

UNIVERSITY of
HOUSTON
Solutions to Homework #4

Due 11:59pm, Monday, April 20, 2020
Multiple submissions accepted, last one graded.
100 points total.

Name: _____

PeopleSoft ID: _____

1. (20 points) Truth tables. You will find some discussion in lecture 16.
Convert the following equation into a truth table.

$$d = a * b * c + \neg a + a * \neg b * c$$

$\neg a$ means “not a”.

Use <https://web.stanford.edu/class/cs103/tools/truth-table-tool/> to learn how to build a table, and work out how to express the equation above.

- a. With n inputs, what equation do you use to calculate the number of rows you'll need?

Copy and paste the table you get from the Stanford simulator here:

a	b	c	$((a \wedge (b \wedge c)) \vee (\neg a \vee (a \wedge (\neg b \wedge c))))$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

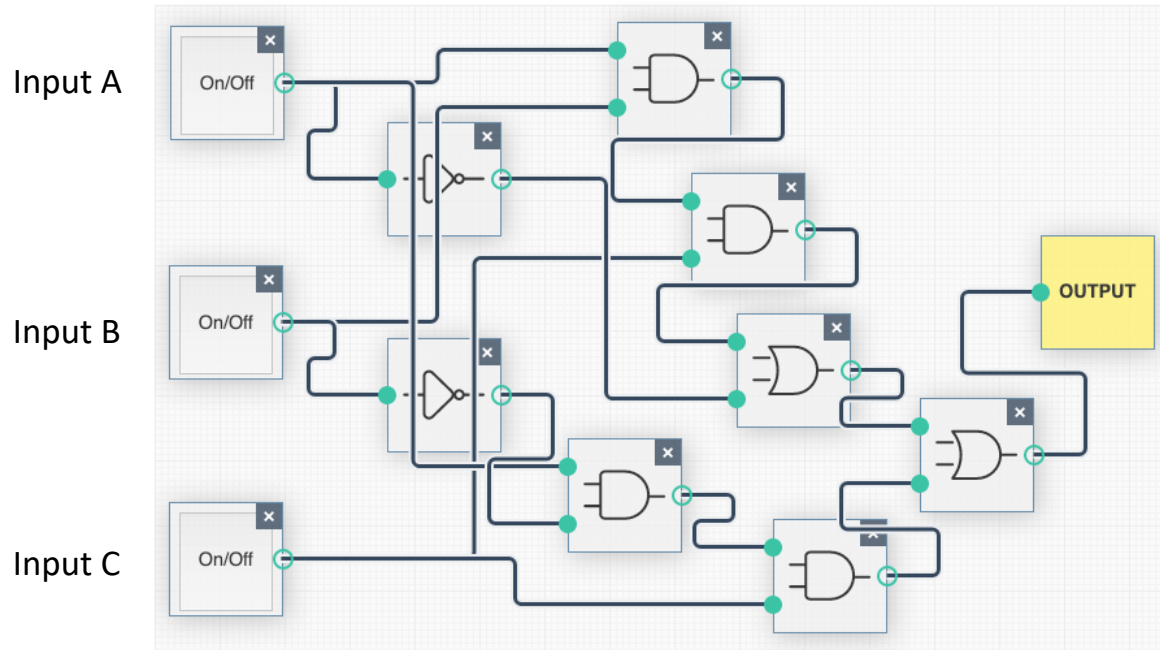
b. Consider the following rules for simplifying equations:

AND rules	OR rules	Name of rule
$AB = BA$	$A+B=B+A$	Commutative
$AA=A$	$A+A=A$	Idempotent
$A \cdot \neg A = 0$	$A + \neg A = 1$	Complement
$A \cdot 1 = A$	$A + 0 = A$	Identity
$A \cdot 0 = 0$	$A + 1 = 1$	Annulment
$\neg(\neg A) = A$		Double Negation
$A(BC)=(AB)C$	$A+(B+C)=(A+B)+C$	Associative
$A(B+C)=AB+AC$	$A+BC=(A+B)(A+C)$	Distributive
$A+(AB)=A$	$A(A+B)=A$	Absorptive 1
$A + (\neg A \cdot B) = A + B$		Absorptive 2
$\neg(A + B) = \neg A \cdot \neg B$		DeMorgan's 1
$\neg(A \cdot B) = \neg A + \neg B$		DeMorgan's 2

We will use these rules to reduce the equation in part (a) to “ $\neg A$ or C ”.
Label each step with the rule used.

- i. $d = a \cdot b \cdot c + \neg a + a \cdot \neg b \cdot c$ original equation
- ii. $d = a \cdot c \cdot (b + \neg b) + \neg a$ _____ Distributive
- iii. $d = a \cdot c \cdot (1) + \neg a$ _____ Complement
- iv. $d = \neg \neg a \cdot c + \neg a$ _____ Double Negation
- v. $d = c + \neg a$ _____ Absorptive 2

c. I built the original equation in b.i. above with a logic diagram simulator at
<https://academo.org/demos/logic-gate-simulator/>:



When the simulator opens, $a=b=c=0$ because they are grey, but the output is 1 because it is yellow. There's a video in the homework folder, here:
<https://www.dropbox.com/s/dwl46d5zva85c71/HW4P1c%20video.mov?dl=0>
 In the video, I step through all possible permutations of a , b , and c , pausing for a moment between each. When a box turns yellow, it means I've clicked on it and set it to "1". When I click again and it reverts to grey, it represents "0".

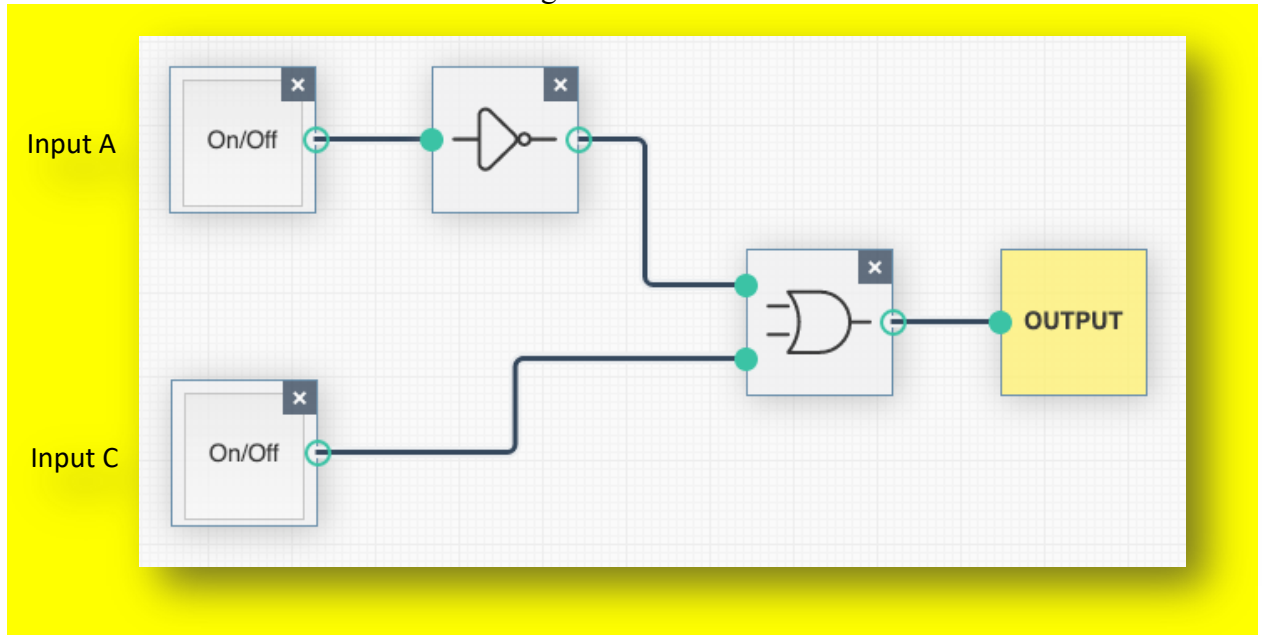
Build a truth table for the simulation based on the movie. Your answers should match what you calculated in part "a" but will have a different order. Leave the final column blank.

A	B	C	$d = a * b * c + \neg a + a * \neg b * c$	$d = c + \neg a$
0	0	0	1	1
1	0	0	0	0
0	1	0	1	
1	1	0	0	
0	0	1	1	1
1	0	1	1	1
0	1	1	1	
1	1	1	1	

- d. Your task is now to create a circuit diagram using this tool for the simplified version of the equation: $d = c + \neg a$. You must have only two inputs, one for a and one for c. Copy and paste your resulting design here:

<paste your image here>

Answer: it needs to look something like this:



Finally, go back up to the table and fill in the values this circuit produces with just the two inputs, which is going to be for four rows only, when “ac” are 00, 01, 10 and 11.

2. (20 points) Cache Simulation – from lecture 23
Visit the block replacement simulator at this site:
<http://www.ecs.umass.edu/ece/koren/architecture/Cache/frame1.htm>

Start with the following settings:

- Set the cache size to 32
- Set the # of Sets to 4
- Leave the Replacement Policy to LRU
- In another window go to <https://www.random.org/integers/> and generate 40 random integers between 0 and 64 in 1 column. They do not need to be unique among themselves; in other words, don't worry if some are repeated. Your numbers must be different than everyone else's in the course, so generate your own random sequence.
- Copy and the 40 integers you created and paste them into the box in the block replacement simulator
- Set Repeat to 3 cycles
- Turn off Limit Query at the bottom

- a. (1 pts) Copy and paste your numbers that you used here. It must be text, not a scan or a screen capture; the TA needs to be able to copy and paste your numbers and reproduce your results.

<paste your numbers here>

- b. (12 pts, ½ point per cell) Run the simulator by clicking on “Show Cache” and fill in the following table.

Answer: will depend on the student’s random numbers. Re-run their simulation if you like by copying and pasting in the numbers if you need to.

# of sets	Words per Set	Hit Rate Cycle 1	Hit Rate Cycle 2	Hit Rate Cycle 3
1				
2				
4				
8				
16				
32				

Next, download the spreadsheet attached to the homework – it is on Blackboard and on the Google Drive, named “Cache Simulator.xlsx”. There is also a google sheets version [here](#). These will open in read-only mode, so you can copy it to your own location to edit.

- c. (1 pts) Transfer your answers from (a) into the table in the Cache Simulator spreadsheet and let the graph generate, and screen grab it and paste it below.

<paste your Excel graph here>

- d. (1 pts) Take a screen snapshot of your browser window with the Block Replacement Simulator (that’s the web site). Insert it below. Take a picture with the # of sets = 8. The screen capture must show everything in the window – the settings and the results.

< paste your screen snapshot here>

Answer the following questions:

- e. (1 pts) Which # of Sets corresponds to a fully associative cache? _____

Answer: 1

- f. (1 pts) Which # of Sets corresponds to a direct-mapped cache? _____

Answer: 32

- g. (1 pts) What is as set called with 8 sets, each with 4 entries?

☐Direct-Mapped ☐4-way Set Associative ☐8-way Set Assoc. ☐Fully-Assoc.

Answer: 8-way set associative.

- h. 1 pts) What’s your highest hit rate? _____

Answer: Inspect their graph and see if it is the highest.

- i. (1 pts) In terms of hit rates, what type of cache design is the winner?

_____ Direct-Mapped _____ Set Associative _____ Fully-Associative

Answer: Normally it’s fully-associative, the first column. Inspect their graph.

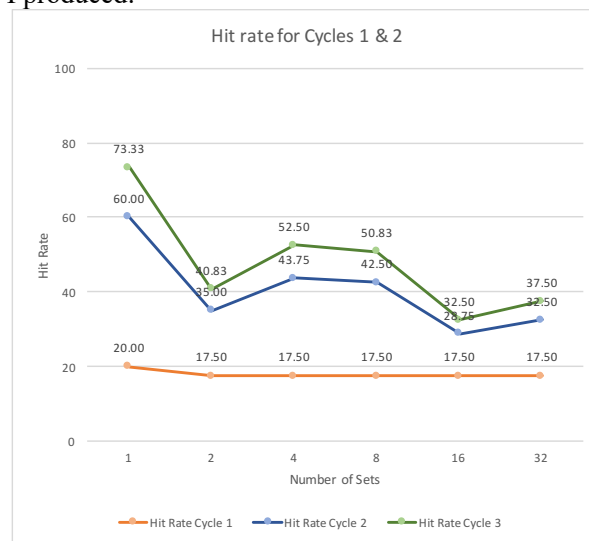
Here is an extended example:

- I generated the following 40 random numbers:
74 89 60 24 97 63 77 63 75 41 47 23 22 42 13 46 20 47 20 88 95 33 65 21 71 58
79 19 100 2 10 60 58 52 70 10 34 4 74 77

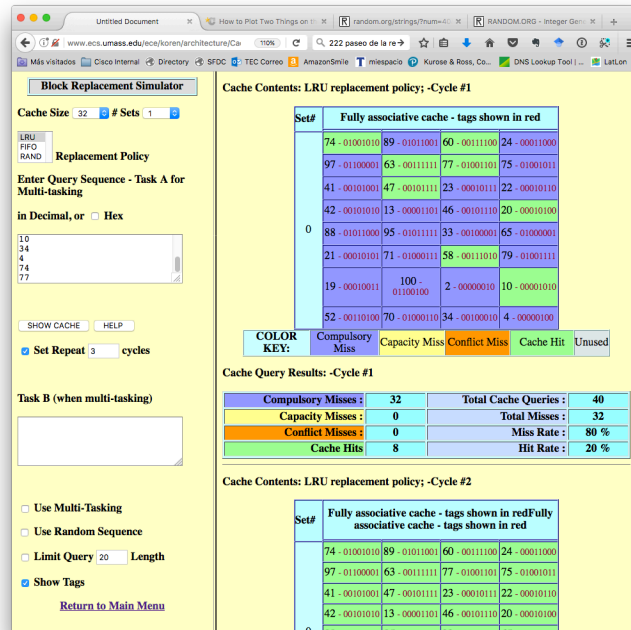
- Here's part of the table I filled out (I am leaving some cells blank):

# of sets	Words per Set	Hit Rate Cycle 1	Hit Rate Cycle 2	Hit Rate Cycle 3
1		20	60	73.33
2		17.5	35	40.83
4		17.5	43.75	52.5
8		17.5	42.5	50.83
16		17.5	28.75	32.5
32		17.5	32.5	37.5

- Here's the graph I produced:



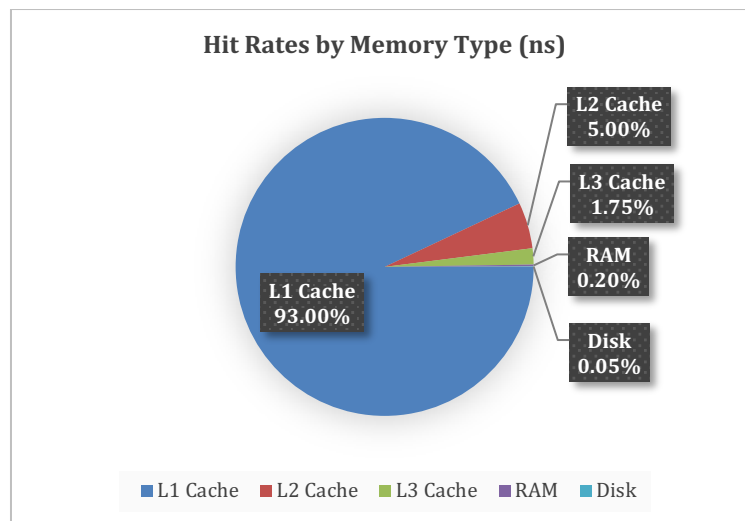
- Here's a snapshot of one of the 3-cycle simulations:



3. (20 pts) AMAT - Average Memory Access Time

Consider the cache system shown below. Memory access is sequential as discussed in lecture. Some quick definitions:

- Isolated time** is the time required to search in that level of memory. It's how fast that level is for a single search, and that speed never changes.
- Cumulative time** is the total time spent searching for an item so far if it is ultimately found in the row you're calculating. Just keep adding in the isolated time as you go.
- The **global hit rate** is the % of the time the answer is found in that category as a proportion of 100% of of the searches overall. There are other ways of calculating this, namely, the % out of the time left, but we're not doing it that way. So the way to think of the way we're doing it is as slices of a pie, where they have to all add up to 100% like this example. 93% of items are found in L1 cache, 5% in L2, and so on.



- The **weighted time** is the cumulative time multiplied by the global hit rate of the overall

searches that are hits in that level. If 5% of the time you find an item in L2 and the cumulative time for L2 is 10ns, then on average it will add $10 \times .05 = 0.5\text{ns}$ to the overall average, so you'll write "0.5" in the weighted time column.

- AMAT is the sum of weighted times.

(20 pts) Using this cumulative approach, complete the following table to calculate AMAT for a system with 5 levels of memory: 3 cache plus RAM and Disk.

Mem Hierarchy	Isolated Time (ns)	Cumulative Time (ns)	Global Hit Rate	Weighted Time (ns)
L1 Cache	1	1	91.00%	
L2 Cache		3		0.18
L3 Cache	5		2.25%	
RAM		18	0.60%	
Disk	82		0.15%	
			AMAT	

Answer:

Mem Hierarchy	Isolated Time (ns)	Cumulative Time (ns)	Global Hit Rate	Weighted Time (ns)
L1 Cache	1	1	91.00%	0.91
L2 Cache	2	3	6.00%	0.18
L3 Cache	5	8	2.25%	0.18
RAM	10	18	0.60%	0.108
Disk	82	100	0.15%	0.15
			AMAT:	1.53

4. (20 pts) RAID – from lecture 22
- Which RAID level is best if you want to minimize the up-front investment in drives? _____
 - Why not always use this level of RAID? _____
 - At 47:44 of the lecture recording, we examine the difference between writes with RAID 4 and 5. What was the compelling reason for choosing 5 over 4 we discussed?
 - ___ RAID 5 is better for large block sizes per write
 - ___ RAID 5 improves performance by removing a bottleneck of parity writes
 - ___ Leaving parity bits on 1 drive is risky – if you lose that drive you're dead

- d. Can all of the RAID versions survive a disk outage without permanent loss of data? Explain.

Answer: No: Raid 0 has no redundancy.

5. (20 pts) In Lecture 24 (coming up) we will review the Hamming Calculator.
- Convert the last 2 digits of your student ID to an 8-bit binary number, and use it as the input into a Hamming table and show the parity bits and resulting 12-bit data you will transmit.

Preparing for Transmission															
Data to Send															
		bit position		p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8
Coverage Matrix	p1		x		x		x		x		x		x		x
	p2			x	x			x	x			x	x		
	p4					x	x	x	x						x
	p8										x	x	x	x	x
	p16														
	p32														
	p64														
Data with Codes															

- Calculate the last 4 digits of your student ID mod 12. What is that value?

Take the output from a, invert the bit in the position calculated above, and use that as input into the received data table. Show how the process correctly identifies the bit in error.

Checking Received Data for Errors													
Data Received		100001101111											
bit position		p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8
Check Bits	c1		x		x		x		x		x		x
	c2			x	x			x	x			x	x
	c4					x	x	x	x				x
	c8									x	x	x	x
	c16												
	c32												

