# Electrodynamics Lecture 1-2

mathmatical knowledge and skills

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# 目录

# 曲线坐标

- $(q_1, q_2, q_3) \rightarrow$ 一个三维空间中的点　　与直角坐标相互转换
- 等值面
- 坐标曲线 $\rightarrow$ 正交曲线坐标系

# 曲线坐标

- 三个方向上微元弧的长度和方向 $\vec{e_1}, \vec{e_2}, \vec{e_3}$用$\vec{i}, \vec{j}, \vec{k}$表示
  $|dl_1| = \sqrt{\left(\frac{\partial x}{\partial q_1}\right)^2 + \left(\frac{\partial x}{\partial q_2}\right)^2 + \left(\frac{\partial x}{\partial q_3}\right)^2}$
- $\vec{e_1} \to \frac{\partial \vec{r}}{\partial q_1}$
  反过来 $\vec{i}(\vec{e_1}, \vec{e_2}, \vec{e_3})$
- 应用：柱坐标系和球坐标系

# 曲线坐标中的场微分

- 梯度的表达式（引出∇算子）
- $\nabla = \vec{e_1}\frac{1}{H_1}\frac{\partial}{\partial q_1} + \vec{e_1}\frac{1}{H_2}\frac{\partial}{\partial q_2} + \vec{e_1}\frac{1}{H_3}\frac{\partial}{\partial q_3}$
- 散度和旋度的计算公式

**球坐标系**。 $d\boldsymbol{l} = dr\hat{\boldsymbol{r}} + r d\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$； $d\tau = r^2\sin\theta dr d\theta d\phi$

梯度：$\nabla t = \frac{\partial t}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\boldsymbol{\phi}}$

散度：$\nabla\cdot\boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$

旋度：
$$\nabla\times\boldsymbol{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

拉普拉斯算子：$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$

**柱坐标系**。 $d\boldsymbol{l} = ds\hat{\boldsymbol{s}} + s d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$； $d\tau = s ds d\phi dz$

梯度：$\nabla t = \frac{\partial t}{\partial s}\hat{\boldsymbol{s}} + \frac{1}{s}\frac{\partial t}{\partial\phi}\hat{\boldsymbol{\phi}} + \frac{\partial t}{\partial z}\hat{\boldsymbol{z}}$

散度：$\nabla\cdot\boldsymbol{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial\phi} + \frac{\partial v_z}{\partial z}$

旋度：$\nabla\times\boldsymbol{v} = \left[\frac{1}{s}\frac{\partial v_z}{\partial\phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{\boldsymbol{s}} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial\phi}\right]\hat{\boldsymbol{z}}$

拉普拉斯算子：$\nabla^2 t = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial t}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}$

# $\nabla$ 符号与简单场论

- $\nabla \cdot (\nabla \times \vec{A}) \equiv 0$
- $\nabla \times (\nabla \phi) \equiv 0$

# 积分计算回顾

- 二重积分
- 三重积分
- 第一类曲线积分——线段元参数化为一定区间内的定积分
- 第一类曲面积分——曲面元参数化为一定区域的二重积分
- 第二类曲线积分——先化为第一类曲线积分
- 第二类曲面积分——先化为第一类曲面积分