Exercise. lecture 1.

1.1 we let $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$ for all element in B

$b_{ij} \in \mathbb{C} \quad 1 \le i,j \le 4.$

(a) 1.

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \\ b_{41} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{4} b_{1k} b_{k1} \\ \sum_{k=1}^{4} b_{2k} b_{k1} \\ \sum_{k=1}^{4} b_{3k} b_{k1} \\ \sum_{k=1}^{4} b_{4k} b_{k1} \end{bmatrix}$$

2.

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ \frac{b_{31}}{2} & \frac{b_{32}}{2} & \frac{b_{33}}{2} & \frac{b_{34}}{2} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}.$$

3.

$$\begin{bmatrix} b_{11}+b_{31} & b_{12}+b_{32} & b_{13}+b_{33} & b_{14}+b_{34} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix}$$

4.

$$\begin{bmatrix} b_{14} & b_{12} & b_{13} & b_{11} \\ b_{24} & b_{22} & b_{23} & b_{21} \\ b_{34} & b_{32} & b_{33} & b_{31} \\ b_{44} & b_{42} & b_{43} & b_{41} \end{bmatrix}$$

5.

$$\begin{bmatrix} b_{11}-b_{21} & b_{12}-b_{22} & b_{13}-b_{23} & b_{14}-b_{24} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31}-b_{21} & b_{32}-b_{22} & b_{33}-b_{23} & b_{34}-b_{24} \\ b_{41}-b_{21} & b_{42}-b_{22} & b_{43}-b_{23} & b_{44}-b_{24} \end{bmatrix}$$

6.

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{13} \\ b_{21} & b_{22} & b_{23} & b_{23} \\ b_{31} & b_{32} & b_{33} & b_{33} \\ b_{41} & b_{42} & b_{43} & b_{43} \end{bmatrix}$$

7.

$$\begin{bmatrix} b_{12} & b_{13} & b_{14} \\ b_{22} & b_{23} & b_{24} \\ b_{32} & b_{33} & b_{34} \\ b_{42} & b_{43} & b_{44} \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \qquad C = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$

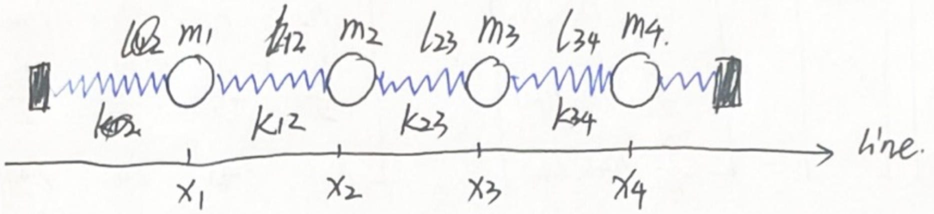
for all entries in A, B belongs to $\mathbb{C}$

$$ABC = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix}$$ to get result

$$= \begin{bmatrix} \sum_{i=1}^{4} a_{1i} b_{i1} & \sum_{i=1}^{4} a_{1i} b_{i2} & \sum_{i=1}^{4} a_{1i} b_{i3} & \sum_{i=1}^{4} a_{1i} b_{i4} \\ \sum_{i=1}^{4} a_{2i} b_{i1} & \sum_{i=1}^{4} a_{2i} b_{i2} & \sum_{i=1}^{4} a_{2i} b_{i3} & \sum_{i=1}^{4} a_{2i} b_{i4} \\ \sum_{i=1}^{4} a_{3i} b_{i1} & \sum_{i=1}^{4} a_{3i} b_{i2} & \sum_{i=1}^{4} a_{3i} b_{i3} & \sum_{i=1}^{4} a_{3i} b_{i4} \\ \sum_{i=1}^{4} a_{4i} b_{i1} & \sum_{i=1}^{4} a_{4i} b_{i2} & \sum_{i=1}^{4} a_{4i} b_{i3} & \sum_{i=1}^{4} a_{4i} b_{i4} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{21} & c_{22} & c_{23} & c_{24} \\ c_{31} & c_{32} & c_{33} & c_{34} \\ c_{41} & c_{42} & c_{43} & c_{44} \end{bmatrix} \nearrow$$

$$= \begin{bmatrix} \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{1i} b_{ik}) c_{k1} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{1i} b_{ik}) c_{k2} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{1i} b_{ik}) c_{k3} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{1i} b_{ik}) c_{k4} \\ \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{2i} b_{ik}) c_{k1} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{2i} b_{ik}) c_{k2} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{2i} b_{ik}) c_{k3} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{2i} b_{ik}) c_{k4} \\ \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{k1} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{k2} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{3} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{4} \\ \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{4i} b_{ik}) c_{k1} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{k2} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{3i} b_{ik}) c_{3} & \sum_{k=1}^{4}(\sum_{i=1}^{4} a_{4i} b_{ik}) c_{4} \end{bmatrix}$$

1.2



rightward forces

$f_1 = k_{12}(x_2 - x_1 - l_{12})$ $\quad f_2 = k_{12}(x_2 - x_1 - l_{12}) - k_{23}(x_3 - x_2 - l_{23})$

$f_4 = k_{34}(x_4 - x_3 - l_{34})$ $\quad f_3 = k_{23}(x_3 - x_2 - l_{23}) - k_{34}(x_4 - x_3 - l_{34})$

(a) $$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} -k_{12} & k_{12} & 0 & 0 \\ -k_{12} & k_{12}+k_{23} & -k_{23} & 0 \\ 0 & -k_{23} & k_{23}+k_{34} & -k_{34} \\ 0 & 0 & -k_{34} & k_{34} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} -k_{12} l_{12} \\ -k_{12} l_2 + k_{23} l_{23} \\ -k_{23} l_{23} + k_{34} l_{34} \\ -k_{34} l_{34} \end{bmatrix}$$

(b) Spring constant N/kg or kg/sec^2. (c) $(N/kg)^4$ or $(kg/sec^2)^4$

(c) $k = 1000k'$ $\quad 1kg = 10^3 g$ $\quad 1m = 10^2 cm$ $\quad \det(k) = (10^3)^4 \det(k')$

1.3. $B = AR$

$$\begin{bmatrix} b_1 & \cdots & b_n \end{bmatrix} = \begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix} \underbrace{\begin{bmatrix} 1 & \cdots & 1 \\ & \ddots & \vdots \\ & & 1 \end{bmatrix}}_{R}$$

$b_j = A r_j = \sum_{k=1}^{j} a_k$.

R is upper-triangular matrix $r_{ij} = 0$ for all $i > j$

$$\begin{bmatrix} \ne 0 \end{bmatrix}$$

Given this, R is an $m \times m$ nonsingular, upper-triangular matrix

$I = ZR$

$$[e_1, e_2, \cdots, e_m] = \underline{[z_1, z_2, \cdots, z_m]} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ 0 & r_{22} & \cdots & r_{2m} \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \cdots & & r_{mm} \end{bmatrix}$$

we need to show that $[z_1, z_2, \cdots, z_m]$ is upper-triangular

★★ For $B = AC$, each column of B is a linear combination of the columns of A.

$$[b_j] = C_{1j}[a_1] + C_{2j}[a_2] + \cdots + C_{mj}[a_m]$$ we will use this

① $e_1 = r_{11} z_1 \rightarrow z_1 = r_{11}^{-1} e_1 \Rightarrow z_1$ has only one non-zero entry

② Let $C^m(k)$ denote the column space of vectors of dimensionality m which have at most non-zero entries

$C^m(m) = C^m$ and each $C^m(k)$ is a linear subspace of C^m

$z_1 \in C^m(1)$ from ①

$$e_{i+1} = \sum_{k=1}^{m} z_k r_{k(i+1)} = \sum_{k=1}^{i} z_k r_{k(i+1)} + z_{i+1} r_{(i+1)(i+1)}$$

$$z_{i+1} = \underbrace{r_{(i+1)(i+1)}^{-1}}_{\text{scalar}} \underbrace{(e_{i+1} - \sum_{k=1}^{i} z_k r_{k(i+1)})}_{\in C^m(i+1)} \rightarrow C^m(i)$$

1-4. $\sum_{j=1}^{8} C_j f_j(i) = d_i$, $i = 1, \cdots, 8$

(a) we let $FC = D$ for any $D \in C^8$. Range (F) $= C^8$

$$\begin{bmatrix} f_1(1) & f_2(1) & \cdots & f_8(1) \\ f_1(2) & \cdots & \cdots & \vdots \\ \vdots & & & \vdots \\ f_1(8) & \cdots & \cdots & f_8(8) \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_8 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_8 \end{bmatrix}$$

F is a full-rank matrix

So $C \rightarrow d$ is a one-to-one maping

(b) $Ad = C \Rightarrow A^{-1} = F$ the element of A^{-1} is $f_j(i)$