Question (from Xronos 21). If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

Xingjian's solution. By the Mean Value Theorem, there exists $c \in (1, 4)$ such that

$$f(4) - f(1) = f'(c)\,(4 - 1) \geq 2 \cdot 3 = 6.$$

Hence $f(4) \geq 10 + 6 = 16$. The minimum possible value is $\boxed{16}$.

Question (from HW 23). Find the interval where the function $f(x) = x + \dfrac{4}{x}$ is both decreasing and concave up.

Xingjian's solution. Domain: $x \neq 0$.

$$f'(x) = 1 - \frac{4}{x^2}, \qquad f''(x) = \frac{8}{x^3}.$$

Decreasing $\iff f'(x) < 0 \iff 1 - \frac{4}{x^2} < 0 \iff |x| < 2$, so on $(-2, 0)$ and $(0, 2)$. Concave up $\iff f''(x) > 0 \iff x > 0$, so on $(0, \infty)$. Intersection $\Rightarrow$ the function is *both* decreasing and concave up on

$$\boxed{(0, 2)}.$$

Question (from Dr. Keeran review question 23). Determine the intervals on which $f(x) = x + \sqrt{1 - x}$ is concave up and concave down, and all inflection points.

Xingjian's solution. Domain: $x \leq 1$.

$$f'(x) = 1 - \frac{1}{2\sqrt{1 - x}}, \qquad f''(x) = -\frac{1}{4(1 - x)^{3/2}}.$$

For $x < 1$, $(1 - x)^{3/2} > 0$, hence $f''(x) < 0$. Therefore f is

$$\text{concave down on } (-\infty, 1), \qquad \text{concave up on } (1, \infty), \text{that is impossible for our domain of } f(x).$$

Since the concavity does not change sign on the interior of its domain, there are *no inflection points* (note that $x = 1$ is an endpoint, not an interior point).

Xingjian Xu
xingjianxu@ufl.edu
LIT 453
MAC 2311 Webpage: https://people.clas.ufl.edu/xingjianxu/MAC2311/
Office Hours: Tuesday, Periods 4 and 5 on LIT453;
Monday Period 5 on LIT 215.