Georgia Institute of Technology

CS 6601 Artificial Intelligence

Final Exam

Summer 2025

Duration of Exam: July 24th 2025, 8:00 AM (ET) - July 31st 2025, 7:59 AM (ET)

Weight: 20% **Points:** 100 + 2 Extra Credit!

Submission: Via Canvas. Unlimited attempts during exam window, **only last submission will be used for grading**.

Instructions

IMPORTANT*: Please read the instructions in its entirety before beginning the exam.

Academic Integrity

During the week-long exam, NO resources outside of this semester's class materials may be used. It is open-book, open-note, and open-course resources, but it is not open internet (e.g., YouTube, Open Courseware, X-Overflow, Discussions, Forums, LLMs, Search platforms or assistants). We do not allow collaboration with any other intelligent life forms or artificial intelligence (e.g., ChatGPT). Do not discuss or share any materials related to the exam (e.g. this PDF file) anywhere and only acquire the exam through official channels (e.g. Canvas, Edstem, Teaching Staff, GT Github). Sharing on any GT or public/private social media platform (e.g., Ed, Slack) or student communities (e.g. Discord) is a violation of Academic Integrity.

Submitting an attempt to this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia Tech and the Georgia Tech Code of Conduct. Academic misconduct will not be tolerated and and violations will be submitted to the office of student integrity as outlined in the course syllabus.

Communications

From the teaching staff: As the exam period progresses, the teaching staff may discover issues with the exam or find the need to issue clarifications. All communications will be posted on Edstem in the general clarifications thread and under the specific exam section's clarification thread. And so, you should **check Edstem every day** to ensure that no changes / clarifications have been released that would affect your work. There will be a period before the exam deadline after which the teaching staff will not release any further changes / clarifications, this will be announced.

From you: As you complete the exam, you may find that you seek clarification about different exam topics or have general questions. Edstem will be in private mode and you should make a private post on Edstem to ask your questions. It is important that you **(1) have checked the clarification threads first** to see if your question has been answered before making an Edstem post **(2) labeled your post with the correct exam section label** (failure to do so may result in the responsible staff missing your post and a delay in response time) **(3) group your questions based on subject** (e.g. questions related to Search are in one post while questions related to Optimization are in a different post). Do NOT at any point email the teaching staff unless you are explicitly directed by the teaching staff to do so.

The Exam

The exam is submitted through Canvas. Note that there are **two assignments** in Canvas, one is for submitting the exam and the other is for **optionally** uploading any physical work. It is important to recognize that submission of physical work is NOT used for partial credit, it is instead used as evidence that you have not plagiarized in the event that your submission comes under OSI investigation and it is helpful in the event that some technical difficulty causes a loss of your main exam submission.

It is recommended that you download the PDF version of the exam when it is released so that you may have an offline copy in the event you encounter any technical difficulties and for keeping track of your answers that you'd like to input into Canvas as Canvas can be quite finnicky. We also recommend that you periodically submit your answers to Canvas so that you do not accidentally end the exam period with no submission. After you have submitted your final submission and are satisfied, **do NOT open the Canvas exam again** as that will open a new blank submission and leave you with a score of zero.

Question Formats

You may see the following formats in this exam:

- **Numeric Decimal Submission -** Never round intermediate steps. Only round your answer at the end if necessary. For example if the questions says to round to 2 decimal places, 3.141 becomes 3.14 while a whole number like 25 can be submitted as 25 (you do not need to submit 25.00).
- **Relatively Prime Submission -** Several questions in the exam state that the answer can be written in the form of p/q where p and q are relatively prime (coprime) integers and that you should submit the value of $p + q$. Two integers are considered relatively prime (coprime) if they share no common factors other than 1. For example 14 and 15 are relatively prime (coprime) while 6 and 21 are not because they share a common factor of 3. See the Challenge Questions for examples.
- **Single Select -** Also known as multiple choice, exactly one answer among the options is correct.
- **Multiple Select -** At least one answer among the options is correct, select all that apply. We will grade Multiple Select questions with partial credit. For a question with N options worth P points, you will receive P/N points for each answer choice that is correctly checked / unchecked. Note that Canvas does not have an automated option for this and we will run a regrading script after the exam is over to enact this.
- **Matching -** There will be an option bank that you can drag to match with answer choices. Note that an option may be used more than once.

Grading and the Challenge Period

Your last attempt will constitute your graded exam. Do not have an unfinished exam active when the deadline comes, as the system will auto-submit your current working exam. This means that if you had 80 points of completed work on your previous submission, you begin a new attempt, and are only 25% through the exam when the deadline comes, your exam grade may only total 25 points. You will not be able to see the points earned in each submission until after the exam has closed. **Always assume that there will be no partial credit.**

It is your responsibility to ensure that you have submitted the correct answers to Canvas. Again, we will NOT be looking at your physically submitted work for grading purposes other than plagiarism. No partial credit is provided for issues like accidentally selecting the wrong answer, rounding error, and typos. Extensions are also not provided for any reason unless you have reached out with approved ODS accommodations. There have been exceptions in the event of major events (e.g. natural disasters) affecting multiple students, so please reach out to the teaching staff via Edstem if you have a developing situation. Do NOT wait until the last minute to contact the teaching staff.

After the exam period is over, there will be a brief Challenge Period during which the teaching staff will release the answer key and corresponding solutions and students will have the opportunity to challenge the accepted solutions. It is an opportunity for students to outline why they think the accepted solution is incorrect or why another solution should also receive credit based on an alternate interpretation of the question. There is no guarantee that changes will be made, final decision is up to teaching staff discretion. Grades seen in Canvas will not be accurate during this period, you should use the answer key to calculate your expected grade so that you can check that you have the correct grade after the Challenge Period ends and the final regrading script has been run.

Good Luck!

We wish you the best of luck on the exam! This is a great opportunity for you to revisit topics covered in the course material and to consolidate your understanding. And as always, do not hesitate to reach out on Edstem whenever you have questions.

Acknowledgement [1pt]

Please enter your full name below as acknowledgement that you have fully read and understood the exam instructions above.

Respond with your full name.

Contents

1 Search [8pts]

Introduction

You are helping a delivery robot plan routes inside a warehouse represented as a directed graph. Each node represents a room and each edge has a non-negative travel time. The robot uses graph search to find the path with the shortest time from A to I.

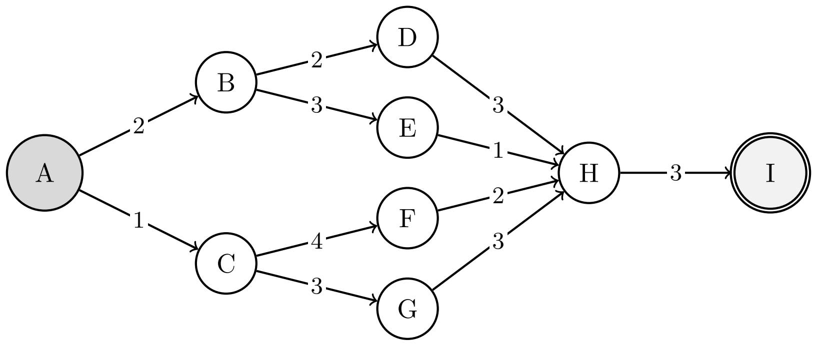


Figure 1: Warehouse graph

The heuristic $h(n)$ from each room to the Goal I is shown in Table 1

Note: Break ties alphabetically.

Room	A	B	C	D	E	F	G	H	I
$h(n)$	6	5	4	4	3	2	2	1	0

Table 1: Heuristic $h(n)$ estimates amount of time required to reach goal I.

1.1 Search Q1 [2pts]

You are running **Uniform Cost Search (UCS)** from **A** to **I**. Which of the following are guaranteed properties of UCS on the warehouse graph in Figure 1?

Multiple select.

(a) UCS will expand all of the nodes in this graph.

(b) UCS will expand fewer nodes than BFS on this graph.

(c) UCS will expand more nodes or the same amount of nodes as A* using the provided heuristic $h(n)$ on this graph.

(d) For all nodes in this graph, the provided heuristic $h(n)$ is *close* to the true cost from the node to the goal. *Close* is defined as $|\, g^*(n) - h(n)\,| \leq 1$. For example if a node **X** has $h(X) = 2$ and $g^*(X) = 5$, $h(X)$ is not considered to be *close* to the true cost from **X** to the goal.

1.2 Search Q2 [2pts]

Which of the following nodes will A* search using the provided heuristic $h(n)$ expand in this graph? *Hint: Nodes expanded are not necessarily on the optimal path returned by A*.*

Multiple select.

(a) Room D

(b) Room E

(c) Room F

(d) Room G

1.3 Search Q3 [1pt]

Which statement below is true about the provided heuristic $h(n)$?

Select one.

(a) The heuristic $h(n)$ is admissible and consistent.

(b) The heuristic $h(n)$ is admissible but NOT consistent.

(c) The heuristic $h(n)$ is consistent but NOT admissible.

(d) The heuristic $h(n)$ is neither admissible nor consistent.

1.4 Search Q4 [2pts]

Suppose a new path is created from room **B** to room **F** with a cost of 1. Which of the following statements below are true?

Multiple select.

(a) The optimal path from **A** to **I** in the new graph is the same as the optimal path in the original graph.

(b) A* search using heuristic $h(n)$ on the new graph will expand fewer nodes than A* search using heuristic $h(n)$ on the original graph.

(c) UCS on the new graph will expand fewer nodes than UCS on the original graph.

(d) BFS on the new graph will expand the same number of nodes as BFS on the original graph.

1.5 Search Q5 [1pt]

Consider the graph in Figure 1 with the additional path between **B** and **F** added in Search Q4. Let us define a new heuristic function $h_2(n)$ where $h_2(n) = h(n) + 2$, Which if the following statements below are true with regards to $h_2(n)$?

Select one.

(a) The heuristic $h_2(n)$ is both admissible and consistent.

(b) The heuristic $h_2(n)$ is admissible but NOT consistent.

(c) The heuristic $h_2(n)$ is consistent but NOT admissible.

(d) The heuristic $h_2(n)$ is neither admissible nor consistent.

2 Game Playing [8pts]

Part I - Minimax Tree

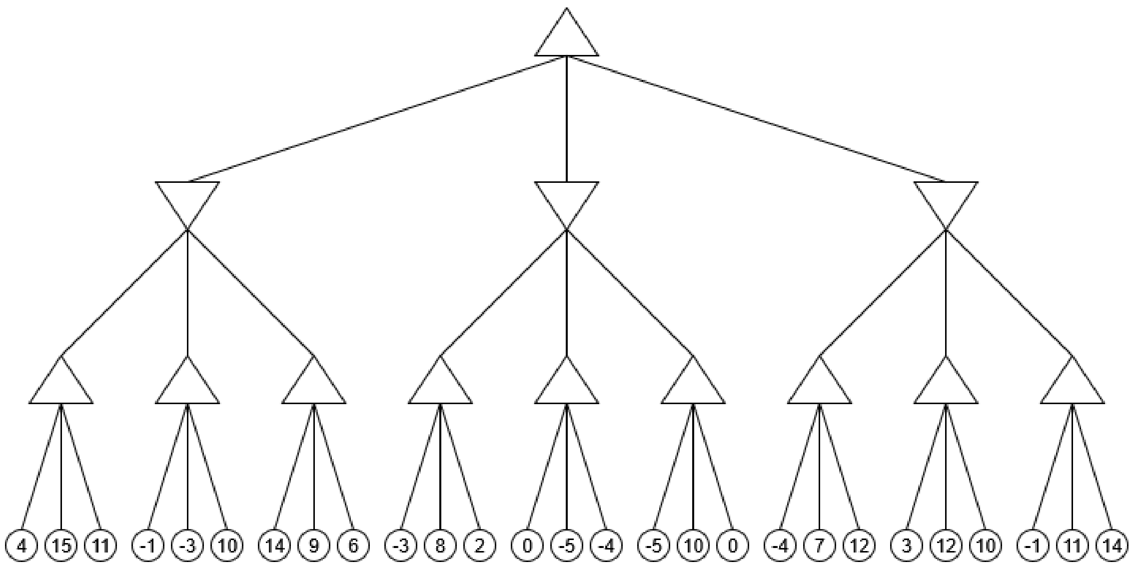


Figure 2: Game tree for Game Playing Q1, Q2

2.1 Game Playing Q1 [2pts]

Perform the minimax algorithm on the game tree seen in Figure 2. What is the utility value of the root node?

Answer as an integer.

2.2 Game Playing Q2 [2pts]

Perform the minimax algorithm with alpha-beta pruning on the game tree seen in Figure 2, evaluating all nodes from left to right. The utility value of each node is bounded by $[-\infty, +\infty]$. What is the total number of branches pruned?

Note that when a branch is pruned, all child branches of that branch are pruned as well (and counted), if any.

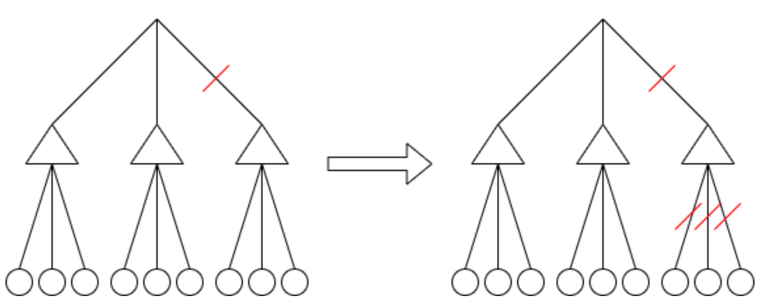
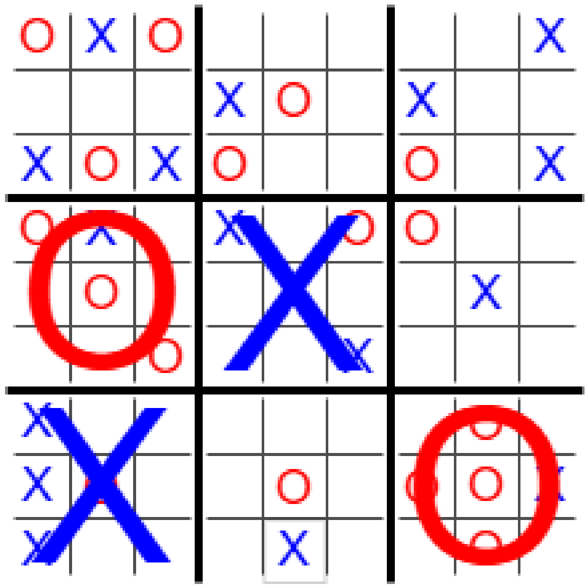


Figure 3: Pruning the parent branch in the example subtree would count as 4 pruned branches.

Answer as a whole number.

Part II - Ultimate Tic-Tac-Toe



Ultimate tic-tac-toe is a game played on a global 3x3 board, where each of its nine large squares contains a local 3x3 tic-tac-toe board. The objective of the game is to win three local boards in a row horizontally, vertically, or diagonally on the global board. A player wins a local board by getting three of their marks in a row just like in standard tic-tac-toe.

Players X and O take turns placing their respective marks in local squares. Suppose player X plays first. In player X's first move, they can place their mark in any of the 81 local squares.

The position of player X's first move dictates where player O can place their mark next - player O cannot place their mark arbitrarily in any of the 80 remaining local squares. The relative position of a player's move within its local board dictates the local board in which their opponent must play a move next.

For example, if player X places their mark in the **top right square** within a local board, player O will be forced to play their next move in the **top right local board** of the global board. If player O then places their mark in the **bottom left square** of that local board, player X will be forced to play their next move in the **bottom left local board** of the global board.

The exception to this rule is that if a player is forced to play in a local board that has already been won by either player or is completely filled, that player may choose to play in any other local board that has not been won or completely filled yet. This rule will not be relevant for the upcoming questions.

Example

The first four moves of a sample game are played to demonstrate the available moves of a player based on their opponent's previous move. X will make the first move.

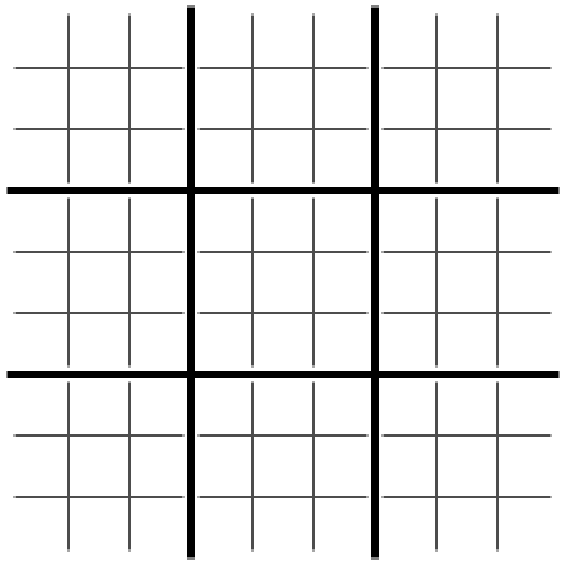


Figure 4: Starting position

For X's first move, X can play in any of the 81 local squares.

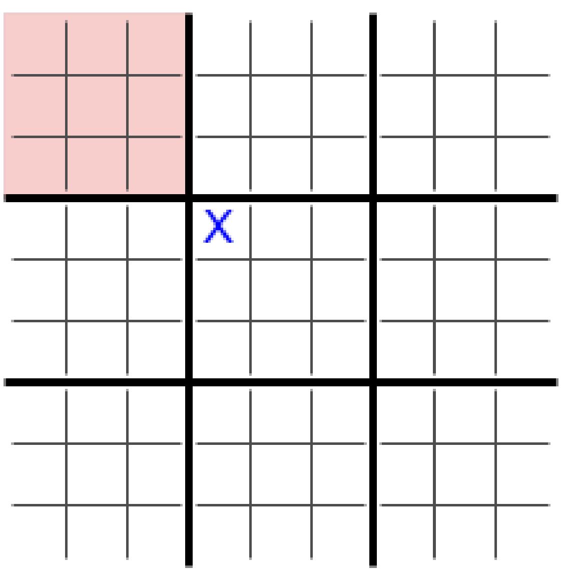


Figure 5: Position after X's first move

X plays in the top left square of the center local board. The relative position of the mark within the local board determines the next local board for the opponent's next move. Since X is played in the top left square of its local board, O's next move must be in the top left local board.

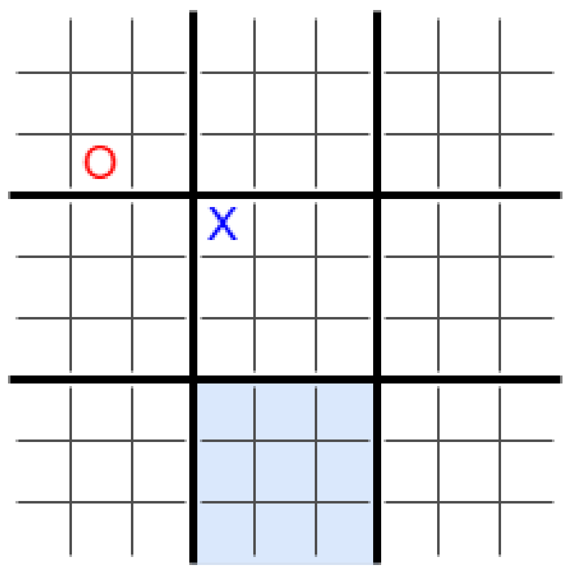


Figure 6: Position after O's first move

O plays in the bottom center square of the top left local board. Since O is played in the bottom center square of its local board, X's next move must be in the bottom center local board.

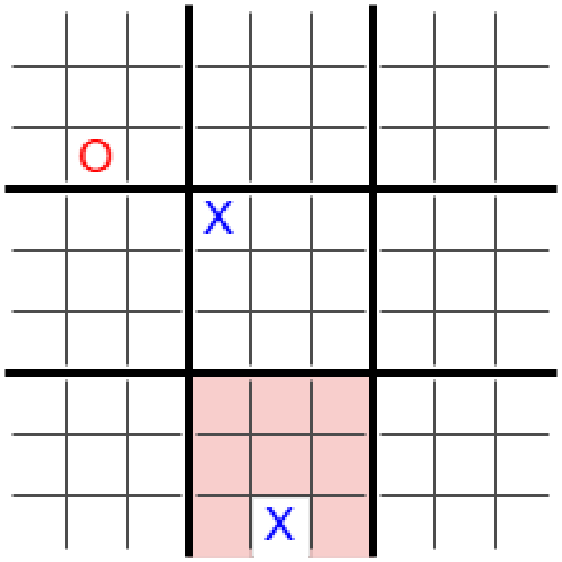


Figure 7: Position after X's second move

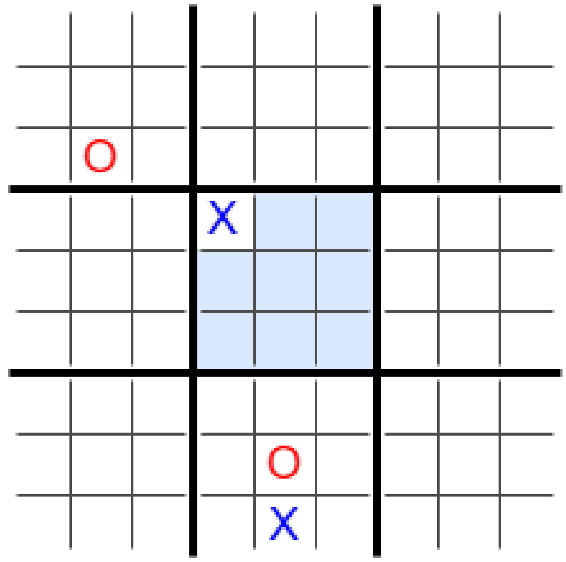


Figure 8: Position after O's second move

X plays in the bottom center square of the bottom center local board. O's next move must be one of the 8 remaining squares of the same bottom center local board.

O plays in the center square of the bottom center local board. X's next move must be one of the 8 remaining squares in the center local board.

2.3 Game Playing Q3 [2pts]

Consider an ultimate tic-tac-toe game tree from the empty starting position. The tree is not simplified using symmetry and there are 81 nodes at depth 1 of the game tree.

How many nodes are at depth 2 of the game tree?

Answer as a whole number.

2.4 Game Playing Q4 [2pts]

If we use minimax with iterative deepening from depth 0 to depth 4 inclusive, how many nodes will be evaluated in total? *Hint:*

- Depth 0 of the game tree has one node (root node).
- Depth 1 of the game tree has 81 nodes.
- Depth 3 of the game tree has 6336 nodes.
- Depth 4 of the game tree has 55,080 nodes.

Answer as a whole number.

2.5 Game Playing Q5 - BONUS! [2pts]

Suppose the game tree is simplified using rotational and reflection symmetry (horizontally, vertically, diagonally). How many nodes will be at depth 1 of the newly simplified game tree?

Answer as a whole number.

3 Optimization [6pts]

Introduction

Round all answers in this section to 6 decimal places.

For Part I of this section, we will be using a genetic algorithm to perform an optimization.

Consider a two-digit number x. When x is represented in decimal (base-10) it is denoted as x_d, and when x is represented in binary (base-2) it is denoted as x_2.

The fitness function can be seen in Eqn. 1, and is defined as one plus the total sum of the number of 1's when each digit in x_d is *individually* represented in binary, all divided by x_d. The binary representations of the digits in 0-9 can be seen in Table 2. For example, when $x_d = 27$, the sum is 4 (see Table 3). For an example of the calculation of the fitness function when $x_d = 27$ see Eqn. 2.

$$f(x_d) = \frac{1 + \sum\limits_{i_2 \epsilon x_d} \text{\# of 1s in } i_2}{x_d} \tag{1}$$

Decimal (Base-10)	Binary (Base-2)	# of 1s
0	0000	0
1	0001	1
2	0010	1
3	0011	2
4	0100	1
5	0101	2
6	0110	2
7	0111	3
8	1000	1
9	1001	2

Table 2: Decimal to binary conversion

Digit (Decimal)	Binary	# of 1s in binary
2	0010	1
7	0111	3
	Sum:	4

Table 3: Example conversion of $x_d = 27$ to the sum of values in binary

$$f(27) = \frac{1 + (4)}{27} = 0.185185 \tag{2}$$

We start with four candidate solutions for x_d: 19, 48, 36, 57. You will need to reuse the candidate solutions and fitness values from the first two parts so be sure to save your results. The best candidate solution is the one with the highest fitness function value.

3.1 Optimization Q1 [1pt]

Consider a crossover that swaps the second digit of two candidate solutions for x_d. Compute the fitness of the children that occur when one round of crossover occurs between parents (19, 48), (48, 36), and (36, 57). Let the child with the highest overall fitness value be A, second highest overall fitness be B, and third highest overall fitness be C. Return the values of A, B, and C.

Answer with the values of A, B, and C as numbers rounded to six decimal places.

3.2 Optimization Q2 [1pt]

Consider a mutation in which the value of candidate solutions for x_d increases by 7. Beginning with the candidate solution set [19, 48, 36, 57] compute the fitness values after mutation. Let the child with the highest overall fitness value be W and the child with the fourth highest overall fitness value be Z. Return the values of W and Z.

Answer with the values of W and Z as numbers rounded to six decimal places.

3.3 Optimization Q3 [1pt]

An important implementation consideration when using a genetic algorithm is the selection process for candidate solutions to be carried forward to the next iteration (i.e. the replacement method). We will consider three different replacement methods for selecting four candidate solutions to be carried forward to the next iteration with respect to the parent and child (from crossover and/or mutation) candidate solutions. Considering the original parent solutions and the children generated through crossover and mutation so far, which of the following methods **guarantees** the individual with the best fitness value so far will continue into the next iteration?

Multiple select.

(a) Keep the best 4 children from crossover/mutation.

(b) Keep the best 4 candidate solutions considering both parents and children.

(c) Randomly select 4 candidate solutions to carry forward.

3.4 Optimization Q4 [1pt]

Considering the original parent solutions and the children generated through crossover and mutation so far, which of the following methods **can** lead to the individual with the best fitness value so far continuing into the next iteration?

Multiple select.

(a) Keep the best 4 children from crossover/mutation.

(b) Keep the best 4 candidate solutions considering both parents and children.

(c) Randomly select 4 candidate solutions to carry forward.

3.5 Optimization Q5 [2pts]

This question is unrelated to previous questions.

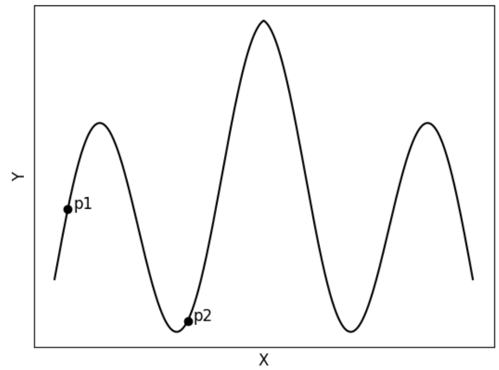


Figure 9: Fitness function (Y) value vs input value (X).

Consider the fitness function $Y = f(X)$ shown in Fig. 9. When using simulated annealing from either Point p1 or Point p2 in that figure, which of the step size shape functions in Fig. 10 would **most** likely find the global maximum value of the fitness function? In Fig. 10, consider the horizontal axis to be the iteration count, the vertical axis to be the step size, and that the scale on each option is the same relative to the others (i.e. one unit on the grid is comparable across each graph).

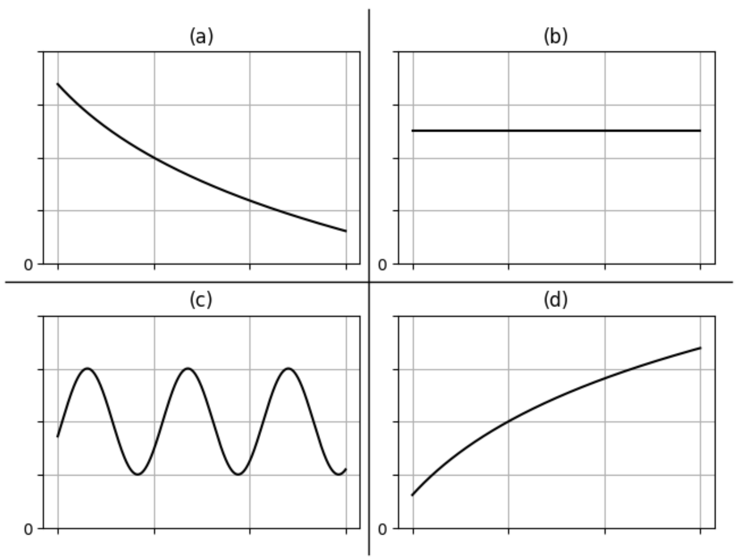


Figure 10: Optimization step size shape functions

Multiple select.

(a) Graph A

(b) Graph B

(c) Graph C

(d) Graph D

4 Constraint Satisfaction Problems (CSPs) [9pts]

Introduction

Figure 11: Seating Positions

You are hosting a dinner with six of your closest friends and family. The guests are conveniently named Anna, Bert, Carol, Damien, Edgar, and Fred. Unfortunately, the guests can be a little particular about the dinner arrangements and so the seating of the guests is subject to the following constraints. Note that all guests must have a seat at the table.

1. Guests cannot share seats.
2. Carol must sit at the end of the table (Position 1 or Position 4).
3. Anna cannot sit next to Bert (e.g. if $A = 4, B \neq 3, 5$).
4. Edgar and Bert must sit directly across from each other (e.g. if $E = 3$, $B = 5$).
5. Damien must sit at Position 3.
6. Edgar must be seated at Position 5 or Position 6.
7. Fred cannot sit on the opposite side of the table from Edgar (e.g. if $F = 2$, $E \neq 5, 6$).

4.1 CSP Q1 [1pt]

How many **complete assignments** are there in total for this CSP problem?

Answer as a whole number.

4.2 CSP Q2 [2pts]

Consistent with all constraints, where must Bert be seated?

Select one.

(a) Position 1

(b) Position 2

(c) Position 3

(d) Position 4

(e) Position 5

(f) Position 6

4.3 CSP Q3 [2pts]

How many **complete and consistent assignments** are there for this CSP problem?

Answer as a whole number.

Domains

Use the table below showing the domains of the variables to answer Q4 and Q5. For these two questions you are to treat the constraint that each guest must have a different seat as a binary constraint.

The variables have the following domains:

Variable	Domain
Anna	2, 4, 5, 6
Bert	2, 4, 5
Carol	1, 4
Damien	3
Edgar	5, 6
Fred	1, 4, 5, 6

Table 4: Starting domains for CSP Q4 and Q5

4.4 CSP Q4 [2pts]

Given the domains of the variables in Table 4, which of variable domains below violate arc consistency?

Multiple select.

(a) Anna

(b) Bert

(c) Carol

(d) Fred

4.5 CSP Q5 [2pts]

Given the domains of the variables shown above, which of these pairs of variables violate path consistency?

Multiple select.

(a) Anna, Fred

(b) Anna, Damien

(c) Damien, Fred

(d) Carol, Fred

5 Probability [8pts]

Introduction

Dungeons & Dragons (D&D) is a tabletop role-playing game (RPG) where players assume the roles of adventurers in a fantasy world. The outcomes of their actions—attacks, spellcasting, stealth, etc.—are often determined by rolling various types of dice.

Dice System

Figure 12: Dice Examples

D&D uses **polyhedral dice**, abbreviated as "**dX**", where **X** is the number of sides:

- **d20**: 20-sided die (most common for attack rolls, saving throws, skill checks)
- **d6**: 6-sided die (used for damage, fireballs, etc.)
- **d8, d10, d12** etc.

All dice in this question are fair and follow a uniform probability distribution from 1 to **X**. Rolling **2d6** means rolling two six-sided dice and taking the sum of the die rolls.

Attack Rolls and Damage Rolls

To hit an enemy, a player rolls a **d20**, adds a **proficiency modifier** (see below), and compares it to the target's **Armor Class (AC)**:

- If the result of the roll $\geq$ **AC**, the attack hits. Then another set of dice, Damage Rolls, (e.g. **1d8**, **2d6**, **1d12**, . . .) are used to dictate how much damage is done by the successful attack. The number and type of dice rolled for damage rolls are specified on a per character basis.
- A **natural 20** (rolling a 20 on the die) is a critical hit, this both guarantees a hit (even if the **AC** is greater than the total) and doubles the damage rolled by the dice. For example: if a 20 is rolled and then a 10 is rolled using **2d6** for damage, the opponent will take 20 damage.
- Similarly, a **natural 1** (rolling a 1 on the die) is a critical failure, this guarantees a miss and often leads to consequences for the player.

Proficiency Modifier

An integer value that is added to Attack **d20** Rolls. Furthermore if a player is proficient in a skill then we add the proficiency modifier to the **d20** roll taken for the skill.

Example

A player is attacking an opponent which has an **Armor Class** of 10, and the player has a **Proficiency** of **+2**. They roll a d20 to attack and get a 12, we add a 2 to this roll to get 14, this is larger than the Armor Class of 10, thus the attack hits. Upon the attack hitting the player must roll how much damage they do, and they use their damage dice rolls. They roll a 4 using a **1d6** damage roll and do 4 damage to their opponent.

Question Setup

Mike, Will, Lucas and Dustin, collectively **The Party**, are playing a game of Dungeons and Dragons the table below gives their character details:

Note: The attack roll is always a **d20** and the damage rolls for each character are specified below.

Name	Armor Class (AC)	Damage Rolls	Proficiency	Health
Mike	14	2d6	+1	22
Will	18	1d8	+1	12
Lucas	16	4d4	+3	20
Dustin	4	2d12	+2	36

Table 5: Character Details

5.1 Probability Q1 [1pt]

What is the expected amount of damage Mike will do in an attack given an opponent with Armor Class of 10? Note that proficiency does not apply to critical actions outlined below (e.g. Mike rolling a 19 does not result in a critical hit, Mike must roll a 20 for it to be a critical hit).

- A critical hit will double the damage dice roll.
- A critical fail will heal the opponent of **1d4** hit points.

The expected amount of damage Mike will do can be expressed in the form of p/q where p and q are relatively prime positive integers. Submit your answer as the value of $p + q$.

Answer as a whole number.

5.2 Probability Q2 [1pt]

What is the expected amount of damage Dustin will do in an attack given an opponent with Armor Class of 5? Note that proficiency does not apply to critical actions outlined below (e.g. Dustin rolling a 18 does not result in a critical hit, Dustin must roll a 20 for it to be a critical hit).

- A critical hit will double the damage dice roll.
- A critical fail will heal the opponent of **2d8** hit points.

The expected amount of damage Dustin will do can be expressed in the form of p/q where p and q are relatively prime positive integers. Submit your answer as the value of $p + q$.

Answer as a whole number.

5.3 Probability Q3 [2pts]

Lucas is sneaking into a castle, and to do so without arousing the suspicion of any guards they must do a stealth check. Lucas and the guard both roll a **d20**, if Lucas ties or rolls higher than the guard then the stealth check succeeds and they are able to sneak in. The following conditions apply:

- Lucas is proficient in stealth and the guard is not. This means we add Lucas' proficiency modifier to his **d20** roll.
- If both Lucas and the guard roll a natural 20, the stealth check passes.
- If both Lucas and the guard roll a natural 1, the stealth check fails.
- If the scores are equal then the stealth check passes.

What is the probability that Lucas is able to sneak in without being discovered?

Answer as a number rounded to six decimal places.

5.4 Probability Q4 [2pts]

Another mechanic in Dungeons and Dragons is having **Advantage** and **Disadvantage**:

- **Advantage**: Roll two **d20s**, take the higher.
- **Disadvantage**: Roll two d20s, take the lower.

Lucas has an advantage on his stealth check while the guard has a disadvantage. All other conditions from Probability Q3 stay the same. What is the probability that Lucas is able to sneak in without being discovered?

Answer as a number rounded to six decimal places.

5.5 Probability Q5 [2pts]

The Party is fighting against a monster which:

- Deals 8d8 damage per attack
- Performs critical hits with rolls of 19 and 20
- Has an attack proficiency of +2

Select all party members who are expected to survive one hit from the monster. Assume no negative consequences for the monster if it critically fails and double the dice damage upon critical hit.

Multiple select.

(a) Mike

(b) Will

(c) Lucas

(d) Dustin

6 Bayes Nets [12pts]

Introduction

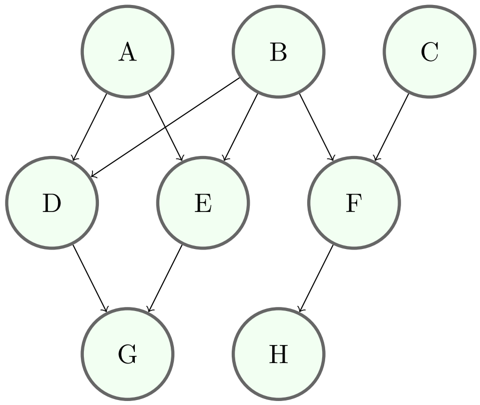


Figure 13: The Bayes Net

The Bayes Nets in Figure 13 contains 8 nodes (variables). Variables A, C, D, F, G, and H have two states each, while variable E has three states and variable B has four states. Note that if a variable X can take on N states, the states will be represented as X_i where $1 \leq i \leq N$. For example variable A has states A_1 and A_2.

6.1 Bayes Nets Q1 [1pt]

If we constructed a joint probability distribution table for these random variables, we'd need 768 parameters without any assumptions on the independence of the random variables. That's a lot of parameters, so we decided to use the Bayes Net in Figure 13. Based on the independence relations originated only by the structure of the Bayes Net in Figure 13, how many parameters are needed to specify this Bayes Net?

Answer as a whole number.

6.2 Bayes Nets Q2 [1pt]

Which of the following nodes are NOT in the **Markov Blanket** of E?

Multiple select.

(a) Node A

(b) Node B

(c) Node C

(d) Node D

(e) Node F

(f) Node G

(g) Node H

6.3 Bayes Nets Q3 [2pts]

Which of the following nodes are NOT independent of F?

Multiple select.

(a) Node A

(b) Node B

(c) Node C

(d) Node D

(e) Node E

(f) Node G

(g) Node H

6.4 Bayes Nets Q4 [2pts]

Which of the following evidence variables will make the following statement true: F is conditionally independent of G?

Multiple select.

(a) Given B and H

(b) Given A and E

(c) Given D and E

(d) Given A and C

6.5 Bayes Nets Q5 [2pts]

D	E	$p(G = G_1 \mid D, E)$
D_1	E_1	0.3
D_1	E_2	0.72
D_1	E_3	0.48
D_1	E_4	0.1
D_2	E_1	0.95
D_2	E_2	0.666
D_2	E_3	0.45
D_2	E_4	0.39

(a) $p(G \mid D, E)$

F	$p(H = H_1 \mid F)$
F_1	0.22
F_2	0.51

(b) $p(H \mid F)$

Figure 14: Probability Tables for Q5

Using Metropolis-Hastings sampling, which of the following states are we most likely to accept over the current state of $\{A = A_1,\ B = B_1,\ C = C_1,\ D = D_1,\ E = E_1,\ F = F_1,\ G = G_1,\ H = H_1\}$ given the probability tables in Figure 14? Assume the acceptance probabilities for State X and State Y below are both less than one.

Select one.

(a) State X = $\{A = A_1,\ B = B_1,\ C = C_2,\ D = D_1,\ E = E_1,\ F = F_2,\ G = G_1,\ H = H_1\}$

(b) State Y = $\{A = A_1,\ B = B_1,\ C = C_2,\ D = D_1,\ E = E_1,\ F = F_2,\ G = G_1,\ H = H_2\}$

6.6 Bayes Nets Q6 [2pts]

B	C	$p(F = F_1 \mid B, C)$
B_1	C_1	2/9
B_1	C_2	3/7
B_2	C_1	3/4
B_2	C_2	3/8
B_3	C_1	6/7
B_3	C_2	1/4
B_4	C_1	2/5
B_4	C_2	3/8

(a) $p(F \mid B, C)$

F	$p(H = H_1 \mid F)$
F_1	1/3
F_2	2/3

(b) $p(H \mid F)$

Figure 15: Probability Tables for Q6

Given $p(F = F_1 \mid B = B_2) = 59/120$, what is the value of $p(C = C_2)$? Do NOT round intermediate steps.

Answer as number rounded to six decimal places.

6.7 Bayes Nets Q7 [2pts]

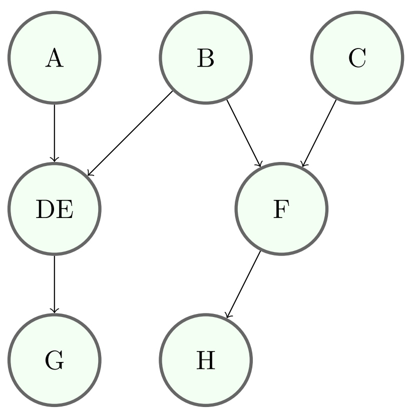


Figure 16: Bayes Net with DE Meganode

Let us replace nodes D and E with a **meganode** (as seen in Figure 16) that takes on six possible values:

$$DE = \{D_1E_1, D_1E_2, D_1E_3, D_2E_1, D_2E_2, D_2E_3\}$$

How many parameters are needed to specify the new Bayes Net seen in Figure 16?

Hint: Your answer should be larger than your answer to Q1.

Answer as a whole number.

7 Machine Learning [12pts]

Introduction

In this question, we will look at backpropagation, a technique used by neural networks to adjust their weights to better predict the data that they are given. Intuitively, backpropagation can be thought of as "learning from error". Backpropagation is possible because the entire network is made up of differentiable units, allowing gradients to pass all the way through the network.

In this question, we will calculate the gradients of each weight with respect to the error function. Hopefully, after this exercise, it will be easier to understand the mathematical workings of training a neural network.

Before we start with the neural network, let us warm up some applications of the chain rule.

7.1 Machine Learning Q1 [1pt]

Let $z = x^2 + y^3$, where $x = w^2 \ln(w)$, $y = w^{1.5}$. What is the derivative of z with respect to w?

The derivative can be expressed in the form of $Aw^B \ln(w)^D + Ew^F \ln(w)^G + 4.5w^{3.5}$ where A, B, C, D, E, and F are whole numbers. What are the values of A, B, C, D, E, and F?

Answer with the values of A, B, C, D, E, and F as whole numbers.

7.2 Machine Learning Q2 [1pt]

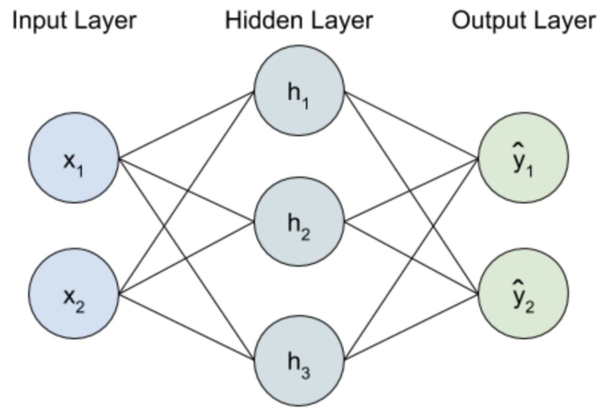
Let $z = x^3 y^2$, where $x = \ln(1 + e^w)$, $y = w^{0.5}$. What is the derivative of z with respect to w?

The derivative can be expressed in the form of $A \ln(1 + e^w)^B + Cw^D \ln(1 + e^w)^E \left(\frac{e^w}{1+e^w}\right)^F$ where A, B, C, D, E, and F are whole numbers. What are the values of A, B, C, D, E, and F?

Answer with the values of A, B, C, D, E, and F as whole numbers.

Interlude

Consider the following neural network and the corresponding notation.



$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad W^{(0)} = \begin{bmatrix} w_{11}^{(0)} & w_{12}^{(0)} & w_{13}^{(0)} \\ w_{21}^{(0)} & w_{22}^{(0)} & w_{23}^{(0)} \end{bmatrix}$$

$$\hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \end{bmatrix}$$

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \qquad W^{(1)} = \begin{bmatrix} w_{11}^{(1)} & w_{12}^{(1)} \\ w_{21}^{(1)} & w_{22}^{(1)} \\ w_{31}^{(1)} & w_{32}^{(1)} \end{bmatrix}$$

We apply a ReLU activation to the hidden layer, so that $h = \text{ReLU}(z_0)$ where $z_0 = (W^{(0)})^T X$ and $\text{ReLU}(z_0) = \max(0, z_0)$. Sigmoid activation is applied to the output layer so that $\hat{y} = \sigma(z_1)$ where $z_1 = (W^{(1)})^T h$ and $\sigma(z_1) = \frac{1}{1+e^{-z_1}}$. The activation functions are element-wise operations when applied to vectors or matrices.

Note that $w_{32}^{(1)}$ refers to the weight of the connection between the third node in the hidden layer and the second node in the output layer. We also ignore the bias term in each layer of the network to simplify computation.

Let the loss function for the neural network be CE, the cross entropy loss for binary classification. The log function here is the natural logarithm. y_i are components of the ground truth vector Y.

$$CE = \sum_{i=1}^{2} -y_i \log \hat{y}_i - (1 - y_i) \log(1 - \hat{y}_i)$$

Round your answers to six decimal places, do NOT round intermediate steps. Make sure to fill in the boxes with numerical answers for all remaining questions.

7.3 Machine Learning Q3 [2pts]

We use the following values for this question.

$$X = \begin{bmatrix} 9 \\ 4 \end{bmatrix} \qquad W^{(0)} = \begin{bmatrix} 0.56 & -0.23 & 0.44 \\ -0.84 & 0.12 & -0.32 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad W^{(1)} = \begin{bmatrix} 0.89 & -0.11 \\ -0.45 & 0.98 \\ -0.41 & -0.67 \end{bmatrix}$$

Using the notation above, we obtain the following equations to compute $\hat{Y}$ given input X.

$$h = \mathrm{ReLU}((W^{(0)})^T X)$$
$$\hat{Y} = \sigma((W^{(1)})^T h)$$

Perform the forward pass to compute $\hat{Y}$ using the provided values, where $\hat{Y} = [\hat{y}_1, \hat{y}_2]$.

Answer with the values of $\hat{y}_1$ and $\hat{y}_2$ as numbers rounded to six decimal places.

7.4 Machine Learning Q4 [2pts]

For this question, use all values provided in Machine Learning Q3 but use the following value for X.

$$X = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

Compute the partial derivative of CE with respect to $\hat{Y}$, where $\frac{\partial CE}{\partial \hat{Y}} = \left[\frac{\partial CE}{\partial \hat{y}_1}, \frac{\partial CE}{\partial \hat{y}_2}\right]$.

Answer with the values of $\frac{\partial CE}{\partial \hat{y}_1}$ and $\frac{\partial CE}{\partial \hat{y}_2}$ as numbers rounded to six decimal places.

7.5 Machine Learning Q5 [3pts]

For this question, use all values provided in Machine Learning Q3 but use the following value for X.

$$X = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Compute the partial derivative of CE with respect to $W^{(1)}$, where $\frac{\partial CE}{\partial W^{(1)}} = \begin{bmatrix} \frac{\partial CE}{\partial w^{(1)}_{11}} & \frac{\partial CE}{\partial w^{(1)}_{12}} \\ \frac{\partial CE}{\partial w^{(1)}_{21}} & \frac{\partial CE}{\partial w^{(1)}_{22}} \\ \frac{\partial CE}{\partial w^{(1)}_{31}} & \frac{\partial CE}{\partial w^{(1)}_{32}} \end{bmatrix}$.

Answer with the values of $\frac{\partial CE}{\partial W^{(1)}}$ as numbers rounded to six decimal places.

7.6 Machine Learning Q6 [3pts]

For this question, use all values provided in Machine Learning Q3 but use the following values for X and $W^{(1)}$.

$$X = \begin{bmatrix} 3 \\ -1 \end{bmatrix} \qquad W^{(1)} = \begin{bmatrix} 0.75 & 0.09 \\ 0.37 & -0.34 \\ -0.62 & -0.76 \end{bmatrix}$$

Compute the partial derivative of CE with respect to $W^{(0)}$, where $\frac{\partial CE}{\partial W^{(0)}} = \begin{bmatrix} \frac{\partial CE}{\partial w_{11}^{(0)}} & \frac{\partial CE}{\partial w_{12}^{(0)}} & \frac{\partial CE}{\partial w_{13}^{(0)}} \\ \frac{\partial CE}{\partial w_{21}^{(0)}} & \frac{\partial CE}{\partial w_{22}^{(0)}} & \frac{\partial CE}{\partial w_{23}^{(0)}} \end{bmatrix}$.

Answer with the values of $\frac{\partial CE}{\partial W^{(0)}}$ as numbers rounded to six decimal places.

If you were doing backpropagation in a project or work setting, you would now using a learning rate α, to move the new weights in the direction of their corresponding negative gradients. This minimizes the error function.

8 Pattern Recognition Through Time [12pts]

Introduction

Professor Ponderosa, a tree ecologist, studies how Rocky Mountain tree species respond to stress when grown as bonsai. Unlike wild trees, bonsai are carefully cultivated in shallow containers, and their growth is tracked over time through trunk thickness, canopy spread, and branch development. He has identified three distinct growth patterns between the following bonsai:

$$bonsai_a\text{: } [18, 21, 24, 23, 25, 20, 17, 15]$$
$$bonsai_b\text{: } [19, 18, 16, 14, 17, 19, 14, 16]$$
$$bonsai_c\text{: } [20, 22, 23, 26, 25, 21, 19, 17]$$

To explore how pruning, watering, and elevation affect growth, he wants to compare these sequences. But differences in growth rates make direct comparison unreliable. He turns to Professor Lodgepole, who suggests using Dynamic Time Warping (DTW), a technique that aligns sequences even when their timing differs.

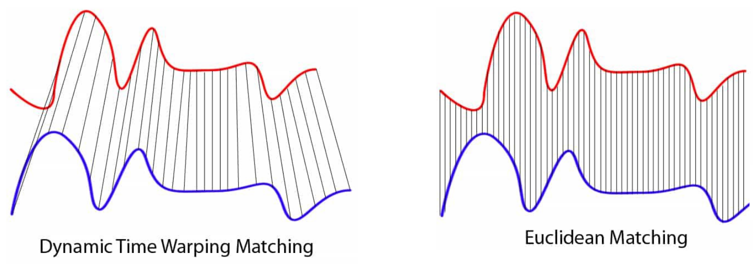


Figure 17: DTW and Euclidean Matching

Professor Lodgepole explains that DTW can be applied to two growth sequences to determine their similarity by filling out a grid using the following recurrence relation:

$$D[i, j] = |A_i - B_j| + \min(D[i-1, j],\ D[i, j-1],\ D[i-1, j-1])$$

This algorithm, illustrated in Figure 18, ensures that each cell accumulates the cost of aligning elements A_i and B_j while preserving the optimal path structure.

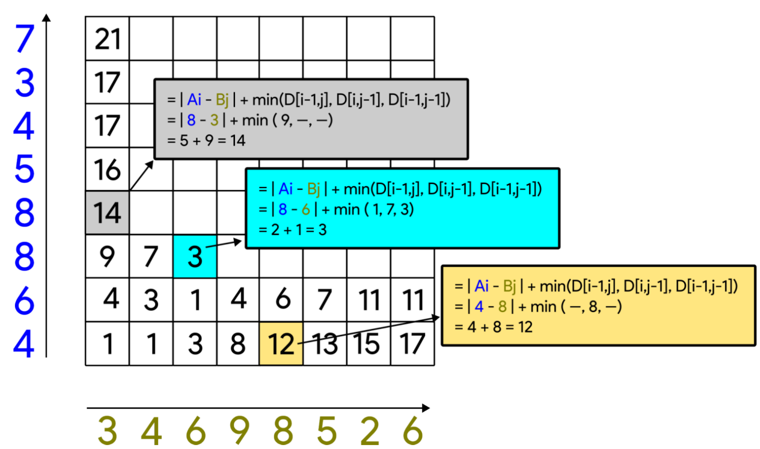


Figure 18: DTW Procedure

Once the grid is filled, the optimal alignment path is traced starting from the **top-right** cell and proceeding to the **bottom-left**, choosing at each step the direction (left, down, or diagonally down-left) that minimizes the cumulative cost. **In the event of a tie, the algorithm prefers a diagonal move.** This approach

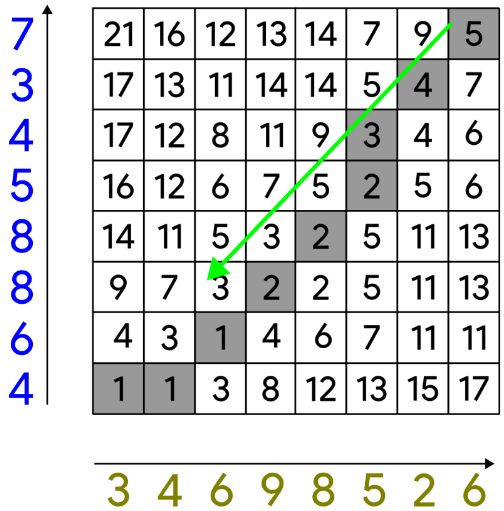


Figure 19: DTW Example

produces the lowest-cost alignment between the sequences. An example of such a path is shown in Figure 19, where the path is 1 - 1 - 1 - 2 - 2 - 2 - 3 - 4 - 5.

We denote the final DTW **distance** between the two sequences to be the value stored in the **top-right** cell. The smaller this value, the more similar the two growth patterns are. The DTW distance in the DTW example in Figure 19 is 5. Apply this algorithm to the bonsai growth sequences below.

8.1 Pattern Recognition Through Time Q1 [1pt]

$bonsai_a$

15			**21**			**28**		
17		**20**			**24**			
20	**19**		**23**			**26**		**34**
25				**B**	**29**		**30**	**C**
23	**12**	**13**				**19**	**24**	**27**
24			**12**		**17**			
21		**4**		**10**			**16**	
18	**1**				**8**		**A**	**15**
	19	**18**	**16**	**14**	**17**	**19**	**14**	**16**

$bonsai_b$

Figure 20: Grid for $bonsai_a$ vs. $bonsai_b$

What are the values of **A**, **B**, and **C** in Figure 20 above ($bonsai_a$ vs. $bonsai_b$)?

Answer with the values of **A**, **B**, and **C** as whole numbers.

8.2 Pattern Recognition Through Time Q2 [1pt]

What is the distance value between $bonsai_a$ and $bonsai_b$ based on Figure 20?

Answer as a whole number.

8.3 Pattern Recognition Through Time Q3 [1pt]

What is the path based on Figure 20?

Answer as a path (fill in the blank) e.g, 1-2-3-4-5...

8.4 Pattern Recognition Through Time Q4 [1pt]

15	**23**			**26**		**16**		
17		**A**			**18**		**8**	
20	**15**		**9**			**6**		**10**
25		**9**		**5**			**15**	
23	**10**		**4**		**8**		**B**	**19**
24		**5**				**10**		**C**
21					**14**			
18	**2**			**19**		**29**		**31**
	20	**22**	**23**	**26**	**25**	**21**	**19**	**17**

$bonsai_a$

$bonsai_c$

Figure 21: Grid for $bonsai_a$ vs. $bonsai_c$

What are the values of **A**, **B**, and **C** in Figure 21 above ($bonsai_a$ vs. $bonsai_c$)?

Answer with the values of **A**, **B**, and **C** as whole numbers.

8.5 Pattern Recognition Through Time Q5 [1pt]

What is the distance value between $bonsai_a$ and $bonsai_c$ based on Figure 21?

Answer as a whole number.

8.6 Pattern Recognition Through Time Q6 [1pt]

What is the path based on Figure 21?

Answer as a path (fill in the blank) e.g, 1-2-3-4-5...

8.7 Pattern Recognition Through Time Q7 [1pt]

$bonsai_b$								
16	**27**		**32**		**43**		**42**	
14		**25**		**34**				
19				**28**		**34**		**30**
17	**16**				**B**		**34**	
14			**18**		**30**			
16		**9**			**C**	**27**		**25**
18	**3**		**9**		**22**		**24**	
19	**A**			**15**				**25**
	20	**22**	**23**	**26**	**25**	**21**	**19**	**17**

$bonsai_c$

Figure 22: Grid for $bonsai_b$ vs. $bonsai_c$

What are the values of **A**, **B**, and **C** in Figure 22 above ($bonsai_b$ vs. $bonsai_c$)?

Answer with the values of **A**, **B**, and **C** as whole numbers.

8.8 Pattern Recognition Through Time Q8 [1pt]

What is the distance value between $bonsai_b$ and $bonsai_c$ based on Figure 22?

Answer as a whole number.

8.9 Pattern Recognition Through Time Q9 [1pt]

What is the path based on Figure 22?

Answer as a path (fill in the blank) e.g, 1-2-3-4-5...

8.10 Pattern Recognition Through Time Q10 [3pts]

Professor Ponderosa believes that two of the bonsai trees exhibit similar growth patterns, while the third shows a distinctly different trajectory. Using Dynamic Time Warping (DTW) distance as a measure of similarity, help him identify the outlier among the three bonsai trees.

Based on the distances, which bonsai is the outlier?

Select one.

(a) $bonsai_a$

(b) $bonsai_b$

(c) $bonsai_c$

9 Deep Learning [12pts]

Please see Module 9 Deep Learning on Canvas as a reference for this section.

Part I

You're part of an engineering team developing an autonomous drone for precision agriculture. The drone flies over crop fields, capturing 64x64 grayscale aerial images of small plots. Your team is building a convolutional neural network (CNN) to classify whether a section of the field is healthy or stressed based on these images.

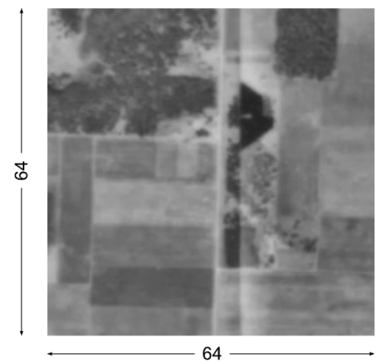


Figure 23: Example aerial image of a crop field.

To start, your model applies a convolutional layer with 16 filters, each of size 5x5 and a learnable bias, stride 1, and no zero-padding, followed by a 2x2 max pooling layer with stride 2. The goal is to compress spatial dimensions while retaining key features for downstream classification.

Your task is to analyze how the input is transformed through the network.

9.1 Deep Learning Q1 [1pt]

Why might we choose zero padding in a convolutional layer?

Select one.

(a) To reduce the number of parameters in the convolutional filter.

(b) To preserve the spatial dimensions of the input after convolution.

(c) To increase the depth (number of channels) of the output feature maps.

(d) To avoid using activation functions after convolution.

9.2 Deep Learning Q2 [1pt]

How does increasing the stride in a convolutional layer affect the output feature map?

Select one.

(a) It increases the spatial resolution of the feature map.

(b) It leaves the spatial resolution unchanged.

(c) It decreases the spatial resolution of the feature map.

(d) It increases the number of output channels.

9.3 Deep Learning Q3 [1pt]

What is the spatial size (height x width) of the output feature maps after the convolution layer?

Select one.

(a) 64 x 64

(b) 62 x 62

(c) 60 x 60

(d) 30 x 30

9.4 Deep Learning Q4 [1pt]

What would be the spatial size (height x width) of the output feature maps after the convolution layer had there been zero padding of size 2 around the entire image?

Select one.

(a) 64 x 64

(b) 62 x 62

(c) 60 x 60

(d) 30 x 30

9.5 Deep Learning Q5 [1pt]

What is the spatial size and depth of the feature maps after the pooling layer?

Select one.

(a) 32 x 32 x 8

(b) 31 x 31 x 16

(c) 30 x 30 x 16

(d) 28 x 28 x 8

9.6 Deep Learning Q6 [1pt]

What is the total number of parameters in this CNN?

Answer as a whole number.

9.7 Deep Learning Q7 [2pts]

What is the total number of parameters in this CNN had there been zero-padding of size 2?

Answer as a whole number.

Part II

A year later, you take up a new role as a machine learning engineer at a startup developing a large language model designed to help medical researchers summarize clinical trial reports.

Your team has implemented a Transformer-based model whose hidden states at each layer have shape (N, D), where N is the batch size and D is the embedding dimension.

However, you've noticed that during training, your model sometimes diverges or trains slowly, especially when batches vary in size due to long documents being truncated or padded differently.

Your colleague suggests trying Layer Normalization instead of Batch Normalization, arguing that it might be better suited for sequences with variable lengths or unstable batch statistics.

You decide to investigate the mathematical differences between Batch Norm and Layer Norm so you can make an informed choice.

9.8 Deep Learning Q8 [1pt]

Consider a fully-connected layer producing an output tensor $X \in \mathbb{R}^{N \times D}$, where N is the batch size and D is the feature dimension. Each row $x_i \in \mathbb{R}^D$ represents the activation of the i-th input sample.

Suppose you apply either Batch Normalization or Layer Normalization to X.

For Batch Normalization, the mean μ_j and variance σ_j^2 for feature j are computed as:

Select one.

(a) $\mu_j = \frac{1}{D}\sum_{k=1}^{D} x_{ik}, \quad \sigma_j^2 = \frac{1}{D}\sum_{k=1}^{D}(x_{ik} - \mu_j)^2$

(b) $\mu_j = \frac{1}{N}\sum_{i=1}^{N} x_{ij}, \quad \sigma_j^2 = \frac{1}{N}\sum_{i=1}^{N}(x_{ij} - \mu_j)^2$

(c) $\mu_j = \frac{1}{ND}\sum_{i=1}^{N}\sum_{j=1}^{D} x_{ij}, \quad \sigma_j^2 = \frac{1}{ND}\sum_{i=1}^{N}\sum_{j=1}^{D}(x_{ij} - \mu_j)^2$

(d) None of the above.

9.9 Deep Learning Q9 [1pt]

For Layer Normalization applied to x_i, the mean μ_i and variance σ_i^2 are computed as:

Select one.

(a) $$\mu_i = \frac{1}{N}\sum_{j=1}^{N} x_{ji}, \quad \sigma_i^2 = \frac{1}{N}\sum_{j=1}^{N}(x_{ji} - \mu_i)^2$$

(b) $$\mu_i = \frac{1}{D}\sum_{j=1}^{D} x_{ij}, \quad \sigma_i^2 = \frac{1}{D}\sum_{j=1}^{D}(x_{ij} - \mu_i)^2$$

(c) $$\mu_i = \frac{1}{ND}\sum_{i=1}^{N}\sum_{j=1}^{D} x_{ij}, \quad \sigma_i^2 = \frac{1}{ND}\sum_{i=1}^{N}\sum_{j=1}^{D}(x_{ij} - \mu_i)^2$$

(d) None of the above.

9.10 Deep Learning Q10 [1pt]

After normalization, both methods apply the transformation:

$$\hat{x}_{ij} = \frac{x_{ij} - \mu}{\sqrt{\sigma^2 + \epsilon}}, \qquad y_{ij} = \gamma \hat{x}_{ij} + \beta \tag{3}$$

Which of the following is true about the learnable parameters γ and β?

Select one.

(a) In both Batch Norm and Layer Norm, γ and β are vectors of size D.

(b) In Batch Norm, γ and β are vectors of size N; in Layer Norm, they are scalars.

(c) In Batch Norm, γ and β are scalars shared across all features, whereas in Layer Norm, they are vectors of size N.

(d) None of the above.

9.11 Deep Learning Q11 [1pt]

Which of the following is an advantage of Layer Normalization over Batch Normalization?

Select one.

(a) It normalizes over the batch dimension, making it faster for large batches.

(b) It does not depend on the batch size and works consistently on individual samples.

(c) It increases the receptive field of convolutional layers.

(d) It completely removes the need for learnable parameters in the network.

10 Planning Under Uncertainty [12pts]

Introduction

During one of their summer adventures, Phineas and Ferb have become separated in their neighborhood in Danville, as shown on the map below. Wishing to be reunited, Phineas begins his journey from location (A,1), seeking to reach Ferb at (E,5).

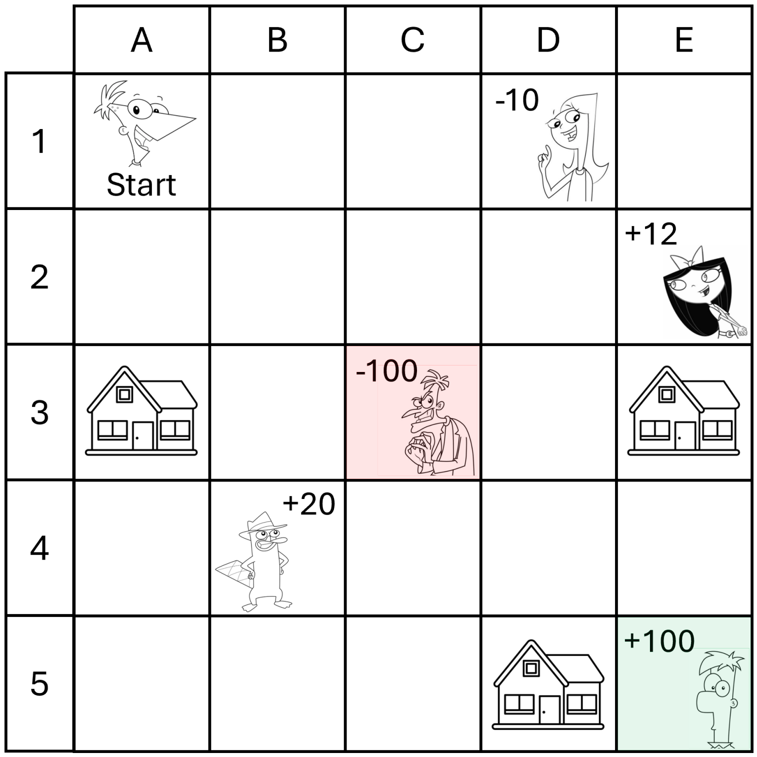


Figure 24: Map and Reward Values in Danville

There are two terminal locations on the map, and if Phineas enters them, he cannot leave, and the journey is over:

- At (C,3) stands Dr. Doofenshmirtz, the arch-nemesis of the entire tri-state area.
- At the goal location (E,5) is Phineas' stepbrother, Ferb.

There are three unreachable and impassable locations where houses are shown on the map: (A,3), (D,5), and (E,3). Three other locations provide rewards if they are met along the way (rewards values are noted on the map). Rewards can be earned multiple times if a location is visited more than once.

- At (D,1) is Phineas' sister, Candace, who would love nothing more than to report his antics to their mother.
- At (E,2) is a dear friend, Isabella.
- At (B,4) is the family's pet platypus, Perry, who is likely putting a stop to whatever mischief Dr Doofenshmirtz has planned.

It's rather hot during the summer, and walking around the neighborhood can be tiresome, so Phineas gets a reward of -1 each time he moves.

Phineas can move in singular cardinal directions: up, down, left and right. However, due to Dr. Doofenshmirtz's "signal scramble-inator", your instructions to Phineas may be corrupted before they reach him. When you select a direction for Phineas to move, there is a 70% chance of him following your command, and a 15% chance of him moving in either orthogonal direction instead. the step cost is charged for every action the agent takes, whether or not the intended move actually changes its position. If an action would move Phineas off the map or into the impassable houses, you can assume that he will stay in his current location, but the -1 action cost still applies. Below are two examples:

- Suppose Phineas is at (B, 2) and the command is to move down. There is a 70% chance of him moving down to (B, 3), a 15% chance of him moving to his left (C, 2), and a 15% chance he moves to his right (A, 2).
- If Phineas is at (A,4) and you tell him to move up, there is an 85% chance he will stay in the same location (70% from hitting the house in (A,3) and 15% from hitting the left side of the map) and a 15% chance he will move to his right, (B,4), where he collects a reward for meeting up with Perry.

10.1 Planning Under Uncertainty Q1 [4pts]

The discount factor is 0.9, and the utility of each location at the beginning of the adventure (at time step 0) is shown below.

	A	B	C	D	E
1	0	0	5	0	0
2	0	0	0	0	0
3	X	0	0	6	X
4	-2	0	14	0	0
5	0	0	0	X	0

Table 6: Initial utility at each location

Compute the utility for each of the following locations at time steps 1 and 2 using the Bellman update equation for value iteration as shown below. (Some values are provided for your reference.)

$$U_{k+1}(s) = R(s) + \gamma \max_{a} \sum_{s'} P(s' \mid s, a) \, U_k(s') \tag{4}$$

	Timestep 1	Timestep 2
(B,1)	2.1500	**W**
(E,4)	**X**	63.0301
(B,5)	**Y**	**Z**

Table 7: Utility at Timesteps 1 and 2

Answer with the values of W, X, Y, and Z as numbers rounded to four decimal places.

10.2 Planning Under Uncertainty Q2 [2pts]

Suppose that the reward of -1 for changing locations is removed and that the initial utility of each location is zero.

Determine if there is a policy that allows for a successful journey from (A,1) to (E,5) while also guaranteeing that no negative rewards will be met along the way. If so, then using such a policy, what is the minimum possible score achieved upon arrival at the goal location (E,5)? If no such policy exists, answer with -1.

Answer as a number rounded to four decimal places.

10.3 Planning Under Uncertainty Q3 [2pts]

Suppose that the reward of -1 for changing locations is removed and that the initial utility of each location is zero.

Phineas is standing at square (C,4). Because it is late in the afternoon he has energy for exactly three moves. After the third move the game ends automatically, no matter where he is. All movements are now deterministic (Perry has disabled the scramble-inator). Let the discount factor satisfy $0 \leq \gamma \leq 1$. For which range of values for γ, is moving right (towards Ferb) the optimal first action from (C,4)? Denote the bounds of the range as A and B where $A \leq \gamma \leq B$. Give the values of A and B.

Answer with the values of A and B as numbers rounded to four decimal places.

10.4 Planning Under Uncertainty Q4 [4pts]

Select **all** statements below that are always true for any finite-state MDP.

Multiple select.

(a) If every immediate reward $R(s)$ is multiplied by a positive constant $k > 0$, the optimal policy remains unchanged for any fixed discount factor γ.

(b) If two MDPs differ **only** in their discount factors $\gamma_1 < \gamma_2 < 1$, they can still induce **different** optimal policies.

(c) Adding the same constant c to all rewards leaves the optimal policy unchanged **and** leaves every Q-value unchanged.

(d) For $\gamma = 1$ and bounded rewards, value iteration is guaranteed to converge on a finite-state MDP.