Question: Find the graph of

$$f(x) = \frac{x}{|x|} + \frac{x-1}{|x-1|}.$$

my solution: Let us say:

$$g(x) = \frac{x}{|x|}$$

and

$$h(x) = \frac{x-1}{|x-1|}$$

. Then we know

$$g(x) = \begin{cases} \frac{x}{x}, & x > 0, \\ \frac{x}{-x}, & x < 0. \end{cases}$$

[why 0 disappear? since $|x|$ is in the denominator.]
similarly:

$$h(x) = \begin{cases} \frac{x-1}{x-1}, & x-1 > 0, \\ \frac{x-1}{-(x-1)}, & x-1 < 0. \end{cases}$$

Let us make it simpler:

$$h(x) = \begin{cases} 1, & x > 1, \\ -1, & x < 1. \end{cases}$$

Then we know $f(x) = g(x) + h(x)$, so let us consider the different domain one by one:
First case: when $x > 1$, $h(x) = 1$ and $g(x) = 1$, thus $f(x) = 2$.
Second case: when $0 < x < 1$, $h(x) = -1$ and $g(x) = 1$, thus $f(x) = 0$.
Third case: when $x < 0$, $h(x) = -1$ and $g(x) = -1$, thus $f(x) = -2$.
Then you may know the answer.
Question 11: Solve the inequality.

$$2x^{-\frac{1}{3}}(x-3)^{\frac{1}{3}} + x^{\frac{2}{3}}(x-3)^{-\frac{2}{3}} \geq 0.$$

my solution: Factorize this left hand side of this inequality:

$$x^{-\frac{1}{3}}(x-3)^{-\frac{2}{3}}(2(x-3)^{\frac{1}{3}+\frac{2}{3}} + x^{\frac{2}{3}+\frac{1}{3}}) \geq 0$$

Then simpify it, you will get:

$$x^{-\frac{1}{3}}(x-3)^{-\frac{2}{3}}(3x-6) \geq 0$$

Look at three critical points:

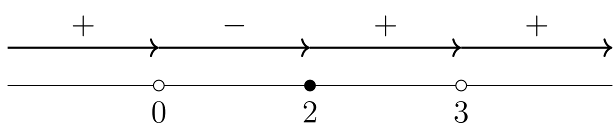
$$x^{-\frac{1}{3}} = \frac{1}{x^{1/3}}, \quad \text{thus } x \neq 0$$

$$(x-3)^{-\frac{2}{3}} = \frac{1}{(x-3)^{2/3}}, \quad \text{thus } x-3 \neq 0, \text{that implies } x \neq 3.$$

$$3x - 6 = 0 \text{ that implies } x = 2.$$

so you can draw the number line and the table then take a look at the value sign.

	$x=-1$	$x=1$	$x=\frac{5}{2}$	$x=4$
$3x-6$	-9 $(-)$	-3 $(-)$	$\frac{3}{2}$ $(+)$	6 $(+)$
$x^{-\frac{1}{3}}$	-1 $(-)$	1 $(+)$	$\left(\frac{5}{2}\right)^{-\frac{1}{3}}$ $(+)$	$4^{-\frac{1}{3}}$ $(+)$
$(x-3)^{-\frac{2}{3}}$	$\frac{1}{\sqrt[3]{4^2}}$ $(+)$	$\frac{1}{\sqrt[3]{2^2}}$ $(+)$	$\frac{1}{\sqrt[3]{\left(-\frac{1}{2}\right)^2}}$ $(+)$	1 $(+)$



Then choose for "+" sign for inequality greater and equal to 0, we will get the answer $(-\infty, 0) \cup [2, 3) \cup (3, \infty)$.

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