Calculus I: MAC2311
Fall 2025
Exam 1 A
09/15/2025
Time Limit: 100 Minutes

Name: Key

Section: ______________

UF-ID: ______________

Scantron Instruction: This exam uses a scantron. Follow the instructions listed on this page to fill out the scantron.

A. Sign your scantron **on the back** at the bottom in the white area.

B. Write **and code** in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UFID Number

3) 4-digit Section Number

C. Under *special codes*, code in the test numbers 1, 1:

• 2 3 4 5 6 7 8 9 0
• 2 3 4 5 6 7 8 9 0

D. At the top right of your scantron, fill in the *Test Form Code* as A .

• B C D E

E. This exam consists of 14 multiple choice questions and 5 free response questions. Make sure you check for errors in the number of questions your exam contains.

F. The time allowed is 100 minutes.

G. WHEN YOU ARE FINISHED:

1) Before turning in your test check for **transcribing errors.** Any mistakes you leave in are there to stay!

2) You must turn in your scantron and free response packet to your proctor. **Be prepared to show your proctor a valid GatorOne ID or other signed ID.**

It is your responsibility to ensure that your test has **19 questions**. If it does not, show it to your proctor immediately. You will not be permitted to make up any problems omitted from your test after the testing period ends. There are a total of 105 points available on this exam.

Part I Instructions: 14 multiple choice questions. Complete the scantron sheet provided with your information and fill in the appropriate spaces to answer your questions. Only the answer on the scantron sheet will be graded. Each problem is worth five (5) points for a total of 70 points on Part I.

1. Find the domain of

$$f(x) = \ln(4 - x^2)$$

(A) $(-2, 2)$ (B) $[-2, 2]$ (C) $(0, 2)$ (D) $(0, 2]$ (E) $(2, \infty)$

Require $4 - x^2 > 0$

$(2-x)(2+x) > 0$

Test intervals

$-3 \quad 0 \quad 3$

X -2 ✓ 2 X

$(-2, 2)$

2. Solve the nonlinear inequality

$$\frac{2x+1}{x-2} \le 3$$

(A) $(-2, 7)$

(B) $(-2, 2) \cup (7, \infty)$

(C) $(-2, 2) \cup [7, \infty)$

(D) $(-\infty, 2) \cup [7, \infty)$

Set right side to zero:

$$\frac{2x+1}{x-2} - \frac{3}{1} \le 0$$

$$\frac{2x+1}{x-2} - \frac{3(x-2)}{(x-2)} \le 0$$

$$\frac{2x+1-3x+6}{x-2} \le 0$$

$$\frac{7-x}{x-2} \le 0$$

Test intervals

$0 \quad 3 \quad 10$

✓ 2 X 7 ✓

$(-\infty, 2) \cup [7, \infty)$

3. Let $f(x) = \frac{x^3+5}{4}$ and $g(x) = \sqrt{x-2}$. Compute $\left(g \circ f^{-1}\right)(8)$.

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

Find $f^{-1}(x)$. Let $y = \frac{x^3+5}{4}$.

$x = \frac{y^3+5}{4}$

$4x - 5 = y^3$

$\sqrt[3]{4x-5} = y^{-1} = f^{-1}(x)$

So, $(g \circ f^{-1})(8)$

$= g(f^{-1}(8))$

$= g\left(\sqrt[3]{4(8)-5}\right)$

$= g\left(\sqrt[3]{27}\right)$

$= g(3) = \sqrt{3-2} = \boxed{1}$

4. Evaluate

$$\lim_{x \to \pi} \frac{\tan(x) + \cos(x)}{x^2}$$

(A) 1 (B) 0 (C) $\frac{1}{\pi^2}$ (D) $-\frac{1}{\pi^2}$ (E) Does not exist.

Plug in π:

$\lim_{x \to \pi} \frac{\tan x + \cos x}{x^2} = \frac{\tan \pi + \cos \pi}{\pi^2} = \frac{0-1}{\pi^2} = -\frac{1}{\pi^2}$

5. Which of the following is equal to $\lim_{x \to 0} \dfrac{\cos(x)}{x}$?

" $\dfrac{\cos 0}{0}$ " = " $\dfrac{1}{0}$ "

(A) 0 (B) 1 (C) $-\infty$ (D) ∞ (E) Does not exist.

Test both sides:

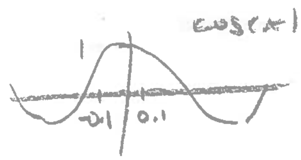
$$\lim_{x \to 0^+} \frac{\cos(x)}{x} \to \frac{(+)}{(+)} \to \infty$$

(Test $x = 0.1$)

The limits do not match.

$$\lim_{x \to 0^-} \frac{\cos(x)}{x} \to \frac{(+)}{(-)} \to -\infty$$

(Test $x = -0.1$)



6. Compute the average velocity of the position function $f(x) = 2x^2 - 4$ over the interval $[2, 4]$.

a b

(A) 12 (B) 6 (C) -18 (D) 24

Slope formula:

$$\frac{f(4) - f(2)}{4 - 2} = \frac{2(4)^2 - 4 - (2(2)^2 - 4)}{2}$$

$$= \frac{28 - (4)}{2} = \frac{24}{2} = 12$$

7. How many of the following functions are continuous for all real numbers?

Check if domain is $\mathbb{R}$.

✓ (i) $f(x) = e^{-x+1}$

✓ (ii) $g(x) = \ln(x^2 + 9)$ $x^2+9 \geq 9$ for all real numbers.

X (iii) $h(x) = \sin^{-1}(x)$ By definition, $x \in [-1, 1]$

X (iv) $k(x) = \frac{x^3+1}{x^2-x+6}$ $= \frac{x^3+1}{(x-3)(x+2)}$ $x \neq 3$, $x \neq -2$

(A) 0 (B) 1 (C) 2

(D) 3 (E) 4

8. Using the Intermediate Value Theorem and given the function $f(x) = 7(x^2+1)e^x - k$, which value of k will ensure that the function has a zero on the interval $[0, 4]$?

a b

(A) $k = -8$ (B) $k = 8$ (C) $k = 0$

(D) $k = -4$ (E) $k = -6$

Let $k = 8$. Then,

$f(x) = 7(x^2+1)e^x - 8$. (continuous function on $[0,4]$)

$f(0) = 7(1)e^0 - 8 = -1 < 0$

$f(4) = 7(4^2+1)e^4 - 8 = 7(17)e^4 - 8 > 0$.

Since the sign changed, $f(x)$ has a zero on $(0,4)$.

9. For the function

$$h(x) = \begin{cases} |x+1|, & x < 0 \\ 1, & x = 0 \\ \sin(\pi x), & 0 < x \le 4 \\ \frac{x^2-6x+9}{x^2-5x+6}, & x > 4 \end{cases}$$

$\frac{(x-3)\cancel{(x-3)}}{\cancel{(x-3)}(x-2)}$ $x=3, x=2$ not in $x>4$

which of the following are true?

✗ (i) The function has a vertical asymptote.

✗ (ii) The function has a removable discontinuity.

✓ (iii) The function has a jump discontinuity.

(*A*) Only (i) (*B*) Only (iii) (*C*) Only (ii) and (iii)

(*D*) Only (i) and (iii) (*E*) (i), (ii), and (iii)

$\lim_{x\to 0^-} h(x) = 1$

$\lim_{x\to 0^+} h(x) = 0$

jump discontinuity

$\lim_{x\to 4^-} h(x) = 0$

$\lim_{x\to 4^+} h(x) = \frac{1}{2}$

10. Evaluate the limit:

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+5x} - x\right)}{1}$$

Use conjugate

(*A*) 0 (*B*) $\frac{5}{2}$

(*C*) $\frac{5}{4}$ (*D*) ∞ (*E*) Does not exist.

$$\lim_{x\to\infty} \frac{\left(\sqrt{x^2+5x} - x\right)}{1} \frac{\left(\sqrt{x^2+5x} + x\right)}{\left(\sqrt{x^2+5x} + x\right)} = \lim_{x\to\infty} \frac{(x^2+5x) - x^2}{\left(\sqrt{x^2+5x} + x\right)}$$

$$= \lim_{x\to\infty} \frac{5x}{\sqrt{x^2+5x}+x} = \lim_{x\to\infty} \frac{5x}{\sqrt{x^2}+x} = \lim_{x\to\infty} \frac{5x}{x+x} = \lim_{x\to\infty} \frac{5x}{2x}$$

$$= \frac{5}{2}$$

11. Evaluate the limit:

$$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}.$$ conjugate.

(*A*) 0 (*B*) 2

(*C*) 4 (*D*) 8 (*E*) Does not exist.

$$\lim_{x\to 4} \frac{(x-4)}{(\sqrt{x}-2)} \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim_{x\to 4} \frac{(x-4)(\sqrt{x}+2)}{\cancel{x-4}}$$

$$\lim_{x\to 4} \sqrt{x}+2 = \sqrt{4}+2 = \boxed{4}$$

12. Let $g(x) = \dfrac{1}{x+1}$. Which of the following limits is equivalent to $g'(2)$ by the definition of the derivative?

(*A*) $\lim\limits_{h\to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h}$

(*B*) $\lim\limits_{h\to 0} \dfrac{\frac{1}{3+h} - \frac{1}{3}}{h}$

(*C*) $\lim\limits_{h\to 0} \dfrac{\frac{1}{h+1} - \frac{1}{2}}{h}$

(*D*) $\lim\limits_{h\to 0} \dfrac{\frac{1}{h+1} - \frac{1}{3}}{h}$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

$$g'(2) = \lim_{h\to 0} \frac{g(2+h)-g(2)}{h}$$

$$g'(2) = \lim_{h\to 0} \frac{\frac{1}{(2+h)+1} - \frac{1}{2+1}}{h}$$

$$g'(2) = \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

13. Consider the rational function given by

$$f(x) = \frac{(a-1)x^2 + 3x + 1}{ax^2 - x - 2}.$$

degree 2
degree 2

For which value of a does f have the horizontal asymptote $y = \frac{1}{2}$?

(A) $a = 2$ (B) $a = 1$ (C) $a = 0$

(D) $a = -2$ (E) $a = -1$

We require $\lim_{x\to\infty} f(x) = \frac{1}{2}$

Set ratio of leading coefficients equal to $\frac{1}{2}$:

$$\frac{a-1}{a} = \frac{1}{2} \iff 2(a-1) = a$$
$$2a - 2 = a$$
$$\boxed{a = 2}$$

14. Let

$$f(x) = \begin{cases} \dfrac{x^2-1}{x-1}, & x < 1, \\ 2x, & x \geq 1. \end{cases}$$

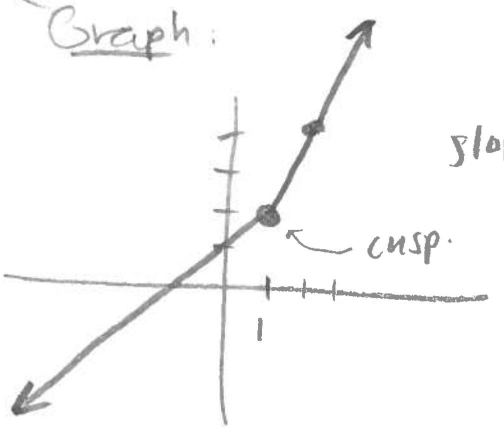
$$\frac{(x-1)(x+1)}{\cancel{x-1}} = x+1$$

(1, 2)

On which set of x-values is f differentiable?

(A) $(-\infty, 1) \cup (1, \infty)$ (B) $(-\infty, \infty)$ (C) At $x = 1$ only (D) $(-\infty, 1)$ only (E) $(1, \infty)$ only

Graph:



slope changes from 1 to 2 at $x = 1$.