1. Let $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 0} g(x) = -5$. Evaluate

$$\lim_{x\to 0} \frac{f(x+2)}{(g(x))^2} + \lim_{x\to 1} \frac{f(x+1)}{g(x-1)}$$

.

Xingjian Solution: First we can write

$$\frac{\lim_{x\to 0} f(x+2)}{\lim_{x\to 0}(g(x))^2} + \frac{\lim_{x\to 1} f(x+1)}{\lim_{x\to 1} g(x-1)}$$

We handle difficult terms separately.

For $\lim_{x\to 0} f(x+2)$,let $z = x+2$, since $x \to 0$, thus $z \to 2$, then we can change the limit to

$$\lim_{x\to 0} f(x+2) = \lim_{z\to 2} f(z) = \lim_{x\to 2} f(x) = 3.$$

For second identity, just like typo change. The value does not change.

For $\lim_{x\to 1} f(x+1)$, let $y = x+1$, since $x \to 1$, then $y \to 2$. Thus, we can change the limit to

$$\lim_{x\to 1} f(x+1) = \lim_{y\to 2} f(y) = \lim_{x\to 2} f(x) = 3.$$

For $\lim_{x\to 1} g(x-1)$, let $p = x-1$, since $xto1$, then $p \to 0$. Thus, we can change the limit to

$$\lim_{x\to 1} g(x-1) = \lim_{p\to 0} g(p) = \lim_{x\to 0} g(x) = -5.$$

Thus the limit question just becomes

$$\frac{3}{(-5)^2} + \frac{3}{-5} = \frac{3}{25} - \frac{15}{25} = \frac{-12}{25}.$$

So the answer is $\boxed{\frac{-12}{25}}$.

2. (5 pts) Does $f(x) = 3x^2 - x\tan(x) + 2$ has a solution in this interval $[1, 1.5]$? Why?

Xingjian Solution: First we know $f(x)$ is continuous function in that [1,1.5] even though $\tan(x)$ has period discontinuous points. [do you remember $\tan(x)$ is continuous in $(-\frac{\pi}{2}, \frac{\pi}{2})$?Actually $(-\frac{\pi}{2}, \frac{\pi}{2}) \approx (-1.57, 1.57)$. That is true.

Then what we do here is try to plug in the value 1 and 1.5 to see the sign of value.

$$f(1) = 3 - \tan(1) + 2 = 5 - \tan(1) > 0$$
$$f(\frac{3}{2}) = 3 \times \frac{9}{4} - \frac{3}{2}\tan(\frac{3}{2}) + 2 < 0.$$

Then using intermediate value theorem, you will get a root in the interval $[1, 1.5]$. So the answer is $\boxed{\textit{yes.}}$

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