1. Let $f(x) = x^{\ln(2^x)}$. Use the linear approximation of $f(x)$ at $x = 1$ to approximate the value of $(1.01)^{\ln(2^{(1.01)})}$.

   Xingjian Solution:
$$\ln f(x) = \ln\left(x^{x \ln 2}\right) = (\ln 2)\, x \ln x.$$
   Then
$$\frac{f'(x)}{f(x)} = (\ln(2))(1 \ln(x) + 1).$$
   Thus,
$$f'(x) = x^{\ln(2^x)}(\ln(2))(\ln(x) + 1)$$
   For the value at $x = 1.01$, a faithful first-order approximation should be taken at the nearby point $x = 1$ (where f is smooth):
$$f(1) = 1^{\ln 2} = 1, \qquad f'(x) = (\ln 2)(1 + \ln x)\, x^{\ln(2^x)} \;\Rightarrow\; f'(1) = \ln 2.$$
   Thus the linearization at $a = 1$ is
$$L_1(x) = f(1) + f'(1)(x - 1) = 1 + (\ln 2)(x - 1),$$
   and hence
$$(1.01)^{\ln(2^{1.01})} = f(1.01) \approx L_1(1.01) = 1 + (\ln 2)(0.01) \approx 1 + 0.006931 = \boxed{1.00693}.$$

2. Find the absolute maximum and minimum values of $f(x) = \arctan(x) - x$ on [-1,1] ($\pi \approx 3.14$).

Xingjian Solution: f is continuous on $[-1, 1]$ and differentiable on $(-1, 1)$. Critical points satisfy
$$f'(x) = \frac{1}{1 + x^2} - 1 = 0 \implies x^2 = 0 \implies x = 0.$$
Evaluate at the endpoints and the critical point:
$$f(-1) = \arctan(-1) - (-1) = -\frac{\pi}{4} + 1 = 1 - \frac{\pi}{4} \approx 1 - 0.785 = 0.215,$$
$$f(0) = \arctan(0) - 0 = 0,$$
$$f(1) = \arctan(1) - 1 = \frac{\pi}{4} - 1 \approx 0.785 - 1 = -0.215.$$
Therefore,
$$\text{Absolute maximum} = f(-1) = \boxed{1 - \tfrac{\pi}{4}} \quad (\approx 0.215),$$
$$\text{Absolute minimum} = f(1) = \boxed{\tfrac{\pi}{4} - 1} \quad (\approx -0.215).$$

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