\alpha 1$	$\delta 2 = \ln \alpha 3/\alpha 2$	$\delta 3 = \ln \alpha 4/\alpha 3$	$\delta 4 = \ln \alpha 5/\alpha 4$	$\delta_{n-1} = \ln \alpha_n/\alpha_{n-1}$
R_1					
R_2					
R_3					
R_4					
R_5					
R_6					

Draw the graphic δ=f(n)

3.4. Analysis and Conclusions

The followings will be taken into consideration when doing this laboratory:

- Be careful when handling the components of the circuit and trying to determine with accuracy the balance moment
- Compare the graphics from Figure 4.2 to the ones that you have realized from the tables
- Comparing the logarithmic decrement results with the ones from Figure 4.4
- Interpret the results from the above results
- Compare and interpret the values of sensitivities for different values of the measure ratio