MAC 2311 Quiz 5: Lectures 11-12

1. Calculate the derivative of the following function:

$$f(x) = (xe^x + 1)(x - 2)x$$

Xingjian Solution: $\boxed{\textit{Method 1}}$: Some of you choose to combine $(x-2)x$. That is a great idea here! So I will follow this to do it. So now $f(x) = (xe^x + 1)(x^2 - 2x)$.

Apply the product rule here, we can get:

$$f'(x) = (xe^x + 1)'(x^2 - 2x) + (xe^x + 1)(x^2 - 2x)'$$

Here,

$$(x^2 - 2x)' = 2x - 2.$$

But this following must be careful for xe^x, you are supposed to use product rule again!

$$(xe^x + 1)' = x(e^x)' + x'e^x + (1)' = xe^x + 1e^x + 0 = (xe^x + e^x).$$

So the answer is like the following:

$$\boxed{f'(x) = (xe^x + e^x)(x^2 - 2x) + (xe^x + 1)(2x - 2)}.$$

If you keep this, I will give you full points.

$\boxed{\textit{Method 2}}$: Let $g(x) = xe^x + 1$, $h(x) = (x - 2)$ and $s(x) = x$. Define $F(x) = f(x)h(x) = (xe^x + 1)(x - 2)$. Then do $F'(x)$ first, we can get:

$$F'(x) = (xe^x + 1)'(x - 2) + (xe^x + 1)(x - 2)'.$$

Here, $(xe^x + 1)' = (e^x + xe^x)$ and $(x - 2)' = 1$. Thus,

$$F'(x) = (e^x + xe^x)(x - 2) + (xe^x + 1) \times 1.$$

Then we do $f(x) = F(x)s(x)$.

$$f'(x) = F'(x)s(x) + F(x)s'(x) = ((e^x + xe^x)(x-2) + (xe^x + 1) \times 1)x + (xe^x + 1)(x-2) \times 1.$$

If you keep this, I will give you full points.

2. Find the equations of the tangent line to $f(x) = \frac{x}{\sqrt{x}+2}$ at the point where $x = 4$.

Xingjian Solution:
We need to compute $f'(x)$ first. Here please remember this

$$\boxed{(\sqrt{x})' = \frac{1}{2\sqrt{x}}}$$

Then we need to use quotient rult to compute here:

$$f'(x) = \frac{x'(\sqrt{x} + 2) - x(\sqrt{x} + 2)'}{(\sqrt{x} + 2)^2} = \frac{(\sqrt{x} + 2) - x\frac{1}{2\sqrt{x}}}{(\sqrt{x} + 2)^2} = \frac{\frac{1}{2}\sqrt{x} + 2}{(\sqrt{x} + 2)^2}$$

Then take $x = 4$ into this $f'(x)$, we can get:

$$f'(4) = \frac{\frac{1}{2} \times 2 + 2}{(2+2)^2} = \frac{3}{16}.$$

Also take $x = 4$ into $f(x)$, we can get: $f(4) = \frac{4}{2+2} = 1$.
Then the tangent line will be

$$\boxed{y - 1 = \frac{3}{16}(x - 4)}.$$

Xingjian Xu
xingjianxu@ufl.edu
LIT 453
MAC 2311 Webpage: https://people.clas.ufl.edu/xingjianxu/MAC2311/
Office Hours: Tuesday, Periods 4 and 5 on LIT453;
Monday Period 4 on LIT 215.