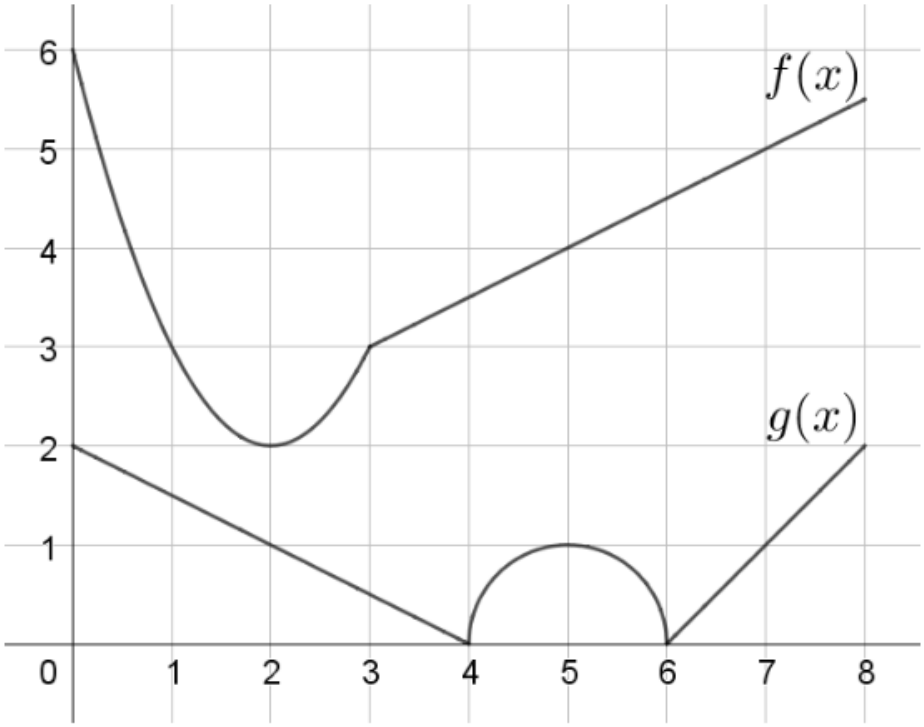
Question:(from HW12) The graphs of two functions $f(x)$ and $g(x)$ are shown below. If $h(x) = f(x)/g(x)$, then what is $h'(2)$?



Xingjian's solution:

Okay, by the quotient rule, we can get $h'(x)$ here:

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}.$$

From the graphs, we see $g(2) = 1$ in the $g(x)$ curve and $f(2) = 2$ in the $f(x)$ curve. Then we can substituting into $h'(2)$:

$$h'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{g(2)^2} = \frac{f'(2) - 2g'(2)}{1^2} = f'(2) - 2g'(2).$$

So, now we only need $f'(2)$ and $g'(2)$.

From the graph, $g(x)$ is linear in the interval $[0, 4]$. In addition, $g'(x)$ is the slope of the linear curve. Thus, we can get

$$g'(x) = \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \text{ for given points} (x_1, y_1) \text{ and } (x_2, y_2).$$

We can select $(0, 2)$ and $(2, 1)$ to calculate the slope, $g'(2)$ is supposed to be equal to the slope:

$$g'(2) = \frac{1 - 2}{2 - 0} = -\frac{1}{2}.$$

For $f'(2)$, we have learned that the tangent line is like a straight line that crosses the point. if we draw the tangent line for $(2, 2)$ here, we can actually see that it is assumed to be $y = 2$. (here $y = 2$ is the line with slope equal to 0.) Thus, $f'(2) = 0$.

Then

$$h'(2) = f'(2) - 2g'(2) = 0 - 2 \times (-\frac{1}{2}) = 1.$$

Thus, the final answer is $\boxed{h'(2) = 1}$.

Question:(from Xronos 12) Calculate the derivative of the following function:

$$f(x) = (x^2 + 3)(x^2 - 3)x^2$$

Xingjian's solution:

Let $g(x) = x^2 + 3$, $h(x) = x^2 - 3$ and $s(x) = x^2$. Also, let $G(x) = g(x)h(x)$. Thus, $f(x) = G(x)s(x)$.

First, apply the product rule to $f(x)$, then we can get:

$$f'(x) = G'(x)s(x) + G(x)s'(x).$$

Second, differentiate $G(x)$ again:

$$G'(x) = g'(x)h(x) + g(x)h'(x).$$

Thus, we can change to

$$f'(x) = (g'(x)h(x) + g(x)h'(x))s(x) + g(x)h(x)s'(x).$$

we also know $g'(x) = 2x$, $h'(x) = 2x$, and $s(x) = 2x$. Then we can get

$$f'(x) = (2x(x^2 - 3) + (x^2 + 3)2x)x^2 + (x^2 + 3)(x^2 - 3)2x.$$

Actually, That is enough.

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