Question:(from HW16) Calculate y'' (the second derivative) given $x^4 + y^4 = a^4$ where a is a constant.

Xingjian's solution:

Differentiate both sides implicitly with respect to x:

$$\frac{d}{dx}(x^4 + y^4) = \frac{d}{dx}(a^4)$$

$$4x^3 + 4y^3\frac{dy}{dx} = 0$$

Then solve for $\frac{dy}{dx}$ (denoted as y'):

$$4y^3y' = -4x^3 \quad \Rightarrow \quad y' = -\frac{x^3}{y^3}$$

Now differentiate both sides again to find y'':

$$y'' = \frac{d}{dx}\left(-\frac{x^3}{y^3}\right)$$

Use the quotient rule:

$$y'' = -\frac{d}{dx}\left(\frac{x^3}{y^3}\right) = -\left(\frac{3x^2y^3 - x^3 \cdot 3y^2y'}{y^6}\right)$$

Substitute $y' = -\frac{x^3}{y^3}$ into the expression:

$$y'' = -\left(\frac{3x^2y^3 - x^3 \cdot 3y^2 \cdot \left(-\frac{x^3}{y^3}\right)}{y^6}\right) = -\left(\frac{3x^2y^3 + \frac{3x^6y^2}{y^3}}{y^6}\right)$$

Simplify:

$$y'' = -\left(\frac{3x^2y^3 + 3x^6/y}{y^6}\right) = -\left(\frac{3x^2}{y^3} + \frac{3x^6}{y^7}\right)$$

Then the final answer for this question is

$$\boxed{y'' = -\left(\frac{3x^2}{y^3} + \frac{3x^6}{y^7}\right).}$$

Question:(from Xronos 16) Compute the following derivative:

$$\frac{d}{dx}(-2\text{arcsec}(x+1)).$$

Xingjian's solution: Let

$$y = \text{arcsec}(x+1) \quad \Rightarrow \quad \sec(y) = x + 1$$

Differentiate both sides with respect to x:

$$\frac{d}{dx}[\sec(y)] = \frac{d}{dx}[x+1] \quad \Rightarrow \quad \sec(y)\tan(y)\cdot\frac{dy}{dx} = 1$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{1}{\sec(y)\tan(y)}$$

Now, express everything in terms of x. Since $\sec(y) = x+1$, use the identity $\tan^2(y) = \sec^2(y) - 1$:

$$\tan(y) = \sqrt{(x+1)^2 - 1} = \sqrt{x^2+2x}$$

Now plug back into the derivative:

$$\frac{dy}{dx} = \frac{1}{(x+1)\cdot\sqrt{x^2+2x}}$$

Now go back to the original expression:

$$\frac{d}{dx}\left(-2\text{arcsec}(x+1)\right) = -2\cdot\frac{dy}{dx} = -2\cdot\frac{1}{(x+1)\sqrt{x^2+2x}}$$

Thus final Answer is:

$$\boxed{\frac{d}{dx}\left(-2\text{arcsec}(x+1)\right) = -\frac{2}{(x+1)\sqrt{x^2+2x}}}$$

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