Question:(from HW15) If $f(x) = (x+1)^5(x-1)^3$, then $f'(x)$ is equal to what value?
Xingjian's solution:
In my opinion, the most straightforward way is to apply the product rule. Thus,

$$f'(x) = \big((x+1)^5\big)'(x-1)^3 + (x+1)^5\big((x-1)^3\big)'.$$

By the chain rule, let us first consider $(x+1)^5$. We can think of it as a composition of two functions: the outer function $(\cdot)^5$ and the inner function $(x+1)$. The derivative of the outer function is $5(\cdot)^4$, and substituting back the inner function gives $5(x+1)^4$. Multiplying by the derivative of the inner function (which is 1), we obtain

$$\big((x+1)^5\big)' = 5(x+1)^4.$$

Similarly, one can compute

$$\big((x-1)^3\big)' = 3(x-1)^2.$$

Thus,

$$f'(x) = 5(x+1)^4(x-1)^3 + 3(x+1)^5(x-1)^2.$$

Next, let us factorize this expression:

$$f'(x) = (x+1)^4(x-1)^2\big[5(x-1) + 3(x+1)\big].$$

Simplifying the last factor:

$$f'(x) = (x+1)^4(x-1)^2(5x-5+3x+3),$$

$$f'(x) = (x+1)^4(x-1)^2(8x-2).$$

Finally, factoring out the constant gives the compact form:

$$\boxed{f'(x) = 2(x+1)^4(x-1)^2(4x-1)}.$$

Question:(from HW14) If $f(x) = \sec(x)$, find an expression of $f''(x)$.
Xingjian's solution: For this question, if you can remember the derivative of $\sec(x)$, then you can directly use that: $\sec'(x) = \sec(x)\tan(x)$. Then we can use product rule to use it here:

$$\sec''(x) = \big(\sec'(x)\big)' = (\sec(x)\tan(x))' = (\sec(x))'\tan(x) + \sec(x)(\tan(x))'$$

Then I also know $(\tan(x))' = \sec^2(x)$, let us try it:

$$\sec''(x) = \sec(x)\tan^2(x) + \sec(x)\sec^2(x)$$

Thus,

$$\boxed{\sec''(x) = \sec(x)\big(\tan^2(x) + \sec^2(x)\big)}$$

Remark: If you didn't remember clearly for the derivative of $\sec(x)$, that is fine and we can also solve it. To compute the derivative of $\sec(x)$, we note that

$$\sec(x) = \frac{1}{\cos(x)} = \big(\cos(x)\big)^{-1}.$$

Here, we can regard $\big(\cos(x)\big)^{-1}$ as a composition of two functions: the outer function $(\cdot)^{-1}$ and the inner function $\cos(x)$.

First, for the outer function $(\cdot)^{-1}$, its derivative is $-(\cdot)^{-2}$. Substituting the inner function $\cos(x)$ back, this becomes

$$-(\cos(x))^{-2}.$$

Next, we multiply by the derivative of the inner function $\cos(x)$, which is $-\sin(x)$. Thus,

$$\frac{d}{dx}\sec(x) = -(\cos(x))^{-2} \cdot (-\sin(x)).$$

Simplifying:

$$\frac{d}{dx}\sec(x) = \frac{\sin(x)}{\cos^2(x)}.$$

Finally, we rewrite this in terms of $\sec(x)$ and $\tan(x)$:

$$\frac{\sin(x)}{\cos^2(x)} = \sec(x)\tan(x).$$

Therefore,

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x).$$

Then you can do the same step with method I proivided.

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