

# Electrodynamics Lecture 1-2

mathematical knowledge and skills

Yuxuan Zhang

School of Physics Zhejiang University

May 4th 2023

# 目录

曲线坐标

曲线坐标中的场微分

$\nabla$  符号与简单场论

积分微分回顾

# 曲线坐标

- ▶  $(q_1, q_2, q_3) \rightarrow$  一个三维空间中的点      与直角坐标相互转换
- ▶ 等值面
- ▶ 坐标曲线  $\rightarrow$  正交曲线坐标系

# 曲线坐标

- ▶ 三个方向上微元弧的长度和方向

$\vec{e}_1, \vec{e}_2, \vec{e}_3$  用  $\vec{i}, \vec{j}, \vec{k}$  表示

$$|dl_1| = \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial x}{\partial q_2}\right)^2 + \left(\frac{\partial x}{\partial q_3}\right)^2}$$

- ▶  $\vec{e}_1 \rightarrow \frac{\partial \vec{r}}{\partial q_1}$

反过来  $\vec{i}(\vec{e}_1, \vec{e}_2, \vec{e}_3)$

- ▶ 应用：柱坐标系和球坐标系

# 曲线坐标中的场微分

- 梯度的表达式 (引出 $\nabla$ 算子)
- $\nabla = \vec{e}_1 \frac{1}{H_1} \frac{\partial}{\partial q_1} + \vec{e}_2 \frac{1}{H_2} \frac{\partial}{\partial q_2} + \vec{e}_3 \frac{1}{H_3} \frac{\partial}{\partial q_3}$
- 散度和旋度的计算公式

$$\text{球坐标系: } dl = dr\hat{r} + r d\theta\hat{\theta} + r \sin\theta d\phi\hat{\phi}; \quad d\tau = r^2 \sin\theta dr d\theta d\phi$$

$$\text{梯度: } \nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\text{散度: } \nabla \cdot \mathbf{v} = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r \sin\theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\text{旋度: } \nabla \times \mathbf{v} = \frac{1}{r \sin\theta} \left[ \frac{\partial}{\partial \theta}(\sin\theta v_\phi) - \frac{\partial v_\phi}{\partial \phi} \right] \hat{r} +$$

$$\frac{1}{r} \left[ \frac{1}{\sin\theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[ \frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\text{拉普拉斯算子: } \nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 t}{\partial \phi^2}$$

$$\text{柱坐标系: } dl = ds\hat{s} + s d\phi\hat{\phi} + dz\hat{z}; \quad d\tau = s ds d\phi dz$$

$$\text{梯度: } \nabla t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s} \frac{\partial t}{\partial \phi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

$$\text{散度: } \nabla \cdot \mathbf{v} = \frac{1}{s} \frac{\partial}{\partial s}(s v_s) + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\text{旋度: } \nabla \times \mathbf{v} = \left[ \frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right] \hat{s} + \left[ \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial s} \right] \hat{\phi} + \frac{1}{s} \left[ \frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_r}{\partial \phi} \right] \hat{z}$$

$$\text{拉普拉斯算子: } \nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

## ▽ 符号与简单场论

▶  $\nabla \cdot (\nabla \times \vec{A}) \equiv 0$

▶  $\nabla \times (\nabla \phi) \equiv 0$

# 积分计算回顾

- ▶ 二重积分
- ▶ 三重积分
- ▶ 第一类曲线积分——线段元参数化为一定区间内的定积分
- ▶ 第一类曲面积分——曲面元参数化为一定区域的二重积分
- ▶ 第二类曲线积分——先化为第一类曲线积分
- ▶ 第二类曲面积分——先化为第一类曲面积分