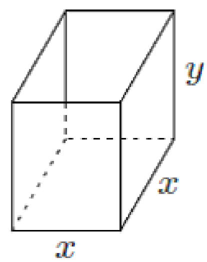
Question 6[from HW 3-4] An open top box with a square base has a surface area of 100 square inches. Express the volume of the box as a function of the length of the edge of the base. What is its domain?



my solution: Be careful—it is an *open-top box*. This means there is 1 bottom surface and 4 side surfaces. From the surface area condition, we have

$$100 = x^2 + 4xy.$$

The problem asks us to compute the volume.
We define the volume as

$$V = x^2 y.$$

To eliminate y, we solve from the surface area equation:

$$4xy = 100 - x^2 \quad \Rightarrow \quad xy = 25 - \tfrac{1}{4}x^2.$$

Thus,

$$V = x^2 y = x\left(25 - \tfrac{1}{4}x^2\right) = 25x - \tfrac{1}{4}x^3.$$

Domain:
Since $x, y > 0$ (box dimensions cannot be negative), and from the surface condition we also know $x^2 \leq 100$. Therefore,

$$\begin{cases} x^2 \leq 100, \\ x > 0, \end{cases}$$

which gives

$$0 < x < 10.$$

$$\boxed{V(x) = 25x - \tfrac{1}{4}x^3, \quad 0 < x < 10}$$

Question 1[from HW 3-4] Let $f(x) = \frac{x}{x+1}$. Find the inverse of $f(x)$ and determine the range of $f(x)$.

Xingjian comment: Be careful, I might ask similar question in quiz 2.

my solution:

let $f(x) = y = \frac{x}{x+1}$, then switch x, y position to compute the inverse function of $f(x)$. Thus we have

$$x = \frac{y}{y+1}.$$

Then

$$xy + x = y$$

Then by moving x to LHS and moving y to RHS, we get

$$(x-1)y = -x$$

$$y = \frac{-x}{x-1} = \boxed{\frac{x}{1-x}}.$$

And this is inverse function. As for range question, we know the range of f(x) is the domain of $f^{-1}(x)$. So look at $\frac{x}{1-x}$, I know the denominator can not be equal to 1. Thus range of f is $\boxed{(-\infty, 1) \cup (1, \infty)}$.

Question 3[from HW 3-4] Find the exact value of $\sin(\tan^{-1}(-2))$.

Xingjian comment: This is a hard and tricky question.

my solution: Let $\theta = \tan^{-1}(-2)$. Then $\tan\theta = -2$ and, since $\tan^{-1}$ returns values in $(-\frac{\pi}{2}, \frac{\pi}{2})$, we have $\theta < 0$ (quadrant IV), so $\sin\theta < 0$.

Use the identity

$$\sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}},$$

which follows from $\sec^2\theta = 1+\tan^2\theta$ and $\sin\theta = \tan\theta\cos\theta$ with $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{\sqrt{1+\tan^2\theta}}$ for $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Substitute $\tan\theta = -2$:

$$\sin(\tan^{-1}(-2)) = \frac{-2}{\sqrt{1+(-2)^2}} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.$$

$$\boxed{\sin(\tan^{-1}(-2)) = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.}$$

Question 2[from lecture quiz 5] Suppose that as an object falls from the top of a cliff, its position in feet above the ground after t seconds is given by $s(t) = 160 - 16t^2$. Find the average velocity from $t = 1$ to $t = 1 + h$ seconds, where $h \neq 0$.

Xingjian comment: Just a reminder that how to compute average velocity

my solution: Firstly, we need to know how to compute average velocity:

$$\text{average velocity} = \frac{\text{distance}}{\text{time range}}.$$

Here in this question distance is supposed to be difference between two different location, so distance $= s(1+h) - s(1)$ and time range is $1 + h - 1 = h$. Thus, we know:

$$\text{average velocity} = \frac{s(1+h)-s(1)}{h} = \frac{(160 - 16 \times (1+h)^2) - (160 - 16 \times 1^2)}{h}$$

$$= \frac{16 - 16(1+h)^2}{h} = \frac{16 - 16(1 + 2h + h^2)}{h} = \frac{-32h - 16h^2}{h} = -32 - 16h$$

So the answer is $\boxed{-32 - 16h}$.

Question 9:[from HW 5] What is

$$\lim_{x \to 1^+} \frac{-2e^{-x}}{(x-1)^3}$$

Xingjian comment: I know some of you might be scared of the limit computation. I may have an easiest way to do that. I will also take this question as an example.

my solution: I saw if $x \to 1$ from the right. and when equaling 1, the denominator will be undefined. So my plan is to plug in different values which are close to 1 from the right.

I will take 4,3,2,3/2,5/4 and make a table.

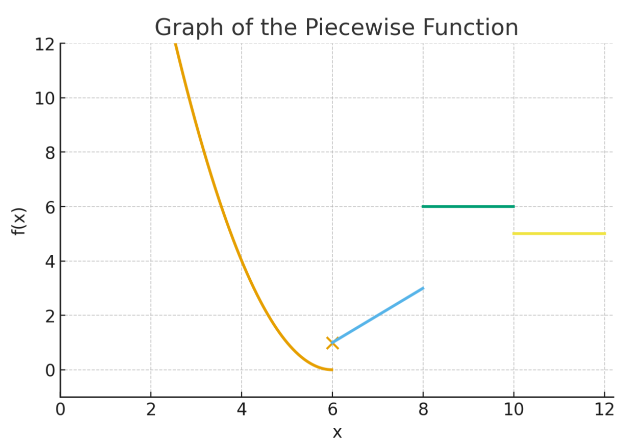
	$x = 4$	$x = 3$	$x = 2$	$x = \frac{3}{2}$	$x = \frac{5}{4}$
$\frac{-2e^{-x}}{(x-1)^3}$	$\frac{-2e^{-4}}{27}$	$-2e^{-3}/8$	$-2e^{-2}/1$	$-16e^{-3/2}$	$-2e^{-5/4} \times 64$

When x decreases from 4 to 5/4, the denominator $(x-1)^3$ becomes very small and positive, while the numerator is always negative. As a result, the value of $\frac{-2e^{-x}}{(x-1)^3}$ decreases without bound. Hence, the limit tends to $-\infty$. So the answer is $\boxed{-\infty}$.

Question 11:[from Xronos homework 5] Let $f(x) = \begin{cases} x^2 - 12x + 36 & x < 6; \\ x - 5 & 6 \le x < 8; \\ 6 & 8 < x < 10; \\ 5 & x > 10. \end{cases}$

evaluate $\lim_{x \to 5^+} f(x), \lim_{x \to 8^-} f(x), \lim_{x \to 8^+} f(x)$.

my solution: When you draw the graph for this function $f(x)$, you will get the graph like the following:



Now for $\lim_{x \to 5^+} f(x)$ should be fallen into the orange curve. You can directly plug in 5 then

$$\lim_{x \to 5^+} 5^2 - 12 \times 5 + 36 = 1$$

. For $x = 8$, we can see from the definition and graph. It is not defined. but we indeed have

$$\lim_{x \to 8^-} f(x) = \lim_{x \to 8^-} 8 - 5 = 3$$

and

$$\lim_{x \to 8^+} f(x) = \lim_{x \to 8^+} 6 = 6$$

The answer has already been performed.

Xingjian comment: But $\lim_{x \to 8^-} f(x) \neq \lim_{x \to 8^+} f(x), \lim_{x \to 8} f(x)$ does not exist.

.

Xingjian Xu
xingjianxu@ufl.edu
LIT 453
MAC 2311 Webpage: https://people.clas.ufl.edu/xingjianxu/MAC2311/
Office Hours: Tuesday, Periods 4 and 5 on LIT453;
Monday Period 4 on LIT 215.