1. UF wants to determine the number of fan cutouts they should offer for current football season. The cost per unit to produce $x$ hundred cutouts is $C(x) = x^3+3x^2-9x+2025$ dollars per cutout for $x > 0$. How many cutouts should they produce to minimize the cost?

   *Xingjian's Solution:*

   This question asks us to find the cutout that minimizes the cost. In mathematical terms, we are looking for the minimum value of $C(x)$. To find this minimum, we take the derivative of $C(x)$ and set it equal to zero to locate any critical points. Start by computing the derivative:
$$C'(x) = 3x^2 + 6x - 9.$$
   Then set $C'(x) = 0$, we can get:
$$3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x + 3)(x - 1) = 0.$$
   Basically, you will get two values for $x = -3$ and $x = 1$, However, since we are only considering positive values of $x$, we discard $x = -3$. Therefore, the value that minimizes the cost occurs at: $\boxed{100}$ cutouts.

2. Suppose $f(x) = \sqrt{e^{2x}}$, compute the value of $f'(0)$.

   *Xingjian's Solution:* We have two ways to solve this problem.

   **Method 1:**

   Since $f(x) = \sqrt{e^{2x}} = e^{(2x)^{1/2}} = e^x$, $f'(x) = e^x$. Thus, $\boxed{f'(0) = e^0 = 1}$. In this method, we avoid using the chain rule altogether. This is a clever trick that simplifies the computation.

   **Method 2:**

   We can use regular chain rult to solve it. Let $g(x) = e^{2x}$
$$f(x) = \sqrt{g(x)}$$
   By chain rule: we know
$$f'(x) = \frac{1}{2}\frac{1}{\sqrt{g(x)}}g'(x).$$
   Since $(g(x))' = (e^{2x})' = 2e^{2x}$. (You can use chain rule or product rule for I said before.) Then,
$$f'(x) = \frac{1}{2}\frac{1}{\sqrt{e^{2x}}}2e^{2x} = \sqrt{e^{2x}}.$$
   Thus, $\boxed{f'(0) = \sqrt{e^0} = \sqrt{1} = 1}$.

3. Suppose $f(x) = \sin(\tan(x))$, compute the value of $f'(0)$.

*Xingjian's Solution:* by using chain rule again. Let $u(x) = \tan(x)$

$$f'(x) = \big(\cos(u(x))\big)' u'(x)$$

Here $u'(x) = \sec^2(x)$, thus

$$f'(x) = \big(\cos(\tan(x))\big)\sec^2(x).$$

Thus we can directly plug in $x = 0$.

$$f'(0) = \cos(\tan(0))\sec^2(0) = \cos(0)\frac{1}{\cos^2(0)} = 1 \times 1 = 1.$$

Thus

$$\boxed{f'(0) = 1}.$$

*Xingjian Xu*
xingjianxu@ufl.edu
LIT 453
MAC 2311 Webpage: https://people.clas.ufl.edu/xingjianxu/MAC2311/
Office Hours: Tuesday, Periods 4 and 5 on LIT453;
Monday Period 4 on LIT 215.