

1. Let  $f(x) = \frac{2x+3}{x-4}$ . Find the inverse of  $f(x)$  and determine the range of  $f(x)$ .

*Xingjian Solution:*

Let  $f(x) = y = \frac{2x+3}{x-4}$ . Then we exchange the variable, then we get

$$x = \frac{2y+3}{y-4}.$$

Multiply  $y - 4$  to both sides:

$$x(y-4) = 2y+3$$

$$xy - 4x = 2y + 3$$

$$-3 - 4x = 2y - xy$$

$$3 + 4x = xy - 2y$$

$$3 + 4x = (x-2)y$$

$$\boxed{y = \frac{3+4x}{x-2}}.$$

Since the range of  $f(x)$  is the domain of  $f^{-1}(x)$  [let us say that this is an inverse function.] Thus the range of  $f(x)$  is  $(-\infty, 2) \cup (2, \infty)$ .

2. Suppose  $f(x) = \frac{x}{|x|} + \frac{x-1}{|x-1|}$ , evaluate  $\lim_{x \rightarrow 0^-} f(x)$ ?

(Hint: think about the graph of  $f(x)$  from your homework 1 and 2.)

*Xingjian Solution:*

If you are looking at the selected problem for what I give you before. Then I get

$$f(x) = \begin{cases} 2, & x > 1 \\ 0, & 0 < x < 1 \\ -2, & x < 0 \end{cases}$$

so  $\boxed{\lim_{x \rightarrow 0^-} f(x) = -2}.$

*Xingjian Xu*  
xingjianxu@ufl.edu  
LIT 453