Calculus I: MAC2311
Fall 2025
Exam 2 B
10/14/2025
Time Limit: 100 Minutes

Name: Key
Section: ______________
UF-ID: ______________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 2, 2:

1 • 3 4 5 6 7 8 9 0
1 • 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as B .

A • C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 100 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors**. Any mistakes you leave in are there to stay!

2) You must turn in your scantron and free response packet to your proctor. **Be prepared to show your proctor a valid GatorOne ID or other signed ID.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

Chain rule.

1. If $y = \cos(\sqrt{x})$, find $y'(\pi^2)$.

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{1}{2\pi}$ (D) $-\frac{1}{2\pi}$ (E) -1

$y = \cos(x^{\frac{1}{2}})$

$y' = -\sin(x^{\frac{1}{2}}) \cdot \frac{1}{2}x^{-\frac{1}{2}}$ Plug in π^2:

$y'(\pi^2) = -\sin(\sqrt{\pi^2})\frac{1}{2}\frac{1}{\sqrt{\pi^2}}$

$y'(\pi^2) = -\sin(\pi) \cdot \frac{1}{2\pi} = 0$

Chain rule (twice)

2. If $h(x) = \cos(\sin(3x))$, then $h'(0) =$

(A) 0 (B) -3 (C) 3

(D) 1 (E) -1

$h'(x) = -\sin(\sin(3x)) \cdot \cos(3x) \cdot 3$ Plug in $x=0$.

$h'(0) = -\sin(\sin(0)) \cdot \cos(0) \cdot 3$

$h'(0) = -\sin(0) \cdot 1 \cdot 3 = 0$

0

Chain rule

3. Let $f(x) = (2x+1)^4$. Then $f'(0) =$

(A) 4 (B) 8 (C) 16

(D) 32 (E) 0

$f'(x) = 4(2x+1)^3 \cdot 2$ Plug in $x=0$.

$f'(0) = 4(1)^3 \cdot 2 = 8$

Implicit diff

4. For the curve $x^2 + y^2 = 25$, find $\frac{dy}{dx}$ at the point $(4,3)$.

(A) $\frac{4}{3}$ (B) $-\frac{4}{3}$ (C) 3 (D) $-\frac{3}{4}$ (E) $\frac{3}{4}$

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$

$2x + 2y \cdot y' = 0$

$2yy' = -2x$

$y' = \frac{-2x}{2y}\Big|_{\substack{x=4\\y=3}} = \frac{-8}{6} = \frac{-4}{3}$

5. Let $f(x) = x\sin(3x)$. Find $f'(\pi/3)$. product rule

(A) 0 (B) $-\pi$ (C) π (D) $\frac{\pi}{3}$ (E) $-\frac{\pi}{3}$

$f'(x) = (1)\sin(3x) + x\cos(3x)\cdot 3$ Plug in $x = \frac{\pi}{3}$

$f'(\frac{\pi}{3}) = (1)\sin(\pi) + \frac{\pi}{3}\cos(\pi)\cdot 3$

$f'(\frac{\pi}{3}) = 0 + \pi(-1) = -\pi$

6. For which value of k will the function $f(x) = kx^4 - 3x^2 + 8$ have a horizontal tangent at $x = -2$?

constant

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $-\frac{3}{4}$ (E) $\frac{2}{3}$

We need $f'(-2) = 0$.

$f'(x) = 4kx^3 - 6x$

$f'(-2) = 4k(-2)^3 + 12 = -32k + 12$

If $f'(-2) = 0$, Then $-32k + 12 = 0$, so $k = \frac{12}{32}$

$k = \frac{3}{8}$

7. If $f(x) = 3^x(\sqrt{2})^x$, what is $f'(2)$?

(A) $9\sqrt{2}\ln 3 + 6\ln\sqrt{2}$ (B) $9\sqrt{2}(\ln 3)(\ln\sqrt{2})$ (C) $3\sqrt{2}(\ln 3 + \ln\sqrt{2})$

(D) $18(\ln 3 + \ln\sqrt{2})$ (E) None of the above.

Rewrite:

$f(x) = 3^x(\sqrt{2})^x = (3\sqrt{2})^x$

$f'(x) = (3\sqrt{2})^x \cdot \ln(3\sqrt{2})$ Plug in $x=2$

$f'(2) = (3\sqrt{2})^2 \ln(3\sqrt{2})$

$f'(2) = 9(2)(\ln(3) + \ln(\sqrt{2}))$ (log rule)

$f'(2) = 18(\ln(3) + \ln(\sqrt{2}))$

8. If $f(x) = \frac{\sqrt{x}-1}{x+1}$, which of the following is $f'(4)$? Quotient rule

$\frac{x^{1/2}-1}{x+1}$

(A) $\frac{1}{20}$ (B) $\frac{1}{25}$ (C) $\frac{1}{50}$

(D) $\frac{1}{100}$ (E) $-\frac{1}{200}$

$$f'(x) = \frac{(x+1)\left(\frac{1}{2}x^{-1/2}\right) - \left((x^{1/2}-1)(1)\right)}{(x+1)^2}$$ Plug in $x=4$.

$$f'(4) = \frac{(5)\left(\frac{1}{2\sqrt{4}}\right) - \left((\sqrt{4}-1)\right)}{(4+1)^2} = \frac{\frac{5}{4} - 1}{25} = \frac{\frac{1}{4}}{25} = \frac{1}{100}$$

9. A particle moves along a straight line with a position function given by $s(t) = (2t^2 - 12t + 18)e^{2t}$. During which of the following intervals is the velocity strictly negative?

↑ need $s'(t) < 0$

(A) $[0, 3)$ (B) $(2, 3)$ (C) $(1, 4)$

(D) $(4, 6)$ (E) $(2, \infty)$

product rule

$s'(t) = (4t-12)e^{2t} + (2t^2-12t+18)(e^{2t} \cdot 2)$ factor

$s'(t) = e^{2t}[4t-12 + 4t^2 - 24t + 36]$

$s'(t) = e^{2t}[4t^2 - 20t + 24]$

$s'(t) = 4e^{2t}[t^2 - 5t + 6]$

$s'(t) = 4e^{2t}(t-3)(t-2)$ Test around $t=2$, $t=3$

0 2.5 5

(+) 2 (−) 3 (+)

(2,3) is negative

10. Let $h(x) = (x+2)^5(2x-1)^4$; for which of the following values of x will the function have a horizontal tangent line? need $h'(x) = 0$.

(A) 0 (B) $-\frac{11}{18}$ (C) $-\frac{5}{6}$

(D) $-\frac{23}{18}$ (E) $-\frac{7}{6}$

(Product rule)

$h'(x) = 5(x+2)^4(2x-1)^4 + (x+2)^5 \cdot 4(2x-1)^3 \cdot 2$

$h'(x) = (x+2)^4(2x-1)^3[5(2x-1) + 8(x+2)]$

$h'(x) = (x+2)^4(2x-1)^3[18x + 11]$

$h'(x) = 0$, so $x = -2$, $x = \frac{1}{2}$, $x = \frac{-11}{18}$

11. Calculate the derivative of

$$f(x) = \cos^{-1}\left(3x^2 + x\right).$$

Chain rule

(Here, $\cos^{-1}(x) = \arccos(x)$)

(A) $f'(x) = \frac{-6x-1}{\sqrt{1-(3x^2+x)^2}}$ (B) $f'(x) = \frac{3x^2-x}{\sqrt{1+(6x+1)^2}}$

(C) $f'(x) = \frac{-6x}{\sqrt{1-(3x^2+1)^2}}$ (D) $f'(x) = \frac{-3x^2+1}{\sqrt{1+(3x^2+x)^2}}$

$$f'(x) = \frac{-1}{\sqrt{1-(3x^2+x)^2}} \cdot (6x+1)$$

12. If g is the inverse of $f(x) = 3x - \sin(x)$, calculate $g'(3\pi)$.

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$

(D) $\frac{1}{2}$ (E) 0

$f(\pi) = 3\pi - \sin(\pi) = 3\pi$.
This means $g(3\pi) = \pi$.

$$g'(3\pi) = \frac{1}{f'(g(3\pi))} = \frac{1}{f'(\pi)} = \frac{1}{4}$$

$f'(x) = 3 - \cos(x)$
$f'(\pi) = 3 - (-1)$
$f'(\pi) = 4$

$h = \frac{1}{2}d$

13. Sand pouring from a chute forms a conical pile with height equal to half the diameter. If the height increases at a constant rate of 5 feet per minute, at what rate is the sand pouring from the chute when the pile is 10 feet high? Volume of a cone is $V = \frac{1}{3}\pi r^2 h$.

(A) $V' = 500\pi$ ft^3 per min. (B) $V' = 250\pi$ ft^3 per min. (C) $V' = 125\pi$ ft^3 per min.

(D) $V' = \frac{125\pi}{2}$ ft^3 per min. (E) $V' = \frac{125\pi}{4}$ ft^{32} per min.

$\frac{dh}{dt} = 5$ ft/min

$\frac{dV}{dt} = ?$

$h = 10$

$h = \frac{1}{2}d = \frac{1}{2}(2r) = r$

$r = h$

$V = \frac{\pi}{3} r^2 h$

$V = \frac{\pi}{3}(h)^2 h$

$V = \frac{\pi}{3} h^3$

$\frac{dV}{dV} = \pi h^2 \frac{dh}{dt} = \pi(10)^2(5)$

$= 500\pi$ ft^3/min

14. A particle moves along the curve $y = \sqrt{1+x^3}$. As it reaches the point $(2,3)$, the y-coordinate is increasing at a rate of 10 cm. per second. How fast is the x-coordinate of the point changing at that instant?

(A) $\frac{dx}{dt} = 20$ cm. per second (B) $\frac{dx}{dt} = 15$ cm. per second (C) $\frac{dx}{dt} = 12$ cm. per second

(D) $\frac{dx}{dt} = 8$ cm. per second (E) $\frac{dx}{dt} = 5$ cm. per second

$y = (1+x^3)^{\frac{1}{2}}$ $x = 2,\ y = 3,\ \frac{dy}{dt} = 10$ cm/sec

$\frac{dy}{dt} = \frac{1}{2}(1+x^3)^{-\frac{1}{2}} \cdot (3x^2) \frac{dx}{dt}$

$10 = \frac{1}{2}(1+2^3)^{-\frac{1}{2}}(3 \cdot 2^2) \frac{dx}{dt}$

$20 = \frac{1}{\sqrt{9}} \cdot 12 \frac{dx}{dt}$

$20 = 4 \frac{dx}{dt}$

$\boxed{5 = \frac{dx}{dt}}$