

TreeTracker Join: Turning the Tide When a Tuple Fails to Join

[Extended Technical Report]

Zeyuan Hu

University of Texas at Austin
Austin, TX, USA
zeyuan.zack.hu@gmail.com

Daniel P. Miranker

University of Texas at Austin
Austin, TX, USA
miranker@cs.utexas.edu

ABSTRACT

Many important query processing methods proactively use semi-joins or semijoin-like filters to delete dangling tuples, i.e., tuples that do not appear in the final query result. Semijoin methods can achieve formal optimality but have high upfront cost in practice. Filter methods reduce the cost but lose the optimality guarantee.

We propose a new join algorithm, TreeTracker Join (TTJ), that achieves the data complexity optimality for acyclic conjunctive queries (ACQs) without semijoins or semijoin-like filters. TTJ leverages *join failure* events, where a tuple from one of the relations of a binary join operator fails to match any tuples from the other relation. TTJ starts join evaluation immediately and when join fails, TTJ identifies the tuple as dangling and prevents it from further consideration in the execution of the query. The design of TTJ exploits the connection between query evaluation and constraint satisfaction problem (CSP) by treating a join tree of an ACQ as a constraint network and the query evaluation as a CSP search problem. TTJ is a direct extension of a CSP algorithm, TreeTracker, that embodies two search techniques *backjumping* and *no-good*. We establish that join tree and plan can be constructed from each other in order to incorporate the search techniques into physical operators in the iterator form. We compare TTJ with hash-join, a classic semijoin method: Yannakakis's algorithm, and two contemporary filter methods: Predicate Transfer and Lookahead Information Passing. Favorable empirical results are developed using standard query benchmarks: JOB, TPC-H, and SSB.

CCS CONCEPTS

- Information systems → Join algorithms.

KEYWORDS

optimal join algorithm, join operator, acyclic conjunctive queries, join ordering, sideway information passing

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1 INTRODUCTION

Removing *dangling tuples*, tuples that do not contribute to the final output of a query [27], has been central in improving both formal and practical join query execution speed [8, 12, 17, 22, 23, 26, 31, 34, 37, 44, 46, 50, 54, 56, 59, 69–71, 73]. However, a trade-off exists as the cost of dangling tuples removal may offset the join performance improvement. Yannakakis's algorithm (YA) is a representative of semijoin methods [12, 17, 59, 69–71] for acyclic conjunctive queries (ACQs) evaluation. YA executes a sequence of semijoins called full reducer (F_Q) as a preprocessing step and removes the dangling tuples from the input relations completely before join evaluation [12, 69]. As a result, YA provides optimal data complexity guarantee. However, in practice, using semijoins introduces high upfront costs [24, 30, 59, 64]. On the other hand, filter methods [22, 23, 26, 31, 33, 34, 37, 46, 50, 54, 56, 73] usually trade off optimal data complexity guarantee for reduction of dangling tuple removal cost by replacing semijoins with semijoin-like filter structures, e.g., Bloom filters [14] and removing dangling tuples by proactively checking base relations against filters. Efficient filter implementation allows these methods to work well in practice. Both semijoin and filter methods are *eager* approaches because they preemptively remove dangling tuples, aiming to prevent possible join failures (events where a tuple from one of the relations of a binary join operator fails to match any tuples from the other relation) from happening. Those methods rely on the efficient amortization of the upfront cost, incurred by dangling tuple removal, over the resulting join time reduction. If few dangling tuples exist, the upfront cost of the methods cannot be sufficiently amortized and the cost of dangling tuple reduction is more likely to outweigh its benefits. In an extreme case where no dangling tuples exist in the input relations, dangling tuple removal operations induce extra costs with no benefits. Common existing mitigations of this problem rely on heuristics such as disabling the filters based on selectivity estimation of the underlying relations [22, 24, 26, 56], which require workload-specific assessment on the trade-off between the execution cost and the potential speed improvement.

TreeTracker Join (TTJ) is the first join algorithm that leverages join failure events to remove dangling tuples with minimal overhead while maintaining the optimal data complexity for ACQs. TTJ is a *lazy* approach. The signature feature of TTJ is to start

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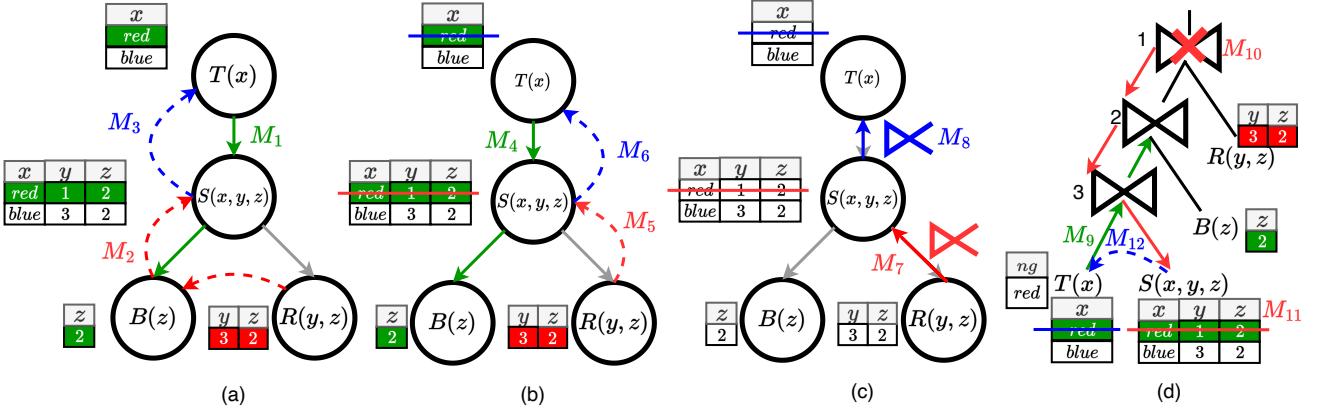


Figure 1: Illustration of the identification and removal of two dangling tuples by different algorithms: (a) join evaluation viewed as solving a CSP; (b) TTJ using CSP search techniques (backjumping and no-good) on the join tree \mathcal{T}_Q ; (c) Yannakakis's algorithm (YA); and (d) TTJ packed into physical operators on a left-deep query plan. We explain the details in Example 1. M_i are execution moments referenced throughout the paper.

join evaluation immediately without any preprocessing and perform two additional operations only on join failure: (1) identifying which tuple from which relation (*guilty relation*) causes a join failure at another relation (*detection relation*), and (2) subsequently removing the tuple from the guilty relation. The goal of TTJ is to remove a sufficient number of dangling tuples in the minimal amount of time to achieve a satisfactory level of join time reduction. Comparing with YA, TTJ does not aim to remove all dangling tuples, but the optimal guarantee still holds.

Fundamentally, TTJ exploits the equivalence between *constraint satisfaction problem* (CSP) and conjunctive query processing [15, 40] by treating query evaluation as a search problem. The intuition is that *join tree* \mathcal{T}_Q , the graph representation of ACQ, can be interpreted from CSP perspective as a *constraint network*. For example, consider a binary join between $A(x, y)$ and $B(y, z)$, which is acyclic and its \mathcal{T}_Q is $A - B$. Interpreting \mathcal{T}_Q as a constraint network, we view both A and B as variables. Tuples in each relation are possible *assignments* to each variable. Our goal is to find all possible assignments to A and B such that *constraint* $A.y = B.y$ is satisfied. \mathcal{T}_Q , when viewed as a constraint network, can be evaluated using search techniques such as *backjumping* and *no-good*, which are commonly-used in both database [5, 18, 35] and AI communities [9, 21, 28]. TTJ is a direct extension of a CSP algorithm, Tree-Tracker [9], that embodies the aforementioned two search techniques. We show \mathcal{T}_Q and query plan can be easily constructed from each other. Thus, the aforementioned search techniques can be integrated into a query plan. In this paper, we directly encode the two search techniques into physical operators in iterator interface [27], utilizing the form of *sideways information passing* (SIP). To help understand how TTJ works, we illustrate the CSP view of query evaluation, and the unique features of TTJ using Example 1.

EXAMPLE 1. Consider a join of 4 relations $T(x)$, $S(x, y, z)$, $B(z)$, and $R(y, z)$ with the database instance shown in Figure 1. All four plots show how the same two dangling tuples from the database instance are identified and removed by different algorithms.

(a) presents how evaluating a \mathcal{T}_Q can be viewed as solving a CSP by recursively assigning variables one by one until all variables are successfully assigned. The evaluation starts to assign T with a tuple from its instance $T(\text{red})$ and then moves on to S (moment M_1). Since $S(\text{red}, 1, 2)$ agrees with $T(\text{red})$ on attribute x , $S(\text{red}, 1, 2)$ can be assigned to S . This assignment is the same as obtaining a join result $(\text{red}, 1, 2)$ for $T \bowtie S$. The process continues to B and assigns B with $B(2)$. $R(3, 2)$ cannot be assigned to R given all the previous assignments because $(y, z) = (3, 2)$ in $R(3, 2)$ but $(y, z) = (1, 2)$ in $S(\text{red}, 1, 2)$. Since no other tuples from R can be assigned, the search process has to *backtrack* to B to try a different value given the existing assignments on T and S . Since no other tuples from B can be assigned, the search backtracks to S at M_2 . The same behavior repeats at S and the process further backtracks to T at M_3 . Then, T is assigned with the next tuple $T(\text{blue})$ and the process continues. When all variables are successfully assigned, we obtain one solution to the CSP by joining all the current assignments to the variables. The solution to the CSP is exactly a join result to the query. The search process for the next solution continues until all the solutions to the CSP are found.

(b) shows how TTJ improves the solving process in (a) with the two search techniques and removes two dangling tuples. The process (M_4) is identical to (a) until it fails to assign a tuple to R . Unlike (a) where the process backtracks to the previously assigned variable B , TTJ directly *backjumps* to S (M_5), the parent of R in \mathcal{T}_Q . Relations skipped due to backjumping are called *backjumped relations*, e.g., B . Once the search backjumps to S , the current assignment to S is marked as *no-good*, i.e., $S(\text{red}, 1, 2)$ is a dangling tuple. TTJ removes $S(\text{red}, 1, 2)$ from the instance of S and the removed tuple will not be considered again for future assignments. Since no other tuples from S can be assigned, backjump happens again (M_6) and $T(\text{red})$ is removed.

(c) highlights how YA removes the same dangling tuples as TTJ in a different way. YA executes the full reducer F_Q , a sequence of semijoins, before join starts: At M_7 , $S' = S \bowtie R$ and $S(\text{red}, 1, 2)$ is removed. Then, at M_8 , $T \bowtie S'$ and $T(\text{red})$ is removed. Unlike TTJ

that removes dangling tuples while performing join, YA removes all dangling tuples before join starts.

(d) illustrates the same join process as (b) on a left-deep query plan using demand-driven pipelining with operators implemented in iterator interface consisting of `open()` and `getNext()`. The evaluation starts with recursive `open()` calls on the join operators and builds hash tables on S , B , and R . To obtain the first query result, the join process first calls \bowtie_1 's `getNext()`, which calls its left child \bowtie_2 's `getNext()`, and such pattern repeats until the left most relation T 's `getNext()` is called and returns $T(red)$ (M_9). \bowtie_3 probes into \mathcal{H}_S , the hash table on S , and finds a matching tuple $S(red, 1, 2)$. The joined result $(red, 1, 2)$ is returned to \bowtie_2 . Then, the matching tuple $B(2)$ from \mathcal{H}_B joins with $(red, 1, 2)$ and the joined result $(red, 1, 2)$ is returned to \bowtie_1 . Probing into hash tables to find a matching tuple is the same as assigning a tuple to a variable in CSP. No tuples from \mathcal{H}_R join with $(red, 1, 2)$ (M_{10}); hence, join fails at R and R is the detection relation. Thus, TTJ performs backjumping making additional method calls to reset the evaluation flow to S , the guilty relation, because S is the parent of R in \mathcal{T}_Q . Subsequently, $S(red, 1, 2)$ is removed from \mathcal{H}_S (M_{11}), which is logically equivalent to removing the tuple from the instance of S . Since no tuples from S join with $T(red)$, TTJ backjumps to T and implicitly removes $T(red)$ by adding it to a no-good list ng (M_{12}). The no-good list will be used in future steps to filter out dangling tuples from T .

The rest of the paper fills the missing details from Example 1 such as how to construct \mathcal{T}_Q from a query plan (and vice versa), how TTJ packs backjumping and no-good techniques into a physical operator in the form of SIP, and formally show the correctness and optimality guarantee of TTJ. In summary, this paper makes the following contributions:

- (1) We use CSP search techniques to design a lazy join algorithm TTJ that removes dangling tuples if they cause join failures (§ 3).
- (2) We propose an algorithm to construct join tree from query plan, and vice versa (§ 3.1).
- (3) We formally show TTJ works correctly and runs optimally in data complexity for ACQ (§ 4).
- (4) We deduce a general condition called clean state that enables optimal evaluation of ACQ while permitting the existence of dangling tuples (§ 4).
- (5) We conduct extensive experiments to compare TTJ with four baseline algorithms on three benchmarks and perform detailed analysis to understand the features of TTJ (§ 5).

2 PRELIMINARIES

We review related background on acyclic conjunctive query evaluation, formulate the problem, introduce baseline algorithms, and summarize the notation used in this paper.

2.1 Acyclic Conjunctive Query Evaluation

We consider a relational database consisting of k relations under bag semantics. A *full conjunctive query* (CQ) is a natural join of k relations:

$$Q(\mathbf{a}) = R_1(\mathbf{a}_1) \bowtie R_2(\mathbf{a}_2) \bowtie \dots \bowtie R_k(\mathbf{a}_k) \quad (1)$$

For each relation $R_i(\mathbf{a}_i)$, \mathbf{a}_i is a tuple of variables called *attributes*. We define $attr(R_i) = \mathbf{a}_i$. Q is full because \mathbf{a} includes all the attributes appearing in the relations, i.e., $attr(Q) = \bigcup_{i=1}^k attr(R_i)$.

Query graph. The literature contains a number of different graph representations of Q . The most common choice is hypergraph [29, 47]. To better emphasize TTJ view of the connection between CSP and query evaluation, we use an equivalent [21] alternative, *query graph* [16] (also known as *join graph* [68]¹, *dual constraint graph* [21], or *complete intersection graph* [43]). The *query graph* of Q is a graph where there is a bijection between nodes in the graph and relations in the query. Two nodes v_1, v_2 are adjacent if their corresponding relations R_1, R_2 satisfy $attr(R_1) \cap attr(R_2) \neq \emptyset$. For clarity, we use the relations to label the nodes in the query graph.

Join Tree. Q is *acyclic* if its query graph contains a spanning tree called *join tree* \mathcal{T}_Q , which satisfies the *connectedness property* [10, 21]: for each pair of distinct nodes R_i, R_j in the tree and for every common attribute a between R_i and R_j , every relation on the path between R_i and R_j contains a . For the rest of the paper, we assume Q is a full acyclic CQ (ACQ). For ACQ, one can find a maximum-weight spanning tree from the query graph, where the weight of an edge (R_i, R_j) is $|attr(R_i) \cap attr(R_j)|$. Such tree is guaranteed to be a join tree [43]. A *rooted join tree* is a join tree converted into a directed tree with one of the nodes chosen to be the root. We assume \mathcal{T}_Q is a rooted join tree.

Query Plan. Physical evaluation of ACQ is commonly done using query plan. A *query plan* is a binary tree, where each internal node is a join operator \bowtie , and each leaf node is a scan operator (we use table scan by default) associated with one of the relations $R_i(\mathbf{a}_i)$ in Query (1). The plan is a *left-deep query plan*, or *left-deep plan*, if the right child of every join operator is a leaf node [52]. For example, $((T \bowtie S) \bowtie B) \bowtie R$ in Figure 2 (c) is a left-deep plan. In the paper, we focus on the left-deep plan and expand to the other plan shape in Appendix C. As a shorthand [66], we represent a left-deep plan, labeled from bottom to top, $(\dots ((R_k \bowtie R_{k-1}) \bowtie R_{k-2}) \dots) \bowtie R_1$ as $[R_k, R_{k-1}, \dots, R_1]$.

EXAMPLE 2. Consider an ACQ

$$Q(x, w, z) = T(x) \bowtie S(x, y, z) \bowtie B(z) \bowtie R(y, z) \quad (2)$$

Figure 2 illustrates query graph, join tree, and query plan of Q . \mathcal{T}_Q in (b) is obtained from the query graph in (a) by removing edge (B, R) . B and R satisfy the connectedness property because S , the only relation on the path between B and R , also shares their common attribute z . From CSP perspective, removing edge (B, R) from the query graph does not impact the query result because the constraint $B.z = R.z$ is enforced via an alternate path $B - S - R$, i.e., $B.z = S.z \wedge S.z = R.z$.

Complexity measurement. We assume a standard RAM complexity model [4]. Following the convention of research in the formal study of conjunctive query processing [3, 39, 61], we use data complexity (big- \mathcal{O} notation) as the measure of TTJ theoretical performance, which assumes that the size of a query, k , is a constant, but data size n varies [7]. We also determine TTJ performance in combined complexity [65] (big- O notation), which considers both k and n as variables. Under data complexity, the lower bound of any join algorithm is $\Omega(n+r)$ [61] (r is the output size) because the

¹Join graph is defined in CSP and database theory with a slightly different definition: a spanning subgraph of query graph that satisfies the connectedness property [21, 43].

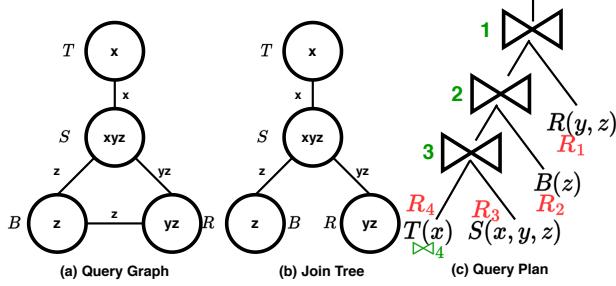


Figure 2: (a) query graph, (b) join tree , and (c) query plan of Q in Example 2. R_1, \dots, R_4 show the relation numbering and $\bowtie_1, \bowtie_2, \bowtie_3, \bowtie_4$ denote the join operator numbering. \bowtie_4 represents the table scan operator associated with the left-most relation R_4 , which is T in this example.

algorithm has to read input relations and produce join output. A join algorithm is *optimal* if its performance upper bound matches the aforementioned lower bound.

Physical Operators. Operators in the query plan of Q are physical operators, commonly implemented in an iterator interface [27] consisting of `open()`, `getNext()`, and `close()`. `open()` prepares resources (e.g., necessary data structures) for the computation of the operator; `getNext()` performs the computation and returns the next tuple in the result; and `close()` cleans up the used resources. In this paper, evaluation of a query plan is done using *demand-driven pipelining* (or *pipelining*): it first calls `open()` of each operator and then keeps calling `getNext()` of the root join operator of the plan, which further recursively calls `getNext()` of the rest of the operators, until no more tuples are returned [57].

2.2 Problem Definition

With the above background, we are ready to define the problem that TTJ tries to solve.

Problem. Given an ACQ Q , we want to evaluate a left-deep query plan of Q consisting of physical join operators implemented in iterator interface using demand-drive pipelining with formal optimality guarantee and practical efficiency.

2.3 Baselines

We compare TTJ with in-memory hash-join (HJ), one classic semi-join method: Yannakakis’s algorithm (YA), and two representative filter methods: Lookahead Information Passing (LIP) and Predicate Transfer (PT).

HJ evaluates Q using pipelining on a left-deep plan with in-memory hash-join operators [31]. In `open()`, each hash-join operator builds a hash table \mathcal{H} from its right child R_{inner} . In `getNext()`, a tuple t from the left child of the join operator, R_{outer} , probes into \mathcal{H} to find a set of joinable tuples denoted as *Matching-Tuples*. `getNext()` returns the join between t and the first tuple from *MatchingTuples*. The join between t and the rest of the tuples will be returned in the subsequent `getNext()` calls.

YA [69] is an optimal join algorithm for ACQ. The algorithm consists of two phases: a *full reducer phase* and a *join phase*. In the full reducer phase, YA makes two passes over \mathcal{T}_Q . The first pass, called *reducing semijoin program* [12] HF_Q , traverses the join tree bottom-up and applies $R_p \bowtie R_c$ where R_p is a parent relation and

R_c is one of its children. The possibly reduced R_p further semijoins with its other children. The resulting relations after HF_Q are denoted as R'_i . For example, in Figure 1 (c), two semijoins $S' = S \bowtie R$ and $T' = T \bowtie S'$ are part of the bottom-up pass. In the second pass, the algorithm traverses \mathcal{T}_Q top-down applying $R'_i \bowtie R'_p$ ². The fully reduced relations are denoted as R_i^* for $i \in [k]$ ³ and they are free of dangling tuples. In the join phase, YA makes the third pass of \mathcal{T}_Q to produces the join output by again traversing \mathcal{T}_Q bottom-up and performing pairwise joins.

LIP [26, 72, 73] leverages a set of Bloom filters to evaluate star schema queries consisting of a fact table and dimension tables. In `open()`, LIP computes filters from R_{inner} of each join operator and passes those filters downwards along the left-deep plan to the fact table, which is the left-most relation of the plan. In `getNext()` of the left-most table scan operator, LIP checks the tuples from the fact table against the filters and propagates those pass the check upwards along the plan.

PT [68] is the state-of-the-art filter method that generalizes the idea of LIP to queries not limited to star schema queries. Similar to YA, PT divides query evaluation into two phases. First, in predicate transfer phase, PT passes filters over the predicate transfer graph, a directed acyclic graph built from the query graph, of a query in two directions: forward and backward, which is similar to the first two passes over \mathcal{T}_Q in YA. Relations are gradually reduced as filters are being passed. Once the predicate transfer phase is done, the join phase begins where the reduced relations are joined.

2.4 Notation

We summarize the notation used in the paper in Table 1. We omit standard relational algebra notation in the table, e.g., antijoin \bowtie and semijoin \bowtie . We further define some terminologies used throughout the paper. We call a relation *internal* if it appears as an internal node [20, 53] in \mathcal{T}_Q . For relations corresponding to non-root internal nodes of \mathcal{T}_Q , we call them *internal^o relations*. Similarly, a *leaf relation* means the relation appears as a leaf node in \mathcal{T}_Q . The *root relation* is defined accordingly. Depending on context, we adapt the following language: If a tuple produced from \bowtie_{i+1} , the R_{outer} of \bowtie_i , cannot join with any tuples from R_i , the R_{inner} of \bowtie_i (*dead-end* in CSP [21]), we call it a *join fails at \bowtie_i* , *join failure happens at \bowtie_i* , or *join fails at R_i* . In such case, R_i is called the *detection relation* (*dead-end variable* in CSP [21]). \bowtie_i is called the *detection operator*. We call the join operator the *removal operator* if its R_{inner} is the parent of the detection relation for a join failure in \mathcal{T}_Q . Such R_{inner} is the *guilty relation* (*culprit variable* in CSP [21]). For example, for the join failure happens at \bowtie_1 in Figure 1 (d), the detection relation is R and the detection operator is \bowtie_1 . S is the guilty relation and \bowtie_3 is the removal operator.

3 TREETRACKER JOIN OPERATORS

Algorithms 3.1 and 3.2 show the formal definition of TTJ. Algorithm 3.1 defines each join operator in a left-deep plan. Algorithm 3.2 defines TTJ *scan*, which replaces the normal left-most table scan operator; the rest of the table scan operators in the plan remains unchanged. We use \mathcal{P}_Q to denote the left-deep plan using TTJ. We

² $R_c \bowtie R'_p$ if R_c is a leaf node because leaf nodes are not reduced in the first pass.

³ $[k]$ is a shorthand for $1, \dots, k$

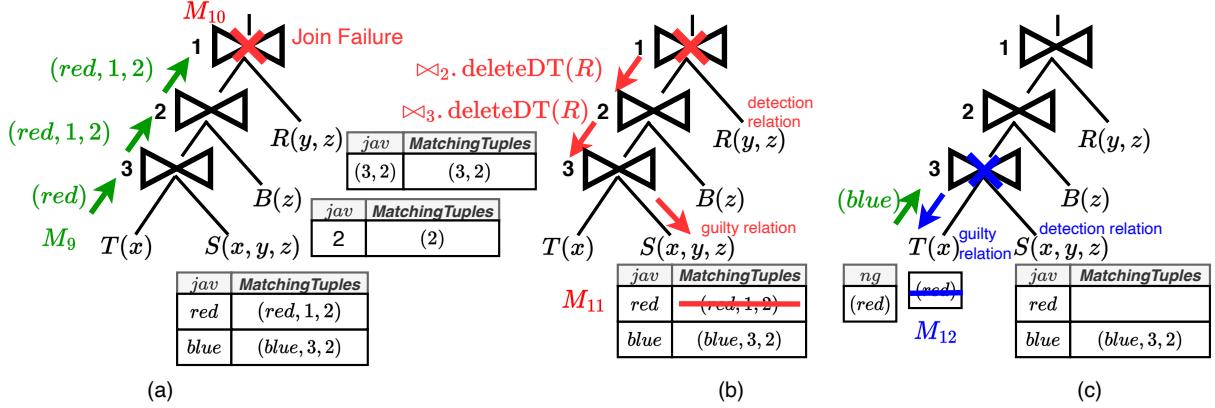


Figure 3: (a) Join fails at \bowtie_1 . (b) A series of $\text{deleteDT}(R)$ is called, which leads to the removal of $S(red, 1, 2)$ from hash table \mathcal{H}_S . (c) Join further fails at \bowtie_3 , which puts $T(red)$ to ng .

Table 1: Summary of common notation

Notation	Definition
Q	a full acyclic CQ
k	number of relations in Q
n	maximum size of the input relations in Q
r	query output size
\mathcal{T}_Q	rooted join tree. See Figure 2 (b).
\mathcal{P}_Q	a left-deep query plan using TTTJ (§ 3)
R_i for $i \in [k]$	relations in \mathcal{P}_Q . Left-most relation is R_k . See Figure 2 (c).
\bowtie_i for $i \in [k]$	join operators in \mathcal{P}_Q . \bowtie_1 is the root operator. \bowtie_k is the table scan operator of R_k . See Figure 2 (c).
$[R_k, R_{k-1}, \dots, R_1]$	a query plan $\dots ((R_k \bowtie R_{k-1}) \bowtie R_{k-2}) \dots \bowtie R_1$
J_u for $u \in [k]$	join result computed from \bowtie_u . Once correctness of TTTJ is proved, $J_u = J_u^*$
J_u^*	join of relations R_k, R_{k-1}, \dots, R_u
j_u	\bowtie_u 's result size, i.e., $ J_u $
R^*	R that is free of dangling tuples w.r.t. Q
$attr(R)$	a function that extracts attributes from R (or from each relation in a set of relations and returns union of the extracted attributes)
$t[a]$	$t[a] = \pi_a(t)$ for tuple t , attribute a , and projection π
$ja(R, S)$	$attr(R) \cap attr(S)$
$R(3, 2)$	tuple $(3, 2) \in R$
$jav(t, R, S)$	join-attribute value $t[attr(R) \cap attr(S)]$
R_{inner}	right child of \bowtie_i
$Router$	left child of \bowtie_i
\mathcal{H}_R (or \mathcal{H}_i)	hash table built from R (or associated with \bowtie_i)
$MatchingTuples$	the list of tuples with the same jav in a hash table
ng	no-good list, a filter in TTTJ scan
\mathbb{R}	physical aspects of R , i.e., a bag of tuples R contains

are now ready to work out Example 1 in full details to highlight the

salient features of TTTJ mentioned in § 1. We expand Figure 1 (d) into Figure 3. All line numbers reference Algorithm 3.1 by default unless noted otherwise.

The following three examples show the execution moments in the first `getNext()` call after `open()` of the pipelining evaluation that leads to the removal of two dangling tuples. Example 3 shows that TTTJ does not schedule any semijoins or semijoin-like filters before query evaluation. The evaluation flow is identical to HJ when no join failure happens.

EXAMPLE 3 (M₁₀ in Figures 1 and 3). After plan evaluation begins, the recursive `getNext()` calls start with \bowtie_1 and end with T 's TTTJ scan operator (Line 4 Algorithm 3.2), which returns $T(red)$. The $jav(x : red)$ is used to look up \mathcal{H}_S (Line 15). Since $T(red)$ joins with $S(red, 1, 2)$, the resulting tuple $(red, 1, 2)$ is further propagated to \bowtie_2 , which probes into \mathcal{H}_B and finds $B(2)$ joinable. The join result $(red, 1, 2)$ is further passed to \bowtie_3 .

Example 4 shows how the backjumping idea from CSP (specifically, *graph-based backjumping* [21]) shown in Example 1 is integrated into physical operators in \mathcal{P}_Q . To do so, we enhance the iterator interface with one more method `deleteDT()` and implements backjumping as a series of `deleteDT()` calls⁴ from the detection operator to the removal operator corresponding to a join failure. `deleteDT()`, under the form of SIP, sends the reference of the detection relation from the detection operator to the removal operator in a fashion that is not explicitly indicated by the plan.

EXAMPLE 4 (M₁₀ and M₁₁ in Figures 1 and 3). Since $(red, 1, 2)$ cannot join with any tuples from \mathcal{H}_R , the goal of TTTJ is to backjump to the guilty relation S and remove the last returned tuple, $S(red, 1, 2)$, from \mathcal{H}_S . To do so, $\bowtie_2, \text{deleteDT}(R)$ is called from Line 20 first. Since \bowtie_2 's R_{inner} , B , is not the parent of R in \mathcal{T}_Q (Line 23), Line 27 is called, e.g., $\bowtie_3, \text{deleteDT}(R)$. In \bowtie_3 's `deleteDT()`, since S is the parent of R (Line 23), Line 24 is executed: $S(red, 1, 2)$ is removed from \mathcal{H}_S .

Example 4 shows that removing tuples from internal⁵ relations is implemented as removing the tuples from their index representations. Example 5 illustrates another CSP technique, *no-good list*

⁴We omit argument to `deleteDT()` when reference it generically.

⁵No tuples are removed from the leaf relations because they cannot be guilty relations, i.e., by leaf definition, they are not parent of any relations in \mathcal{T}_Q .

Algorithm 3.1: TTJ Join Operator

Purpose: An iterator returns, one at a time, the join result of R_{outer} and R_{inner} .

Output: A tuple $t \in R_{outer} \bowtie R_{inner}$

```

1 TTJOperator
2   void open()
3     // router references a tuple from Router
4     // MatchingTuples references a set of tuples from
5     // Rinner that are joinable with router
6     Initialize router, MatchingTuples to nil
7     Rinner.open()
8     Build hash table H: Insert each tuple, rinner, from
9     Rinner into H using the join attribute value(s),
10    jav(rinner, Router, Rinner) as the key
11    Router.open()
12
13   Tuple getNext()
14     if MatchingTuples ≠ nil ∧ MatchingTuples ≠ ∅ then
15       // If there are more matching tuples left, return
16       // the join of router and the next matching tuple
17       if (aMatchingTuple ← MatchingTuples.next())
18         ≠ nil then
19         return the join of router and
20         aMatchingTuple
21       // No matching tuples are left. Get a new router
22       router ← Router.getNext()
23       if router = nil then return nil
24
25     if router = nil then router ← Router.getNext()
26     while router ≠ nil do
27       // Find tuples from Rinner joinable with router
28       MatchingTuples ←
29       H.get(jav(router, Router, Rinner))
30       if MatchingTuples ≠ nil then
31         aMatchingTuple ← MatchingTuples.next()
32         return the join of router and
33         aMatchingTuple
34       else
35         // Join failure identified; start the
36         // backjumping to the guilty relation, parent
37         // of Rinner in TQ
38         router ← Router.deleteDT(Rinner)
39
40   return nil
41
42 Tuple deleteDT(Detection Relation R)
43   if Rinner is the parent of R in TQ then
44     // Rinner is the guilty relation; join failure was
45     // identified at R because the join between router
46     // and aMatchingTuple was eventually returned to
47     // R and cannot join with any tuples from R
48     Remove aMatchingTuple from MatchingTuples
49     and H
50
51   else
52     // Has not reached the guilty relation for R;
53     // backjumping continues
54     MatchingTuples ← nil
55     router ← Router.deleteDT(R)
56     if router = nil then return nil
57
58   return getNext()

```

Algorithm 3.2: TTJ Table Scan Operator for R_k

Purpose: Table scan operator for R_k that returns tuples not in ng .

```

1 TTJScan
2   void open()
3     Initialize ng to an empty set
4
5   Tuple getNext()
6     while (t ← Rk.next()) ≠ nil do
7       if jav(t, Rk, Ri) ∉ ng for all children Ri of Rk in
8       TQ then
9         return t
10
11   return nil
12
13 Tuple deleteDT(Detection Relation R)
14   // Rk is the guilty relation; t contributes to the
15   // tuple that caused the join failure at R
16   Insert jav(t, Rk, R) into ng
17
18   return getNext()

```

(ng), that TTJ incorporates to filter out dangling tuples from the left-most relation R_k .

EXAMPLE 5 ([M12](#) in Figures 1 and 3). Removal of $S(red, 1, 2)$ causes $T(red)$ to become dangling. TTJ adds it to ng , effectively removing it from T . After removing $S(red, 1, 2)$, $getNext()$ of \bowtie_3 is called (Line 29). Since $MatchingTuples$ is now empty and $r_{outer} = T(red)$, Line 15 is executed. No tuples from S joins with $T(red)$. Thus, $T.deleteDT(S)$ is called (Line 20) and Algorithm 3.2 Line 10 adds $jav(x : red)$ to ng . Once ng is non-empty, it will work like a filter to prevent future dangling tuples with the same jav from returning to $\bowtie_3.getNext()$ of T is called (Algorithm 3.2 Line 11). The next tuple $T(blue)$ then probes into ng (Algorithm 3.2 Line 6). Since T has only one child S , $jav(x : blue)$ is computed and it is not in ng . Thus $T(blue)$ is safe to further propagate upwards towards \bowtie_3 .

3.1 Construction of Query Plan or Join Tree

TTJ operates on a left-deep query plan, which represents the join order of the input relations of the query. In addition, TTJ requires a T_Q to find the parent of the detection relation, i.e., the guilty relation, for a join failure. Thus, if either the plan or the T_Q is missing, we need to construct it from the other one. A constraint exists for such construction to ensure TTJ can function correctly. Since $deleteDT()$ always sends a reference of the detection relation downwards along the plan, when the plan is missing, we need to construct a plan such that the guilty relation must sit below the detection relation. For the same reason, when T_Q is missing, we need to construct a T_Q such that for any detection relation in a plan, exactly one of the relations below it must be its parent in the tree. In this section we formalize the constraint and describe how to properly construct a T_Q or a plan given the other input.

Given a left-deep query plan, Definition 1 defines the aforementioned constraint on the T_Q .

Definition 1 (join tree assumption). Suppose $\mathcal{P}_Q = [R_k, R_{k-1}, \dots, R_1]$. TTJ assumes T_Q satisfies the following property: for a given relation R_i in \mathcal{P}_Q , its parent in T_Q is one of the relations $R_k, R_{k-1}, \dots, R_{i+1}$. The root of T_Q is the left-most relation R_k .

EXAMPLE 6. Consider \mathcal{P}_Q in Figure 2 (c), B is labeled as R_2 . TTJ expects that B 's parent in \mathcal{T}_Q has to be either R_3 or R_4 . As shown in Figure 2 (b), B 's parent is S , which corresponds to R_3 . Thus, \mathcal{T}_Q in (b) satisfies the assumption.

The next lemma states that we can easily construct a required \mathcal{T}_Q from any left-deep query plan that does not have cross-product.

LEMMA 3.1. *For any left-deep plan without cross-product for acyclic queries, there exists a \mathcal{T}_Q satisfies the join tree assumption (Definition 1).*

We defer the construction step and proof to Appendix A. The key idea is as follows: We construct \mathcal{T}_Q following the order of relations in \mathcal{P}_Q from left to right. Suppose R_k, \dots, R_{j+1} are already added to \mathcal{T}_Q . For R_j , we want to find a relation R_i that is already in \mathcal{T}_Q such that $\text{attr}(R_j) \cap (\bigcup_{u=j+1}^k \text{attr}(R_u)) \subseteq \text{attr}(R_i)$. Left-deep query plan without cross-product for acyclic queries guarantees such R_i exists. We add R_j in \mathcal{T}_Q through an edge (R_i, R_j) .

EXAMPLE 7. Suppose $\mathcal{P}_Q = [R_3(x, y), R_2(x, y, z), R_1(y, z)]$. The left-most relation $R_3(x, y)$ has to be the root of \mathcal{T}_Q . For the next relation $R_2(x, y, z)$, since only R_3 is in \mathcal{T}_Q and $\text{attr}(R_2) \cap \text{attr}(R_3) \subseteq \text{attr}(R_3)$, we add edge (R_3, R_2) . Now, both R_3 and R_2 are in \mathcal{T}_Q and union of their attributes is $\{x, y, z\}$. Since $\text{attr}(R_1) \cap \{x, y, z\} \subseteq \text{attr}(R_2)$, we add edge (R_2, R_1) . The final \mathcal{T}_Q is $R_3 \rightarrow R_2 \rightarrow R_1$.

EXAMPLE 8. Consider a cyclic query, $\mathcal{P}_Q = [R_3(a, b), R_2(b, c), R_1(c, a)]$, the classic triangle query. Let us try to construct \mathcal{T}_Q . $R_3(a, b)$ is the root. $R_2(b, c)$ connects R_3 . $\text{attr}(R_3) \cup \text{attr}(R_2) = \{a, b, c\}$. But, $\text{attr}(R_1) \cap \{a, b, c\} \not\subseteq \text{attr}(R_2)$ and $\text{attr}(R_1) \cap \{a, b, c\} \not\subseteq \text{attr}(R_3)$. R_1 cannot be placed in \mathcal{T}_Q to satisfy the connectedness property while keeping \mathcal{T}_Q being a tree.

EXAMPLE 9. $\mathcal{P}_Q = [T(x), R(y, z), B(z), S(x, y, z)]$ contains a cross-product due to $T(x), R(y, z)$. We cannot construct \mathcal{T}_Q because \mathcal{T}_Q is a subgraph of the query graph and the query graph does not contain (T, R) edge.

Definition 1 can be interpreted as a join order assumption, which defines the constraint on the plan.

COROLLARY 3.2 (JOIN ORDER VIEW OF DEFINITION 1). *Given a \mathcal{T}_Q , TTJ assumes the order of relations in a left-deep query plan satisfies the following property: for a node R_i and its child R_j in \mathcal{T}_Q , R_i is before R_j in \mathcal{P}_Q , i.e., $\mathcal{P}_Q = [\dots, R_i, \dots, R_j, \dots]$.*

Construction of \mathcal{P}_Q is straightforward: performing a top-down pass (not necessarily from left to right) of \mathcal{T}_Q .

EXAMPLE 10. For \mathcal{T}_Q in Figure 2 (b) with T as the root, both $\mathcal{P}_Q^1 = [T, S, B, R]$ and $\mathcal{P}_Q^2 = [T, S, R, B]$ are valid plans for TTJ.

3.2 Additional Practical Considerations

To use TTJ in production environment, additional considerations are required beyond the algorithm itself. We further discuss (1) TTJ cost modeling to determine both \mathcal{T}_Q and \mathcal{P}_Q (Appendix B); (2) using TTJ with bushy plan, including the construction of a bushy plan from a \mathcal{T}_Q and a formal analysis of TTJ performance (Appendix C); and (3) using TTJ for cyclic queries with a formal runtime analysis (Appendix D).

4 CORRECTNESS AND OPTIMALITY OF TTJ

We prove the correctness and the optimality guarantee of TTJ in this section. Due to the space limit, we present the correctness theorem without the proof and focus on the proof of optimality. The omitted lemmas and proofs are in Appendices E to G.

THEOREM 4.1 (CORRECTNESS OF TTJ). *Evaluating an ACQ of k relations using \mathcal{P}_Q , which consists of $k - 1$ instances of Algorithm 3.1 as the join operators and 1 instance of TTJ scan (Algorithm 3.2) for the left-most relation R_k , computes the correct query result.*

The runtime analysis of evaluating \mathcal{P}_Q is done in two steps. First, we propose a general condition for any left-deep plan without cross-product for ACQ called *clean state*. Clean state specifies what tuples can be left in the input relations without breaching the $\mathcal{O}(n+r)$ evaluation time guarantee. In contrast to the common belief that input relations have to be free of dangling tuples to enable $\mathcal{O}(n+r)$ evaluation, clean state permits the existence of dangling tuples. Clean state provides a formal explanation on one reason why YA may have large dangling tuple removal costs – it spends efforts to remove more than necessary tuples. Second, we show \mathcal{P}_Q reaches the clean state and the work done by TTJ between the beginning of the query evaluation and reaching the clean state (*cleaning cost*) is no more than the work done after reaching the clean state. The former takes $\mathcal{O}(n)$ and the latter takes $\mathcal{O}(n+r)$.

Definition 2 (clean state). For a left-deep plan without cross-product for ACQ, we denote the contents of R_i that satisfy the following conditions by $\bar{\mathbb{R}}_i$:

- (i) $\bar{\mathbb{R}}_i = \mathbb{R}_i$ for all the leaf relations R_i of \mathcal{T}_Q ;
- (ii) $(\mathbb{R}_i \bowtie J_{i+1}^*) \overleftarrow{\bowtie} \bar{\mathbb{R}}_u = \emptyset$ for internal $^\circ$ relations R_i and their child relations R_u ; and
- (iii) $\bar{\mathbb{R}}_k \overleftarrow{\bowtie} \bar{\mathbb{R}}_u = \emptyset$ for the root of \mathcal{T}_Q , R_k and its children R_u .

The plan reaches *clean state* if the contents of all R_i equal $\bar{\mathbb{R}}_i$.

LEMMA 4.2. *When the left-deep plan without cross-product for ACQ is in clean state, R_k is fully reduced and free of dangling tuples.*

THEOREM 4.3 (CLEAN STATE IMPLIES OPTIMAL EVALUATION). *Once the left-deep plan without cross-product is in clean state, any intermediate results generated from the plan evaluation will contribute to the final join result and the plan can be evaluated optimally.*

Comparison with full reducer and reducing semijoin program. Relations that are free from dangling tuples are in clean state. Thus, relations after F_Q are in clean state. Relations after HF_Q are in clean state as well. Leaf relations after HF_Q satisfy Condition (i) (by definition of HF_Q) and the root relation after HF_Q satisfies Condition (iii) (by Lemma 4.2 and Lemma 4 of [12]). For an internal $^\circ$ relation R_i , it satisfies $\mathbb{R}_i \overleftarrow{\bowtie} \bar{\mathbb{R}}_u = \emptyset$, which implies the satisfaction of Condition (ii). However, the state of relations after HF_Q or F_Q is stricter than what is required by clean state, i.e., more than necessary tuples are removed for optimal evaluation. Tuples of R_i that are not joinable with J_{i+1}^* will be removed by both F_Q and HF_Q if such tuples are not joinable with tuples from any child relation of R_i . But, those dangling tuples are allowed to present in clean state.

EXAMPLE 11. Consider a \mathcal{T}_Q $R_3(x) \rightarrow R_2(x, y) \rightarrow R_1(y)$ with the following database instance: $R_3(4)$, $R_2(4, 6)$, $R_2(3, 5)$, $R_2(3, 7)$,

$R_2(4, 7)$, and $R_1(7)$. Clean state only requires the removal of one tuple $R_2(4, 6)$. HF_Q removes two tuples $R_2(4, 6)$ and $R_2(3, 5)$. F_Q removes three tuples: $R_2(4, 6)$, $R_2(3, 5)$, and $R_2(3, 7)$.

LEMMA 4.4. *When TTJ finishes execution, \mathcal{P}_Q is in clean state.*

LEMMA 4.5. *TTJ evaluates \mathcal{P}_Q in $\mathcal{O}(n+r)$ once it is in clean state.*

Next, we prove the optimality guarantee of TTJ by bounding the cleaning cost. The key idea is to leverage the fact that whenever a dangling tuple is detected, some tuple has to be removed and there can be at most kn tuples removed. The cost to remove each tuple is $\mathcal{O}(1)$ under data complexity.

THEOREM 4.6 (**DATA COMPLEXITY OPTIMALITY OF TTJ**). *Evaluating an ACQ of k relations using \mathcal{P}_Q , which consists of $k-1$ instances of Algorithm 3.1 as the join operators and 1 instance of TTJ scan (Algorithm 3.2) for the left-most relation R_k , has runtime $\mathcal{O}(n+r)$, meeting the optimality bound for ACQ in data complexity.*

PROOF. By Lemma 4.4, the execution of a plan is in clean state when TTJ execution finishes. The amount of work that makes \mathcal{P}_Q clean, i.e., cleaning cost, is fixed despite the distribution of dangling tuples in the relations. Suppose the execution is in clean state after computing the first join result.

To bound the cleaning cost, we bound the cost of getting the first join result. Cleaning cost of TTJ includes the following components: (1) the cost of `open()`, which is $\mathcal{O}(kn)$; (2) the cost of `getNext()`; and (3) the cost of `deleteDT()`, which is bounded by the cost of `getNext()` as well.

The total cost of `getNext()` is bounded by the total number of loops (starting at Line 14). Within the loop, hash table lookup (Line 15) is $\mathcal{O}(1)$. The total number of loops equals the total number of times that r_{outer} is assigned with a value. r_{outer} assignment happens on Lines 11, 13, 20, and 27. Line 13 is called when `getNext()` is recursively called from \bowtie_1 to start computing the first join result, which in total happens k times. Afterwards, whenever r_{outer} becomes `nil`, execution terminates by returning `nil` (Lines 12, 21, and 28) and Line 13 never gets called.

Each time `deleteDT()` is called from Line 20, exactly one tuple is removed. Thus, r_{outer} is assigned $\mathcal{O}(kn)$ times on Line 20. After a call to `deleteDT()` made in the i th operator ($i \in [k-2]$) from Line 20, `deleteDT()` can be recursively called at most $k-i$ times from Line 27. The number of `deleteDT()` calls with $k-i$ recursive calls is at most n because each relation has size n and each initiation of `deleteDT()` removes a tuple. Thus, the total number of assignment to r_{outer} from Line 27 is $\leq \sum_{i=1}^{k-2} (k-i) \cdot n = \mathcal{O}(k^2 n)$.

If `deleteDT()` is never called during the computation of the first join result, Line 11 is not called. Line 11 can only be called from Line 29 when Line 23 is evaluated to true; any `getNext()` calls (Line 29) from recursive `deleteDT()` calls triggered by Line 20 will not call Line 11 because `MatchingTuples` is set to `nil` on Line 26. Thus, the number of calls on Line 11 equals to the number of `deleteDT()` calls from Line 20, which is $\mathcal{O}(kn)$.

Summing everything together, cleaning cost is $\mathcal{O}(k^2 n)$. Since \mathcal{P}_Q is clean after computing the first join result, with Lemma 4.5, the result follows. \square

The combined complexity of TTJ is $\mathcal{O}(k^2 n + kr)$, which can be further reduced to $\mathcal{O}(nk \log k + kr)$ by imposing an additional constraint on \mathcal{P}_Q . We defer the details to Appendix H.

5 EVALUATION

We compare the performance of TTJ with the baselines (§ 5.3), introduce three parameters that impact TTJ performance, and analyze them through control studies (§ 5.4). We further examine the space consumption of `ng` and the robustness of TTJ (§ 5.4).

5.1 Algorithms and Implementation

We compare TTJ with the baselines (§ 2.3) in an apples-to-apples fashion, where we implement all these methods within the same query engine built from scratch in Java. The engine architecture is similar to the architecture of recent federated database systems [11, 55]. The engine optimizes each algorithm using the same DP procedure [27] with an algorithm-specific cost model (Appendix I). The engine connects two data sources: PostgreSQL 13, which provides the estimation to the terms in the cost models, and DuckDB [51], which serves as the storage manager. All data are stored on disk.

We detail the implementation of `ng` here. Suppose R_k has m children S_1, \dots, S_m . Physically, `ng` is implemented as a hash table $\langle S_i, \ell_i \rangle$ where ℓ_i is a set containing $jav(t, R_k, S_i)$ for dangling tuple t from R_k detected by S_i .

We provide additional implementation details of the baselines that are not described in § 2.3. To implement YA, we introduce a k -ary physical operator *full reducer operator* that executes F_Q . The fully reduced relations, which already reside in memory, are then evaluated by HJ. PT is implemented similarly to YA with a k -ary operator for the predicate transfer phase. PT originally works on the predicate transfer graph, which contains redundant edges compared with \mathcal{T}_Q . Redundant edges may lead to additional unnecessary passes of Bloom filters that may negatively impact PT performance⁶. Thus, we show the results of PT on \mathcal{T}_Q . We use the blocked Bloom filter [49] implementation from [32].

5.2 Experimental Setup

Workload. We use three workloads: Join Ordering Benchmark (JOB) [41], TPC-H [60] (scale factor = 1), and Star Schema Benchmark (SSB) [48] (scale factor = 1). We focus on ACQs in the benchmarks, i.e., we omit cyclic queries, single-relation queries, and queries with correlated subqueries. All 113 JOB queries, 13 TPC-H queries, and all 13 SSB queries meet the criteria.

Environment. For all our experiments, we use a single machine with one AMD Ryzen 9 5900X 12-Core Processor @ 3.7Hz CPU and 64 GB of RAM. We only use one logical core. We set the size of the JVM heap to 20 GB. All the data structures are stored on JVM heap. Benchmarks are orchestrated by JMH [1], which includes 5 warmup forks and 10 measurement forks for each query and algorithm. Each fork contains 3 warmup and 5 measurement iterations.

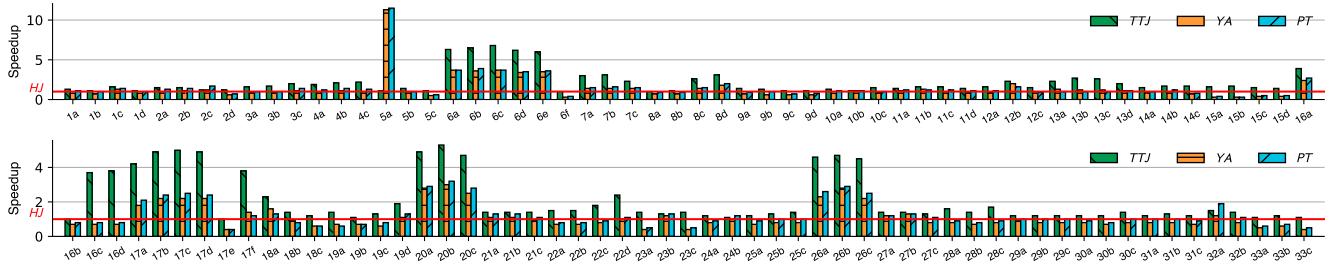


Figure 4: Speedup of TTJ, YA, PT over HJ on all 113 JOB queries

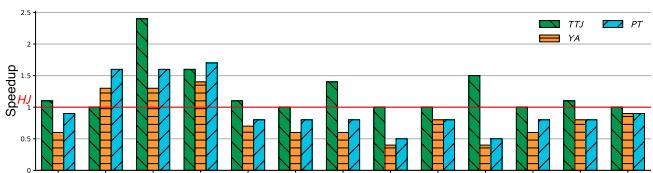


Figure 5: Speedup of TTJ, YA, PT over HJ on 13 TPC-H queries

5.3 Comparison with Existing Algorithms

5.3.1 Query Performance. Figure 4 compares the execution time of TTJ, YA, and PT against HJ on JOB queries. Of all 113 queries, TTJ runs faster than HJ on 112 (99%) of them. The maximum speedup is $6.8\times$ (6.c) and the minimum speedup is $1\times$ (6f). On average (geometric mean), TTJ is $1.8\times$ faster than HJ. YA is faster than HJ on 47 (42%) queries. The maximum, average, and minimum speedup is $11.3\times$ (5a), $1\times$, $0.3\times$ (6f), respectively. PT is faster than HJ on 67 (59%) queries. The maximum, average, and minimum speedup is $11.5\times$ (5a), $1.1\times$, $0.3\times$ (15b), respectively. From the aggregate statistics we can see that (1) TTJ has more steady speedup than YA and PT on the entire workload: TTJ has higher average and minimum speedup than the other two algorithms; (2) YA and PT can outperform TTJ in special cases such as 5a, which returns empty results. 5a is favorable for YA and PT because the query evaluation terminates earlier than TTJ: The first semijoin `movie_companies` \bowtie `company_type` in the bottom-up pass completely removes all the tuples in `movie_companies`, which subsequently terminates the whole query evaluation. In contrast, the two relations appear as the second and the fourth relation in \mathcal{P}_Q , which makes TTJ perform more join computations than YA and PT before it terminates. This exemplifies the importance of join order for TTJ, which we further study in § 5.4.5.

Figure 5 shows the comparison result on TPC-H. TTJ has the maximum speedup $2.4\times$ on Q8, the largest query with $k = 8$ in TPC-H. $2.4\times$ is also the largest speedup among the three algorithms. Similar to its performance pattern on JOB queries, TTJ has steady speedup over the benchmarked TPC-H queries with average $1.2\times$ compared with $0.69\times$ from YA and $0.84\times$ from PT. We further study a few interesting TPC-H queries in § 5.3.2.

For star schema queries, all algorithms share the identical \mathcal{T}_Q and plan, where the fact table is R_k and the dimension tables are

⁶We conducted an empirical study by comparing PT on the predicate transfer graph with the same PT on \mathcal{T}_Q to verify our conclusion. Result shows PT on \mathcal{T}_Q outperforms PT on the predicate transfer graph by $1\times$ (Appendix J).

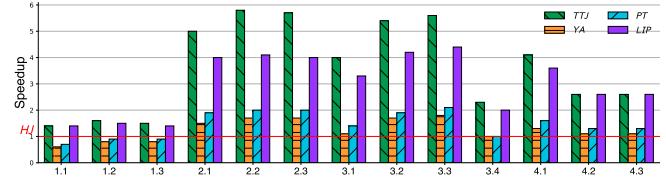


Figure 6: Speedup of TTJ, YA, PT, and LIP over HJ on all 13 SSB queries

the children of R_k ordered from left to right. Figure 6 illustrates TTJ has the largest speedup, $3.2\times$ on average, for all SSB queries and LIP comes in second with average of $2.8\times$. After eliminating the impact of join order and join tree, the performance difference between TTJ and LIP shows that lazily building and probing ng works better than proactively building and probing a set of Bloom filters. Probing Bloom filters at R_k in LIP can be viewed as performing a bottom-up pass of \mathcal{T}_Q . Compared with LIP, YA and PT perform an additional top-down pass of \mathcal{T}_Q . The potential benefit of the top-down pass performed by YA or PT can be very small because the fact table is fully or nearly fully reduced after the bottom-up pass [12] and the dangling tuples in the dimension tables will not or unlikely be matched during join evaluation. A possible performance gain from the top-down pass is from dimension table size reduction, which can speed up hash table operations. Both YA (average $1.2\times$) and PT (average $1.4\times$) are slower than LIP, indicating that the cost of performing the top-down pass of \mathcal{T}_Q outweighs the potential benefit due to dimension table size reduction. PT comes the third and runs faster than YA because Bloom filter probe is faster than semijoin hash table probe.

5.3.2 Trade-off between join time and removing dangling tuple time. All the join algorithms we studied strategically allocate runtime between performing joins and removing dangling tuples. On one end of the spectrum, HJ spends all of its runtime performing joins. On the other end of the spectrum, YA, PT, and LIP spend most of its runtime removing dangling tuples. PT spends less than YA due to the efficiency of Bloom filters. LIP further reduces dangling tuple removal time on star schema queries by eliminating the top-down pass of \mathcal{T}_Q . Due to the laziness nature of TTJ, it aims to stay closer to the HJ side by spending less of its runtime on removing dangling tuples and more time on computing joins. Figure 7 illustrates the patterns by showing the runtime breakdown on TPC-H

queries⁷. The figure shows that each algorithm's overall performance largely depends on its *dominate time*, i.e., join time for TTJ and dangling tuple removal time for YA and PT.

YA and PT are performant when the full reducer can be executed quickly. Consider Q7: A fragment of YA join tree is a chain $\text{orders} \rightarrow \text{lineitem} \rightarrow \text{supplier} \rightarrow \text{nation}$. The first semijoin $\text{supplier} \bowtie \text{nation}$ already removes more than 90% of tuples from supplier because $|\text{nation}| = 1$. The largely reduced supplier speeds up the subsequent semijoin $\text{lineitem} \bowtie \text{supplier}$ and starts a chain reaction on the remaining semijoins. As a result, YA removes close to 100% of the tuples of the input relations (Figure 8) in a small amount of time (Figure 7). PT shares the same join tree as YA and has a similar behavior. On the flip side, YA and PT face challenges when the full reducer executes slowly. A typical example is star schema queries. Figure 9 shows the fraction of input relations tuples removed on SSB. From the figure we see that YA and PT remove almost identical number of dangling tuples as LIP but have much lower speedup (Figure 6). This shows that the top-down pass of \mathcal{T}_Q that YA and PT perform on star schema queries not only incurs additional execution cost but also can hardly reduce dimension table size.

TTJ performs better when its join time is smaller than the dangling tuple removal time of YA and PT. Join time is usually small if a large number of dangling tuples can be removed. Thus, intuitively, TTJ is good if the small amount of dangling tuple removal time spent by TTJ can remove a huge number of dangling tuples. In Q8, a typical example that TTJ greatly outperforms YA and PT, TTJ removes 91% of the dangling tuples removed by YA or PT, while using only 22% of YA's and 27% of PT's dangling tuple removal time. However, the quantity of dangling tuples removed alone is not a decisive factor on explaining the performance of TTJ. For example, in Q15, TTJ spends a negligible amount of time removing the same number of dangling tuples as YA and PT (99% of input tuples) but unlike Q8, the join time is not significantly reduced. As a result, TTJ does not considerably outperform YA and PT. Such observation indicates that the *quality* of dangling tuples removed also matters. A dangling tuple has high quality if removing it can substantially reduce the join time. Directly measuring dangling tuple quality is non-trivial; instead, we use two parameters to measure the effectiveness of the actions to remove certain groups of dangling tuples. The more effectiveness the actions are, the higher quality the removed dangling tuples have.

Duplicate ratio α . ng contains all the unique *javs* of the dangling tuples in R_k . The action taken by TTJ related to ng contains two steps: (1) If R_k is the guilty relation, *jav* is computed and put into ng ; (2) Future tuples from R_k are filtered out if their *jav* appear in ng . We focus on the filtering step of the action. To measure its effectiveness, we can divide the dangling tuples of R_k into two sets: Set A contains dangling tuples that can be filtered out by ng and set B contains the rest of the dangling tuples. We define $\alpha = \frac{|A|}{|A|+|B|}$, which is the fraction of tuples in the dangling tuples of R_k that can be filtered out by ng . The larger α is, the more dangling tuples can

be filtered out by ng . For example, 99% of the tuples in *lineitem* (R_k of Q8) is dangling. Its α is 96%.

Modified Semijoin Selectivity θ . On detecting dangling tuples, $\text{deleteDT}()$ is called. If the guilty relation is an internal⁹ relation, a tuple is removed from its hash table. We denote the action of removing dangling tuples from the hash tables as rm . θ_R measures the fraction of tuples from an internal⁹ relation R that will no longer participate join once the dangling tuples from all of its child relations are removed. The larger θ_R is, the more effective rm is. For example, in Q8, *customer* has the highest θ 6.6%. We provide the formal definition of θ in Appendix L and give an example in § 5.4.2.

With the concept of quality, we can say that TTJ is fast when it can remove a large number of high quality dangling tuples within a small amount of dangling tuple removal time. We introduce a third parameter, *backjumping distance*, which determines how fast a dangling tuple can be removed.

Backjumping distance b_{ij} . When join fails, TTJ backjumps to the guilty relation via $\text{deleteDT}()$ calls. We call the action bj . b_{ij} denotes the number of relations between the detection relation R_i (excluding) and the guilty relation R_j (including) for a join failure. The larger b_{ij} is, the quicker the dangling tuple from the guilty relation can be removed. Join time is also reduced because backjumped relations (relations appear between the detection and the guilty relations in \mathcal{P}_Q) will no longer be probed until a new join result is produced by the guilty relation. In Q8, the largest b_{ij} is 4.

5.4 Detailed Analysis of TTJ

We perform control studies on the parameters introduced in § 5.3.2 to measure the effectiveness of the corresponding TTJ actions.

Result Summary. Query and database instance can lead to a large number of high quality dangling tuple removal if (1) duplicate ratio $\alpha > 50\%$ (§ 5.4.1); (2) modified semijoin selectivity $\theta > 2\%$ (§ 5.4.2). Backjumping is more effective when $b_{ij} > 4$ ($k > 5$). Furthermore, we show that: (1) no-good list takes small spaces on the benchmark workloads (§ 5.4.4); (2) join order has a large impact on TTJ performance, but even with a suboptimal join order, TTJ can still match or outperform HJ.

5.4.1 Impact of α . Consider the following query

$$Q = T(a, b) \bowtie R(a) \bowtie S(b) \quad (3)$$

T is the root of \mathcal{T}_Q and $\mathcal{P}_Q = [T, R, S]$. Let all tuples in T be dangling due to S , i.e., $T \bowtie R = T$ and $T \bowtie S = \emptyset$. $|A| + |B| = |T|$ and $|A| = \alpha|T|$. Column $T.a$ and $R.a$ contain the numbers from 1 to $|T|$. For $T.b$, we first put $|B|$ unique values; then, we append additional $|A|$ values that are sampled from the unique values uniformly at random. We fill in $S.b$ with values that are not in $T.b$. We shuffle all the rows of all the relations at the end. All three relations have equal size of 10 million tuples.

Result Analysis. Figure 10 shows the fraction of ng build and probe time over the overall runtime with different α . The left-most bar shows that ng operations take 8% of runtime when $\alpha = 0\%$, i.e., all dangling tuples in T have unique *javs*. The fraction stays between 8% and 10% when $\alpha \leq 50\%$. Once $\alpha > 50\%$, the fraction

⁷Due to the space limit, we defer the runtime breakdown of LIP on SSB to Appendix K, which illustrates LIP spends less time on dangling tuple removal than YA and PT but more than TTJ.

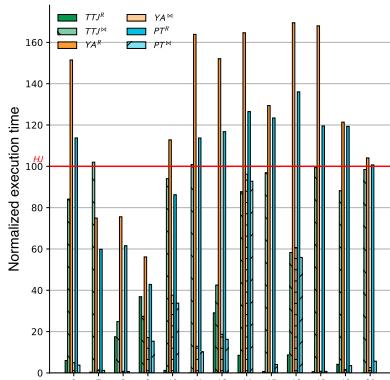


Figure 7: Breakdown of TTJ, YA, and PT execution time into dangling tuples removal (e.g., TTJ^R) and join (e.g., TTJ^Δ) on TPC-H

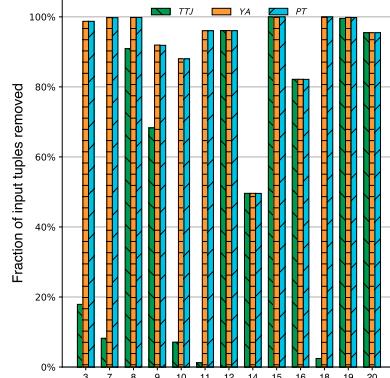


Figure 8: Fraction of tuples removed from the input relations by TTJ, YA, and PT on TPC-H

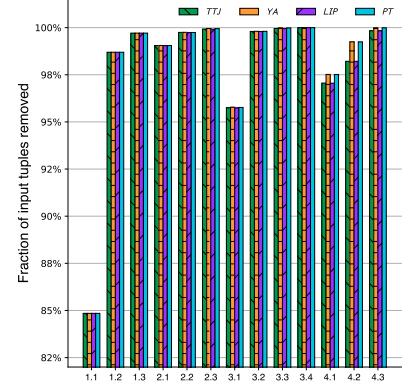


Figure 9: Fraction of tuples removed from the input relations by TTJ, YA, LIP, and PT on SSB

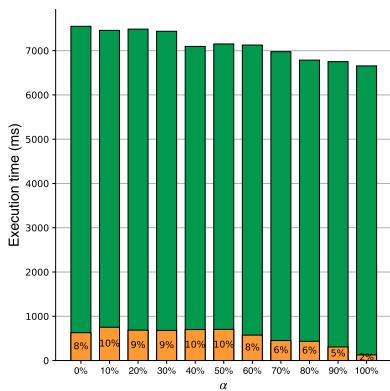


Figure 10: Execution time and profile percentage of runtime spent on building and probing ng across different α on mini-benchmark Query (3)

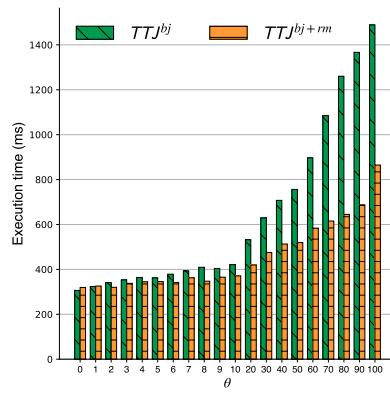


Figure 11: Execution time between TTJ^{bj} and TTJ^{bj+rm} for different θ of mini-benchmark Query (4)

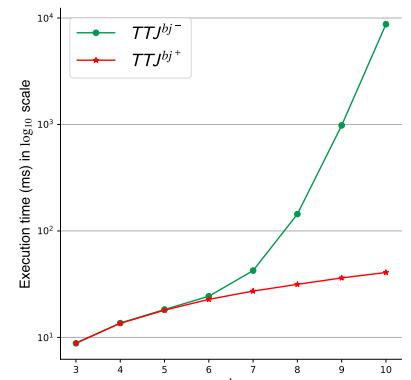


Figure 12: Execution time between TTJ^{bj-} ($bj = 1$) and TTJ^{bj+} ($bj = k - 1$) for different number of input relations k of mini-benchmark Query (5)

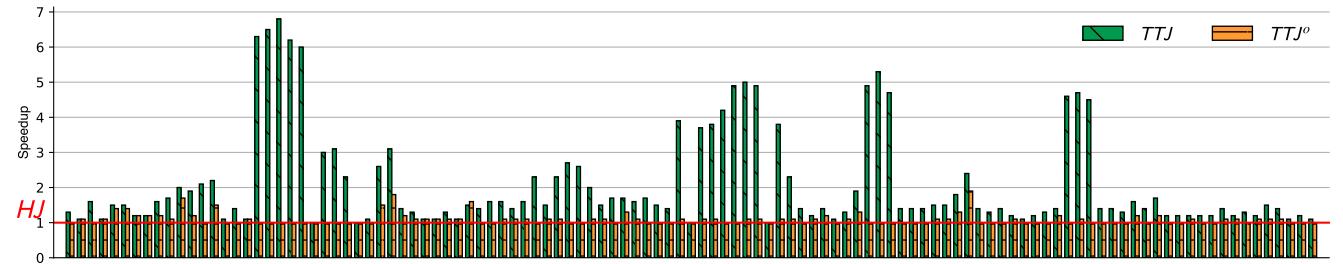


Figure 13: Performance comparison among TTJ on TTJ order (TTJ in the figure), TTJ on HJ order (TTJ^o in the figure), and HJ on HJ order on all 113 JOB queries

starts a steady drop. The right-most bar ($\alpha = 100\%$ ⁸) has 2% fraction and the lowest execution time overall. In general, the larger α , the less time ng operations takes, and the better TTJ performs.

5.4.2 Impact of θ .

$$R(a, c) \bowtie U(c, e) \bowtie V(c, d) \bowtie T(d, g) \bowtie W(d, f) \quad (4)$$

⁸Technically, $\alpha = 99.9\%$ because $|B| \geq 1$, i.e., there has to be at least one tuple in B so that tuples in A can be filtered out by ng .

$\mathcal{P}_Q = [R, U, V, T, W]$. \mathcal{T}_Q starts R as the root and has a chain $R \rightarrow U \rightarrow V$. Both T and W are children of V . R has two tuples $(1, 2), (1, 4)$. U has tuples $(2, 1), (2, 2), \dots, (2, \theta|U|), (3, \theta|U| + 1), \dots, (3, |U|-1), (4, 4)$, where θ is defined below. V has two tuples $(2, 3), (4, 4)$. T has two tuples $(3, 1), (4, 1)$. W has one tuple $(4, 5)$. The query result set is $\{(1, 4, 4, 4, 1, 5)\}$.

We define θ on U as $\theta_U = \frac{|(U \bowtie_R) \bowtie (V \bowtie \tilde{V})|}{|U|}$. \tilde{V} is V in clean state. In words, θ is the fraction of tuples in U that are joinable with R and joinable with the dangling tuples from V . We compare TTJ^{bj} with TTJ^{bj+rm} . TTJ^{bj} only enables bj and disables ng and rm . TTJ^{bj+rm} enables bj and rm , which removes the dangling tuples from the hash tables in addition to backjumping. We fix $|U| = 1$ million.

Result Analysis. Figure 11 shows that (1) the larger θ is, the more beneficial removing dangling tuples from the hash tables becomes; (2) in our implementation, it is always beneficial to remove dangling tuples from the hash tables: When $\theta = 0\%$ ⁹, removing dangling tuples will not reduce subsequent join computations, but in such case, TTJ^{bj+rm} and TTJ^{bj} still have matching performance. To explain the performance difference, first consider TTJ^{bj} , where it does not remove dangling tuples. The evaluation starts with $U(2, 1)$ and does not fail until $W(d, f)$. Then, `deleteDT()` resets the evaluation flow to V . $V(2, 3)$ is not removed. No more matching tuples is left from V given $jav(c : 2)$. `deleteDT()` further sets the evaluation flow to U and moves on to $U(2, 2)$, which is joinable with $R(1, 2)$. Since $V(2, 3)$ still presents, the join result $(1, 2, 2, 3)$ will eventually try W and fail again. `deleteDT()` brings the evaluation flow back to V . Since no more tuples are joinable with $U(2, 2)$, `deleteDT()` resets the flow back to U . Then, $U(2, 3)$ is returned. The same process repeats $\theta|U|$ times in total. In TTJ^{bj+rm} evaluation, $V(2, 3)$ is deleted when `deleteDT()` first resets the evaluation flow to V . The evaluation will finish much earlier because $U(2, 2)$ will be removed immediately once it probes into V , and the same happens to $U(2, 3), \dots, U(2, \theta|U|)$.

5.4.3 Impact of b_{ij} . For this study, we only enable the backjumping action of TTJ (TTJ^{bj}). Consider the following query (for simplicity, we replace \bowtie with comma between relations):

$$Q = R_1(a_1, \dots, a_k), R_2(a_2, a_3), \dots, R_{k-1}(a_{k-1}, a_k), R_k(a_k) \quad (5)$$

The database instance is as follows: $R_1(a_1, \dots, a_k)$ has two tuples $(1, 2, 3, \dots, k-1, k)$ and $(1, 3, 4, 5, \dots, k, k+1)$. $R_i(a_i, a_j)$ has $n-1$ copies of (i, j) and a tuple $(i+1, j+1)$. $R_k(a_k)$ has one tuple $(k+1)$. Query (5) has only one join result $(1, 3, 4, 5, \dots, k+1)$.

We run TTJ^{bj} on two \mathcal{T}_Q : \mathcal{T}_Q^- is a chain shape: $R_1 \rightarrow R_2 \rightarrow \dots \rightarrow R_k$. \mathcal{T}_Q^+ is a star shape: R_1 is the root and the rest of the relations are its children ordered from left to right. We denote TTJ^{bj} on \mathcal{T}_Q^- as TTJ^{bj^-} and on \mathcal{T}_Q^+ as TTJ^{bj^+} . TTJ^{bj^-} has the characteristic that every guilty relation S is immediately before the detection relation R for any join failure, i.e., $b_{ij} = 1$. TTJ^{bj^+} has only one join failure, which happens when the tuple $(k-1, k)$ of R_{k-1} joins with R_k . After the join failure, TTJ^{bj^+} resets the execution flow to R_1 and starts to compute the final join result. Thus, $b_{ij} = k-1$. TTJ^{bj^-} produces $(n-2) \sum_{j=0}^{k-3} (n-1)^j = O(kn^k)$ more dangling intermediate results than TTJ^{bj^+} (Appendix M). We fix $n = 10$ and vary k .

Result Analysis. Figure 12 shows that the performance between TTJ^{bj^+} and TTJ^{bj^-} begins to diverge when $k = 6$ ($b_{ij} = 5$) where

⁹Technically, $\theta > 0\%$ because in our micro-benchmark, U at least has $(2, 1)$ and θ is at least one over 1 million.

TTJ^{bj^-} produces 6560 more dangling tuples than TTJ^{bj^+} does. After that, we see the execution time of TTJ^{bj^-} grows exponentially whereas TTJ^{bj^+} grows logarithmically. The result indicates that the backjumping distance impacts the number of dangling tuples that can be avoided by TTJ , thereby affecting TTJ performance.

Table 2: Number of $javs$ stored in ng . In parenthesis, we list memory percentage consumption taken by ng with respect to total query evaluation memory consumption

Bench.	min	max	avg.
JOB	12 (0%)	4051176 (6%)	908226 (0.3%)
TPC-H	24 (0%)	1470901 (4.2%)	176716 (0.6%)
SSB	2041 (0%)	201343 (1.9%)	65223 (0.4%)

5.4.4 Space Consumption of ng . Table 2 shows the space taken by ng on the benchmark queries. Despite of the relatively large ng size, the memory footprint is negligible, e.g., at most 6% of total memory consumption. The main reason is that ng only stores $javs$ (a few integers), which are tiny compared with other memory consumption, e.g., loading relations into memory.

5.4.5 Robustness against Poor Plans. In this experiment, we study whether TTJ performance is robust against poor plans. We compare three setups: (1) TTJ on HJ order (we call it TTJ^o); (2) TTJ on TTJ order; and (3) HJ on HJ order. We consider HJ order as a poor plan because the order is not specific optimized for TTJ . Figure 13 shows that compared with TTJ , the number of queries that TTJ^o outperforms HJ is smaller (105 vs. 112) and the average speedup goes down ($1.1\times$ vs. $1.8\times$). This result shows that in general, optimizing TTJ specifically can lead to much larger performance gain compared with treating TTJ as HJ. Nevertheless, TTJ still matches or outperforms HJ on HJ order.

6 DISCUSSION AND RELATED WORK

We organize the related work in four categories. (1) *CSP*. The equivalence between CQ evaluation and CSP is established by [15, 40]. TreeTracker in [9] solves a CSP for one solution without preprocessing the CSP. TTJ extends TreeTracker into query evaluation by (a) returning all possible solutions; (b) blending the ideas from TreeTracker into physical operators in a query plan. (2) *Semijoin reduction*. An intensive research has been done on using semijoin to improve query evaluation speed [12, 13, 17, 38, 42, 62, 63, 69]. TTJ achieves a similar effect (clean state) as performing semijoin reduction without explicitly using semijoins. (3) *SIP*. `deleteDT()` of TTJ takes the form of SIP [8, 22, 23, 26, 31, 33, 34, 37, 44, 46, 50, 54, 56, 72, 73]. TTJ is different from the prior approaches in one or more of the following aspects: (a) TTJ does not introduce any preprocessing steps; (b) TTJ does not use Bloom filters, bitmaps, or semijoins; and (c) TTJ provides optimality guarantee. (4) *Worst-Case Optimal Join (WCOJ) algorithms*. A related line of work is to implement WCOJ algorithms efficiently [2, 6, 25, 36, 45, 66, 67]. TTJ is orthogonal to such direction as TTJ focuses on ACQ evaluation (§ 2.2). We show how to use TTJ for cyclic CQ evaluation (Appendix D). Comparing to WCOJ algorithms, which commonly use multi-way join operators, TTJ for cyclic CQs uses binary physical operators in iterator interface.

7 LIMITATIONS AND FUTURE WORK

We propose the first join algorithm that incorporates backjumping and no-good into query evaluation. Gaps remain when consider TTJ with additional requirements from both practical and theoretical aspects, which we discuss next. *Practical aspects.* (1) We focus on estimating the logical cost in our cost model for TTJ. Future extension to the model can include physical cost coefficients such as *ng* probing cost, hash table probing cost, and tuple deletion cost, and so on; (2) we present TTJ using the tuple-based iterator interface. Extending TTJ to work with vectorization has one challenge: Batch processing introduces an additional trade-off because it reduces the number of recursion calls, but potentially loses the opportunity for detecting and deleting dangling tuples; (3) TTJ assumes demand-driven pipelining and requires additional extension to work with asynchronous processing; and (4) TTJ uses *ng* only on the left-most relation R_k . Whether using *ng* on the other relations requires further assessment on the *ng* probing cost versus the potential additional dangling tuple removal. *Theoretical aspects.* (1) The combined complexity of TTJ can be improved because it has an additional $\log k$ term compared with the complexity of YA; (2) the extended TTJ for cyclic queries does not have the same complexity as WCOJ algorithms do, which requires further exploration.

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A PROOF OF LEMMA 3.1

LEMMA 3.1. *For any left-deep plan without cross-product for acyclic queries, there exists a \mathcal{T}_Q satisfies the join tree assumption (Definition 1).*

PROOF. For a left-deep plan without cross-product for an acyclic CQ $[S_k, S_{k-1}, \dots, S_1]$, our proof proceeds by showing the plan permits a rooted join tree that satisfies Definition 1. That is, S_k is R_k , the root of some \mathcal{T}_Q , and for any relation S_i , its parent in \mathcal{T}_Q is S_j with $j \in \{k, k-1, \dots, i+1\}$. For a relation S_i , let attribute set $as(S_i)$ denote the set of attributes appear before it in the plan, i.e., $as(S_i) = attr(S_k) \cup \dots \cup attr(S_{i+1})$. The plan has the property that $attr(S_i) \cap as(S_i) \neq \emptyset$. We want to show there is some relation S_j with $j \in \{k, k-1, \dots, i+1\}$ such that $attr(S_j) \supseteq (attr(S_i) \cap as(S_i))$. If the statement is true, we can construct \mathcal{T}_Q by adding edge (S_j, S_i) . To prove the statement, for a relation S_u , suppose relations before S_u already form a join tree, i.e., we are about to attach S_u to the tree. Suppose the statement is not true and there are two more relations S_i, S_j ($i > j > u$) in the plan such that $attr(S_u) \cap as(S_u) = (attr(S_i) \cup attr(S_j))$ and $attr(S_u) \cap as(S_u) \not\subseteq attr(S_i)$ (correspondingly for $attr(S_j)$ as well). S_i and S_j are connected via a path. To satisfy join tree requirement, one must add two edges (S_i, S_u) and (S_j, S_u) , which form a cycle. \square

B TTJ COST MODEL

Given \mathcal{P}_Q is a transformation of \mathcal{T}_Q , costing TTJ is the same as costing evaluation of \mathcal{T}_Q under TTJ. One simple but effective cost model is the sum of the sizes of intermediate results [19, 22, 41, 58], which comprises two components: the dangling tuples produced and the size of intermediate results that are part of final join result. The former corresponds to the cleaning cost in the optimality proof, which can be estimated based on the clean state. Using clean state, the latter can be estimated easily as well. Since TTJ reduces internal^o relation sizes, like [22], TTJ cost includes the size of inner relations that are in clean state as well.

THEOREM B.1. *The cost of TTJ, i.e., the cost of \mathcal{T}_Q when evaluated by TTJ, is*

$$\sum_{i=1}^m \sum_{t=0}^{|\mathcal{A}^i|-1} b_{R_u^{t+1}}^{R_i} |(\mathbb{R}_i^{[t]} \bowtie_{J_{i+1}^*} \bar{\mathbb{R}}_u^{t+1})| \quad (6)$$

$$+ \sum_{i=1}^s \sum_{t=0}^{|\mathcal{B}^i|-1} b_{R_u^{t+1}}^{R_i} |(\mathbb{R}_i^{[t]} \bowtie_{J_{i+1}^*} \bar{\mathbb{R}}_u^{t+1})| \quad (7)$$

$$+ \sum_{t=0}^{|\mathcal{C}|-1} b_{R_u^{t+1}}^{R_k} |\delta(\pi_{ja(R_k, R_u^t)}(\mathbb{R}_k^{[t]} \bar{\bowtie} \bar{\mathbb{R}}_u^{t+1}))| \quad (8)$$

$$+ \sum_{j=k}^1 |f(S_j)| \quad (9)$$

$$+ \sum_{i=2}^k |\bar{\mathbb{R}}_i| \quad (10)$$

Equations (6) to (8) give the number of dangling tuples. Equation (9) gives the size of intermediate results (including the size of R_k^*) that are part of final join result. Equation (10) is the summation of size of internal^o and leaf relations that are in clean state.

b_{ij} counts the number of additional dangling tuples generated given a dangling tuple from guilty relation R and detection relation S . We define the following three sets over the relations in \mathcal{T}_Q : (1) \mathcal{A} consists of all the leaf relations R_u such that internal^o relation R_i are their parent. We partition \mathcal{A} by leaf relations' parents, $\mathcal{A}^1, \dots, \mathcal{A}^m$ where \mathcal{A}^i is the set of leaf relations that have the parent R_i . Thus, $|\mathcal{A}^i|$ represents the number of leaf relations in \mathcal{T}_Q that are children of R_i . Let us label those leaf relations $R_u^1, \dots, R_u^{|\mathcal{A}^i|}$; (2) \mathcal{B} consists of internal^o relations R_u that their parents R_i are internal^o relations. Similarly to $\mathcal{A}^1, \dots, \mathcal{A}^m$, we partition \mathcal{B} by the parent of R_u : we have $\mathcal{B}^1, \dots, \mathcal{B}^s$. $|\mathcal{B}^i|$ and $R_u^1, \dots, R_u^{|\mathcal{B}^i|}$ are defined similarly as above; and (3) \mathcal{C} comprises all the relations R_u that are children of R_k . The children of R_k are labeled $R_u^1, \dots, R_u^{|\mathcal{C}|}$. Equation (11) defines $\mathbb{R}_i^{[t]}$, which reflects the gradual discovery of dangling tuples of R_i during plan evaluation.

$$\mathbb{R}_i^{[t]} = \begin{cases} \mathbb{R}_i & \text{if } t = 0 \\ \mathbb{R}_i^{[t-1]} - ((\mathbb{R}_i^{[t-1]} \bowtie_{J_{i+1}^*} \bar{\mathbb{R}}_u^t) \bar{\bowtie} \bar{\mathbb{R}}_u^t) & \text{otherwise} \end{cases} \quad (11)$$

Suppose the join order (w.r.t. \mathcal{T}_Q) determined either syntactically from \mathcal{T}_Q or from the DP algorithm is $[S_k, \dots, S_1]$ where $S_j = R_i$ for some $i \in [k]$. $f(S_j)$ in Equation (12) computes the size of intermediate results that are part of the final join result. Given Definition 1, the first relation to apply f is always R_k , root of \mathcal{T}_Q , and its content, per Lemma 4.2, is \mathbb{R}_k^* .

$$f(S_j) = \begin{cases} \mathbb{R}_k^* & \text{if } j = k \\ \bar{\mathbb{R}}_j \bowtie f(S_{j+1}) & \text{otherwise} \end{cases} \quad (12)$$

EXAMPLE 12. Using Example 1, we illustrate how to compute Equations (6) to (8), the most complex terms in the cost equation. Assume $\mathcal{P}_Q = [T, S, B, R]$. Start with Equation (6). $\mathcal{A} = \{B, R\}$. Since both B and R have the same parent, S , $m = 1$. Thus, the cost is $1 \cdot |(S \bowtie T) \bar{\bowtie} B| + 2 \cdot |(S^{[1]} \bowtie T) \bar{\bowtie} R|$. In particular, $|(S \bowtie T) \bar{\bowtie} B| = 0$. Therefore, $S^{[1]} = S$. $|(S^{[1]} \bowtie T) \bar{\bowtie} R| = 1$ due to $S(red, 1, 2)$. Thus, $\bar{S} = \{(red, 3, 2)\}$. Since $\mathcal{B} = \emptyset$, we do not need to compute Equation (7). To compute Equation (8), due to $C = \{S\}$, we have $1 \cdot |\delta(\pi_x(T \bar{\bowtie} \bar{S}))| = 1$. Thus, the number of dangling tuples produced by TTJ is $0 + 2 + 1 = 3$.

C BUSHY PLAN

A common approach to evaluate a bushy plan is to decompose it into a sequence of left-deep subplans: right child of every join operator forms a left-deep subplan and is evaluated first before proceeding with the join [57, 66]. In particular, for in-memory hash-join, build side is a blocking operation, i.e., hash tables can be constructed not just from base relations but also from intermediate results computed from subplans, which are buffered inside the memory [33, 57]. TTJ works with bushy plan exactly as above. The only issue to address is to transform \mathcal{T}_Q into a bushy plan satisfying Corollary 3.2¹⁰ so that when join fails at R , `deleteDT()` can find its parent. We use Algorithm C.1 to control the construction of a bushy plan for TTJ. Such algorithm can be easily adapted into a “reverse-engineer” procedure where one can construct a \mathcal{T}_Q from the given bushy plan: we construct a join tree for each left-deep

¹⁰We use join order view of Definition 1.

subplan using Lemma 3.1 and concatenate all the trees to form the final join tree.

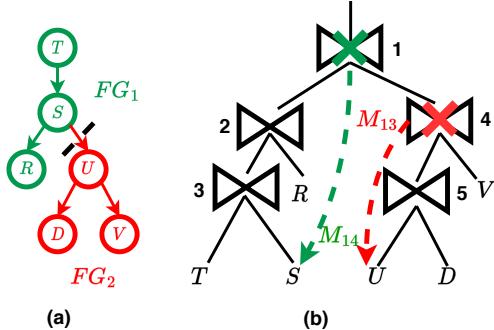


Figure 14: Given \mathcal{T}_Q in (a), there are two fragment groups FG_1 and FG_2 . (b) is a bushy plan constructed from the two fragment groups. Join failures can be categorized into two cases: within FG (M_{13}) and across FGs (M_{14}).

Algorithm C.1: Construct bushy plan for TTJ

- Input:** \mathcal{T}_Q
- Output:** A bushy plan that can be evaluated by TTJ
- 1 Starting from the root of \mathcal{T}_Q , visit each node in pre-order traversal.
 - 2 For each node, decide whether create a new fragment group FG_{i+1} or put it to the fragment group FG_i where its parent node belongs. If for a node R and its left sibling node S has $attr(R) \cap attr(S) = \emptyset$, R has to be in the same group as its parent. Suppose there are fragment groups FG_i for $i \in [m]$ at the end of this step.
 - 3 For each FG_i for $i \in [m]$, create a subplan satisfying the default join order .
 - 4 Start from FG_m and connect it with the subplan from FG_{m-1} with a TTJ join operator. The resulting subplan, a new FG_{m-1} , is connected with the subplan FG_{m-2} and continue. When connecting two subplans FG_i and FG_{i-1} , we always put FG_{i-1} as the left child of TTJ join operator. The step repeats until all the subplans are connected.
-

Fragment group FG is a set of nodes in \mathcal{T}_Q constituting a subplan. We use FG and subplan interchangeably. Any node from \mathcal{T}_Q only belongs to one group. The key idea to form a bushy plan is that we create a TTJ-compatible subplan for each group and connect them altogether using TTJ join operators again. Fragment groups are formed with the property that parent node belongs to the same or lower-numbered group than its child node(s) in \mathcal{T}_Q . Line 2 checks sibling node to avoid cross-product when join two subplans. The resulting plan satisfies Corollary 3.2 and can be evaluated by TTJ directly.

EXAMPLE 13. Consider Q represented in Figure 14 (a). There are two fragment groups $FG_1 = \{T, S, R\}$ and $FG_2 = \{U, D, V\}$. The whole plan is being evaluated by TTJ operators: every join operator is TTJ join operator; T and U are TTJ table scan operators and the rest are normal table scan operators. $deleteDT()$ happens for

two cases: (1) $deleteDT()$ happens inside the subplan. For example, join fails at \bowtie_4 (M_{13}). Since U is the parent of V , a tuple from U is added to ng ; and (2) $deleteDT()$ happens at the join operator connecting two subplans. For example, join fails at \bowtie_1 (M_{14}). In this case, $deleteDT()$ sends the reference to the root of FG_2 , U , downward. The rest of the evaluation is the same as the left-deep plan case in the previous sections.

LEMMA C.1. *The bushy plan constructed from Algorithm C.1 satisfies Corollary 3.2.*

PROOF. Let S be a node and R, U be its children. There are three possible cases: (1) if R and U are all within the same FG as S , by Line 3, the claim holds; (2) if one of its children is in a different FG , say, U . Since S is in the FG with smaller numbering, by Line 4, S is to the left of U . S is to the left of R because they are in the same FG ; and (3) if all of its children are in different FGs , by a similar argument as the previous case, the claim holds. \square

THEOREM C.2 (CORRECTNESS). TTJ evaluates Q correctly under the bushy plan constructed from Algorithm C.1.

PROOF. By Lemma C.1 and Theorem 4.1, TTJ evaluates relations associated with each FG correctly. We only need to focus on TTJ operators that join two different FGs and show it generates the correct join result. W.l.o.g., the two FGs are denoted FG_1 and FG_2 . We want to show $FG_1 \bowtie FG_2$ is correct. If all tuples from FG_1 are joinable with some tuple from FG_2 join result, the claim holds. Suppose $t \in FG_1$ cannot join with any tuple from FG_2 and thus dangling. By Lemma C.1, the parent of the node where join fails must be in FG_1 . Applying the same arguments in Theorem 4.1, the claim holds. \square

Let r_i denote the size of the join result computed from FG_i .

THEOREM C.3 (DATA COMPLEXITY OF TTJ ON BUSHY PLAN). Suppose there are m FGs and each has result size r_1, \dots, r_m , respectively. TTJ runs $\mathcal{O}(n + \max(r_1, \dots, r_m))$.

PROOF. Note that $m \leq k$. Proof by induction on the number of FGs in the plan. Base case FG_m . It takes $\mathcal{O}(n + r_m)$ to evaluate it. Suppose the claim holds for all the fragment groups until i . To evaluate the plan associated with FG_{i-1} , it takes $\mathcal{O}(n + \max(r_{i+1}, \dots, r_m))$ to evaluate the subplan associated with FG_i . By Line 4, FG_i appears as r_{inner} , which TTJ builds the hash table. TTJ takes r_i to build \mathcal{H} corresponding to FG_i . Result follows. \square

D HANDLE CYCLIC CQ

We show a simple approach to allow TTJ to work with cyclic CQs as well. The key challenge for cyclic CQs is that the query does not permit \mathcal{T}_Q , i.e., a graph that has to contain cycles to satisfy the connectedness property. Let G_Q denote the graph that contains cycles but satisfies the connectedness property. As an example, (a) in Figure 15 shows G_Q for a query joining $T(a, b), S(b, c), B(a, c)$, and $R(b)$. The query is a simple extension to the classic triangle query $R(a, b) \bowtie S(b, c) \bowtie T(c, a)$, which makes the query being cyclic. Our solution to the challenge is simple: conceptually, we remove edges from G_Q to obtain \mathcal{T}_Q by renaming necessary attributes. Then, we introduce a select operator at the root of the plan for the query to filter out redundant tuples so that the final result satisfies the

original query semantics. Implementation-wise, both select and renaming can be part of TTJ operator so that there is no need to introduce extra renaming and select operators towards the plan.

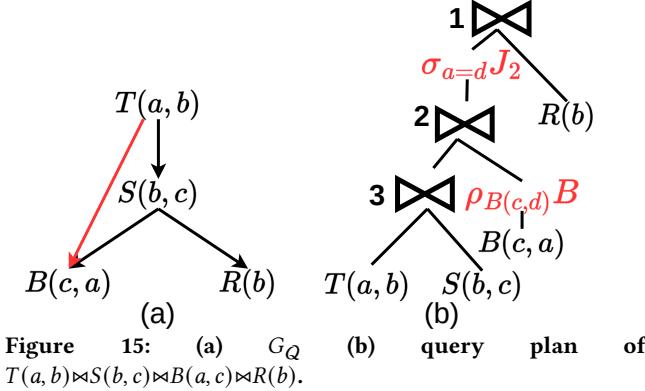


Figure 15: (a) G_Q (b) query plan of

$T(a, b) \bowtie S(b, c) \bowtie B(a, c) \bowtie R(b)$.

EXAMPLE 14. Consider the cyclic query shown in Figure 15. Red color indicates the new operations introduced to handle cyclic CQs.

In the example, we remove edge (T, B) . Since $\text{attr}(B) \cap \text{attr}(T) = \{a\}$, we rename attribute a in one of these two relations. In this case, we rename a in B to d . Thus, in the query plan, we introduce $\rho_{B(c,d)} B$ right above B . The resulting query joining $T(a, b)$, $S(b, c)$, $B(c, d)$, and $R(b)$ is acyclic and can be evaluated using TTJ. Since cycle in G_Q contains T , S , and B , we add $\sigma_{a=d} J_2$ right above \bowtie_2 to filter out those tuples that their a attribute values and d attribute values are different, i.e., adding the removed edge back.

Clearly the simple solution computes the correct join result: the correct join result is subset of the join result compute from \bowtie_1 . To find G_Q for a given join order, one can build \mathcal{T}_Q and introduce necessary extra edges (and thus, form cycles) to satisfy the connectedness property. Those extra edges will be temporarily removed as illustrated in the example above.

Essentially, the approach uses an acyclic query to contain a given cyclic query (in query containment sense [15]) to compute a superset of the cyclic query result set and removes redundant tuples with selections. Thus, the runtime performance of this approach ties to the runtime performance of evaluating the acyclic query, which is $\mathcal{O}(n + r')$ where r' is the output size of the acyclic query.

To detect which edges to remove, there are two ways depending on which structure already exists first: query plan or join tree. If query plan already exists, we can construct \mathcal{T}_Q as usual and if adding a node requires us to introduce more than one edge to satisfy the connectedness property, then those extra edges are to be removed. If G_Q already exists, we can find a spanning tree from it and the edges not in the tree are removed.

E PROOF OF CORRECTNESS

For the correctness (Appendix E) and worst-case runtime analysis (Appendix G), we assume ng in Algorithm 3.2 cannot filter out any tuples from R_k : ng can only guarantee that a dangling tuple will never reappear once it is added to ng . Thus, Algorithm 3.2 Line 6 can be viewed as an implementation optimization that does not matter in the formal analysis. Further, let $iter$ be an iterator on $MatchingTuples$, i.e., when calling $\text{next}()$ on $MatchingTuples$, $iter$

is advanced and returns the next tuple in $MatchingTuples$ if such tuple exists and nil otherwise.

LEMMA E.1. *For every value assignment to r_{outer} , $MatchingTuples$ is initialized with tuples from \mathcal{H} and implicitly, $iter$ is reset. Between each pair of value assignments to r_{outer} , $MatchingTuples$ is never initialized and $iter$ is never reset.*

PROOF. r_{outer} is assigned in four places: Lines 11, 13, 20, and 27. For Lines 11, 13, and 20, $MatchingTuples$ is initialized on Line 15. For Line 27, since $MatchingTuples$ is set to nil (Line 26), $MatchingTuples$ is initialized on Line 15 as well. Since $MatchingTuples$ is never initialized with tuples from \mathcal{H} in the rest of Algorithm 3.1, the claim follows. \square

LEMMA E.2. *A tuple, t , is part of the final join result if and only if it is not marked as dangling during the query evaluation by deleteDT(), i.e., removed from a hash table or added to ng .*

PROOF. We prove the equivalent statement: a tuple t is marked as dangling by $\text{deleteDT}()$ during the query evaluation if and only if t is not part of the final join result. Whenever $\text{deleteDT}()$ is called, a tuple is removed from a hash table or added to ng . $\text{deleteDT}()$ is initiated if and only if $MatchingTuples = nil$, which means r_{outer} contains a dangling tuple, i.e., some tuple is not part of the final join result. \square

THEOREM 4.1 (CORRECTNESS OF TTJ). *Evaluating an ACQ of k relations using \mathcal{P}_Q , which consists of $k - 1$ instances of Algorithm 3.1 as the join operators and 1 instance of TTJ scan (Algorithm 3.2) for the left-most relation R_k , computes the correct query result.*

PROOF. We show $J_1 = J_1^*$ under bag semantics. We first show $J_1 \subseteq J_1^*$. Let $t \notin J_1^*$. Recall

$$J_1^* = \{t \text{ over } \mathcal{P}_{\bowtie_1} \mid t[\text{attr}(R_u)] \in R_u \forall u \in [k]\}.$$

If there doesn't exist a relation R in Q such that $t[\text{attr}(R)] \in R$, it is trivial to see that $t \notin J_1$. Suppose $t[\text{attr}(R_u)] \in R_u$ for $u = \{k, k-1, \dots, i+1\}$ but $t[\text{attr}(R_u)] \notin R_u$ for $u = \{i, i-1, \dots, 1\}$. By default join order (Definition 1), R_i must be a child of some relation R_j with $i < j$ such that $t[\text{attr}(R_j)] \in R_j$ and $t[\text{attr}(R_i)] \notin R_i$. By \mathcal{T}_Q definition, $\text{attr}(R_i) \cap \text{attr}(R_j) \neq \emptyset$. The only non-trivial reason that $t[\text{attr}(R_i)] \notin R_i$ is because $t[\text{attr}(R_i) \cap \text{attr}(R_j)] \notin \pi_{\text{attr}(R_i) \cap \text{attr}(R_j)}(R_i)$. In such case, TTJ will call $\text{deleteDT}()$ from the join operator connected with R_i and $t[\text{attr}(R_j)]$ will be deleted from \mathcal{H}_{R_j} or put onto ng . Thus, t is not in J_1 . If there is a relation R_u with $k \leq u \leq i+1$ such that $t[\text{attr}(R_u)] \notin R_u$, $t \notin J_1$ by the definition of join. The same argument applies if t are duplicated.

To show $J_1^* \subseteq J_1$, suppose $t \in J_1^* \neq J_1$. $t[\text{attr}(R_1)]$ is part of the join result and with Lemma E.2, $t[\text{attr}(R_1)]$ is never deleted. Thus, it must be that $t[\text{attr}(\mathcal{P}_{\bowtie_2})] \in J_2^*$ but $t \notin J_2$. The same argument applies to every operator in the plan. Eventually, we have $t[\text{attr}(R_k)] \in J_k^*$ but $t \notin J_k$. However, this is a contradiction. $t[\text{attr}(R_k)] \in J_k^*$ and joins with the rest of the relations in plan. Thus, with Lemma E.2, $t[\text{attr}(R_k)] \notin ng$ and $\in J_k$.

Next, we show $|J_1| = |J_1^*|$. That is, for a given $t \in J_1^*$, we show the number of tuples t that are in J_1^* equals to the number of tuples t in J_1 . With Lemma E.2, TTJ will not falsely remove a tuple t that is in J_1^* and if t is a dangling tuple, it is removed by $\text{deleteDT}()$.

Further, by Lemma E.1, each tuple from $\bowtie_u \bowtie R_{u-1}$ is enumerated once. Thus, the claim holds. \square

F PROOF OF CLEAN STATE

LEMMA 4.2. *When the left-deep plan without cross-product for ACQ is in clean state, R_k is fully reduced and free of dangling tuples.*

PROOF. Suppose \mathcal{P}_Q is in clean state. Assume there is a dangling tuple $d \in R_k$. Suppose $\{d\} \bowtie R_{k-1} \bowtie \dots \bowtie R_j$ but cannot join with R_{j+1} with $j \in \{k-1, \dots, 2\}$. Given \mathcal{P}_Q satisfying Definition 1, parent of R_{j+1} , R_i , must be one of the relations joinable with $\{d\}$. Thus, R_i is not in clean state. Contradiction. \square

THEOREM 4.3 (CLEAN STATE IMPLIES OPTIMAL EVALUATION). *Once the left-deep plan without cross-product is in clean state, any intermediate results generated from the plan evaluation will contribute to the final join result and the plan can be evaluated optimally.*

PROOF. Proof by induction on the height of \mathcal{T}_Q , h . Base case $h = 0$. Claim trivially holds. Suppose the claim holds for height of $\mathcal{T}_Q < h$. Let R_h be the root of \mathcal{T}_Q with height h . Let R_j be a child of R_h . With Lemma 4.2, no dangling tuples produced when R_h join with R_j . By induction assumption, no dangling tuple produced when further join $R_h \bowtie R_j$ with relations in subtree rooted in R_j . Repeat the same argument for each child of R_h and the result follows. Notice the order of R_j s that invoke proof arguments is specified by the order in \mathcal{P}_Q , which satisfies Corollary 3.2. \square

G PROOF OF RUNTIME

LEMMA G.1. *Algorithm 3.2 Line 10 is executed whenever $\overline{\mathbb{R}}_k \bowtie \mathbb{R}_u \neq \emptyset$ for child relation R_u of R_k . Similarly, Line 24 is executed whenever $\overline{\mathbb{R}}_i \bowtie \mathbb{R}_u \neq \emptyset$ for internal^o relations R_i and its child R_u . \mathbb{R}_u indicates the content of R_u can change during TTJ execution.*

PROOF. We prove the claim on Algorithm 3.2 Line 10; claim on Line 24 can be proved similarly. $t \in R_k$ can be dangling for two reasons: (1) t is dangling at the very beginning of the execution, i.e., $\{t\} \bowtie R_u = \{t\}$. Then, during the execution with t from \bowtie_k , join fails at \bowtie_u , and deleteDT() is initiated (Line 20). Since R_k is the parent of R_u , Algorithm 3.2 Line 10 is executed. (2) t becomes dangling after all tuples from $R_u \bowtie \{t\}$ are removed. After the last tuple in $R_u \bowtie \{t\}$ is removed by Line 24, MatchingTuples becomes empty at \bowtie_u . Line 29 is then called. Since $MatchingTuples = \emptyset$ and $R_u \bowtie \{t\} = \emptyset$, Line 15 is executed and returns nil. deleteDT() is initiated and Algorithm 3.2 Line 10 will be executed. \square

LEMMA 4.4. *When TTJ finishes execution, \mathcal{P}_Q is in clean state.*

PROOF. Satisfaction of Condition (i). Suppose R_i is a leaf relation. Since relations that have tuples removed or put into ng are parent of some other relations in \mathcal{T}_Q , condition holds.

Satisfaction of Condition (ii). Start with internal^o relations R_i that are parent of leaf relations R_u . Then, $\mathbb{R}_u = \widetilde{\mathbb{R}}_u$. By Lemma E.2 and parent-child relation between R_i and R_u , $(\mathbb{R}_i \bowtie J_{i+1}^*) \bowtie \widetilde{\mathbb{R}}_u$ is empty. Thus, $\mathbb{R}_i = \widetilde{\mathbb{R}}_i$ when TTJ finishes execution. Now, let R_i be an internal^o relation and R_u be its child, which is also an

internal^o relation. Start R_u be the parent of leaf relations and apply the same argument from the previous case. $\mathbb{R}_i = \widetilde{\mathbb{R}}_i$. Repeat the same argument all the way till R_u be the grandchild of R_k .

Satisfaction of Condition (ii). By Lemma 4.2, equivalently, we show $\mathbb{R}_k - ng = \mathbb{R}_k^*$. (1) $\mathbb{R}_k^* \subseteq \mathbb{R}_k - ng$. Suppose $t \notin \mathbb{R}_k - ng$. This means t is one of the tuples removed by Algorithm 3.2 Line 10. With Lemma G.1, $t \notin \mathbb{R}_k^*$. (2) $\mathbb{R}_k^* \supseteq \mathbb{R}_k - ng$. Suppose $t \notin \mathbb{R}_k^*$. Then, t has to be a dangling tuple causes a join failure at some relation R . By the proof of Lemma G.1, either deleteDT() is called directly (R is a child of R_k) or indirectly (R causes all tuples from R_u , a child of R_k , joining with t removed). Thus, $t \notin \mathbb{R}_k - ng$. \square

LEMMA 4.5. *TTJ evaluates \mathcal{P}_Q in $\mathcal{O}(n+r)$ once it is in clean state.*

PROOF. By Theorem 4.3, once \mathcal{P}_Q reaches clean state, no dangling tuple is produced by \bowtie_u for $u \in [k]$. Thus, no more calls on deleteDT(). There are k relations and $k-1$ join operators, open() takes $O(kn)$ as each operator is called once and takes $O(n)$ to build \mathcal{H} . It takes $O(k)$ getNext() calls to compute a tuple in J_1^* . Since each getNext() call takes $O(1)$, it takes $O(k)$ to compute one join result and $O(kr)$ for J_1^* . Thus, in total, we have $O(kn + kr) = \mathcal{O}(n+r)$. \square

H IMPROVING TTJ COMBINED COMPLEXITY

Theorem 4.6 gives $O(k^2n + kr)$ combined complexity. We can further improve it to $O(nk \log k + kr)$ by constraining the join order (Corollary 3.2). In particular, to decide join order, one pre-order traverses \mathcal{T}_Q and when multiple subtrees exist for a given relation in \mathcal{T}_Q , one breaks ties by visiting the largest subtree of any relation last [9]. Figure 16 shows an example.

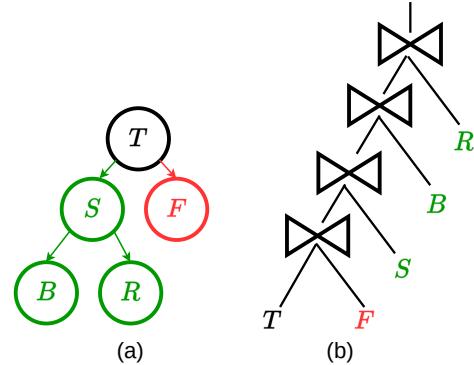


Figure 16: Given \mathcal{T}_Q in (a), one decides join order by pre-order traversing over \mathcal{T}_Q and breaking ties via visiting the largest subtree of any relation last. The resulting order (b) satisfies Corollary 3.2.

THEOREM H.1 (IMPROVING COMBINED COMPLEXITY OF TTJ). *Combined complexity of TTJ can be improved to $O(nk \log k + kr)$ (\log is base 2) if one performs pre-order traversal over \mathcal{T}_Q and break ties by visiting the largest subtree of any relation last.*

PROOF. The new order strategy only changes the total number of deleteDT() calls (Line 27) in Theorem 4.6 proof. For a given \mathcal{T}_Q , let b_i be the backjumping distance where the join failure relation

is R_i . Note that b_i is exactly the same as the number of `deleteDT()` calls generated (Line 27) when join fails at \bowtie_i . $b_i \leq k - i$ for the default join order. Let d_i denote the number of descendants of an internal relation R_i and m_i denotes the number of relations in the largest subtree rooted at one of i 's children, e.g., in Figure 16, $d_T = 4$ and $m_T = 3$. Since the new order satisfies Definition 1, when join fails at R_i , only descendants of R_j (the parent of R_i) could exist between R_i and R_j in the order. The largest number of `deleteDT()` generated when join fails at the root relation of the largest subtree of a relation. Thus, $b_i \leq d_i - m_i + 1$.

Next, we prove $\sum_{i=1}^{k-1} b_i \leq k \log k$. Proof by induction on the size of \mathcal{T}_Q . Base case $k = 1$, the claim holds. Assuming the claim holds for $k - 1$. Suppose there are s subtrees of R_k and each with size k_1, \dots, k_s . Let k_m denote the largest subtree. Then $b_r \leq (k - 1) - k_m + 1 = k - k_m$. Thus, $\sum_{i=1}^{k-1} b_i \leq \sum_{i=1}^s k_i \log k_i + (k - k_m) \leq k \log k_m + (k - k_m) \leq k \log k$ (the last inequality follows Lemma A.1 in [9]). Then, the total number of `deleteDT()` calls on Line 27 is $\leq \sum_{i=1}^{k-1} b_i n = O(nk \log k)$. \square

I BASELINES' COST MODELS

The cost model of HJ is the summation of intermediate results [27, 41]. Query plan and \mathcal{T}_Q are fixed for all compared algorithms on star schema queries. Thus, we do not cost LIP. PT shares the same \mathcal{T}_Q as YA. We detail the cost model of YA below.

The central idea of costing YA is exactly identical to how we cost TTJ in Appendix B. We first deduce the state of relations after F_Q called *full reducer state* (Definition 3), which is similar to clean state (Definition 2). Then, we compute the number of intermediate results produced by F_Q (Equations (13) to (15)), the size of the intermediate results that are part of the final join result (Equation (16)), and the size of the relations that are in full reducer state (Equation (17)) in YA cost equation (Theorem I.1).

Definition 3 (full reducer state). Query plan using F_Q reaches *full reducer state* if the following conditions hold:

- (i) $\mathbb{R}_k \bowtie \widetilde{\mathbb{R}}_u = \emptyset$ for the root of \mathcal{T}_Q , R_k and its children R_u . The content of R_k satisfying the condition is denoted by \mathbb{R}_k^* ;
- (ii) $\mathbb{R}_u \bowtie \mathbb{R}_i^* = \emptyset$ for all the leaf relations R_u of \mathcal{T}_Q and their parent R_i . The content of R_u satisfying the condition is denoted by \mathbb{R}_u^* . Furthermore, $\widetilde{\mathbb{R}}_u = \mathbb{R}_u$; and
- (iii) $(\mathbb{R}_i \bowtie \widetilde{\mathbb{R}}_u) \cup (\mathbb{R}_i \bowtie \mathbb{R}_j^*) = \emptyset$ for internal^o relation R_i , its child relations R_u , and its parent R_j . The content of R_i satisfying the condition is denoted by \mathbb{R}_i^* . If content of R_i satisfies $\mathbb{R}_i \bowtie \widetilde{\mathbb{R}}_u = \emptyset$ only, we denote the content of R_i as $\widetilde{\mathbb{R}}_i$.

THEOREM I.1. *The cost of \mathcal{T}_Q under YA is*

$$\sum_{u=1}^{|\mathcal{A}|} |\mathbb{R}_u^*| \quad (13)$$

$$+ \sum_{i=1}^s (|\mathbb{R}_i^*| + \sum_{t=0}^{|\mathcal{B}^i|-1} |\mathbb{R}_i^{[t]} \bowtie \widetilde{\mathbb{R}}_u^{t+1}|) \quad (14)$$

$$+ \sum_{t=0}^{|C|-1} |\mathbb{R}_k^{[t]} \bowtie \widetilde{\mathbb{R}}_u^{t+1}| \quad (15)$$

$$+ \sum_{i=k}^1 |f(S_i)| \quad (16)$$

$$+ \sum_{i=2}^k |\mathbb{R}_i^*| \quad (17)$$

We define the following three sets over the relations in \mathcal{T}_Q : (1) \mathcal{A} consists of all the leaf relations R_u ; (2) \mathcal{B} consists of R_u whose parents R_i are internal^o relations. We partition \mathcal{B} by the parent of R_u s. Then, we have $\mathcal{B}^1, \dots, \mathcal{B}^s$. $|\mathcal{B}^i|$ indicates the number of relations in \mathcal{T}_Q that are children of R_i . We label those relations $R_u^1, \dots, R_u^{|\mathcal{B}^s|}$; and (3) C comprises all the relations R_u that are children of R_k . The children of R_k are labeled $R_u^1, \dots, R_u^{|C|}$. Equation (18) defines $\mathbb{R}_i^{[t]}$, which reflects the gradual removal of dangling tuples of R_i during semijoins.

$$\mathbb{R}_i^{[t]} = \begin{cases} \mathbb{R}_i & \text{if } t = 0 \\ \mathbb{R}_i^{[t-1]} \bowtie \widetilde{\mathbb{R}}_u^t & \text{otherwise} \end{cases} \quad (18)$$

Suppose the join order (w.r.t. \mathcal{T}_Q) determined either syntactically from \mathcal{T}_Q or from the DP algorithm is $[S_k, \dots, S_1]$ where $S_j = R_i$ for some $i \in [k]$. $f(S_j)$ in Equation (19) computes the size of intermediate results that are part of the final join result.

$$f(S_i) = \begin{cases} \mathbb{S}_i^* & \text{if } i = k \\ \mathbb{S}_i^* \bowtie f(S_{i+1}) & \text{otherwise} \end{cases} \quad (19)$$

J EMPIRICAL STUDY OF PT PERFORMANCE ON THE PREDICATE TRANSFER GRAPH VERSUS \mathcal{T}_Q

Predicate transfer graph is a directed query graph. PT construct the predicate transfer graph from the query graph using a simple heuristic: for an edge of two relations, the head of the edge is the relation with bigger size. For star schema queries in our setup where R_k is the fact table, predicate transfer graph is identical to \mathcal{T}_Q : query graph is identical to the undirected join tree and the heuristic applied on the query graph leads to \mathcal{T}_Q . Thus, PT on the predicate transfer graph (denoted by PTO) has identical performance as PT on \mathcal{T}_Q on star schema queries. PTO can have a different performance compared with PT when one of the following conditions happen: (1) the query graph is not identical to an undirected join tree; (2) when undirected join tree and the query graph are identical, \mathcal{T}_Q created from costing is different from the predicate transfer graph created by the heuristics; and (3) both \mathcal{T}_Q and the predicate transfer graph are identical but the order of passing Bloom filters are different.

Table 3: Speedup of PT and PTO compared with HJ on Q7 and Q8 in TPC-H

Method	Q7	Q8
PT	1.6x	1.6x
PTO	0.6x	0.7x

We empirically compare PT and PTO on Q7 and Q8 in TPC-H. The performance result is shown in Table 3. In Q7, condition (2) happens where PT and PTO have different tree structures. In Q8, condition (3) happens where both PT and PTO share the same \mathcal{T}_Q but Bloom filters are applied in different orders.

K RUNTIME BREAKDOWN OF SSB QUERIES

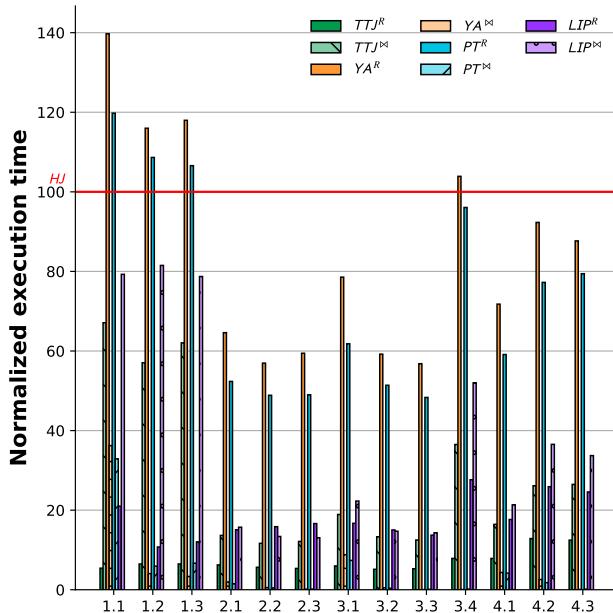
**Figure 17: Breakdown of TTJ, YA, PT, and LIP execution time into dangling tuples removal (e.g., TTJ^R) and join (e.g., TTJ^M) on SSB**

Figure 17 shows the runtime breakdown of the compared algorithms on SSB. The figure illustrates that LIP spends less time on dangling tuple removal than YA and PT but more than TTJ.

L FORMAL DEFINITION ON MODIFIED SEMIJOIN SELECTIVITY θ

Suppose S has m children in \mathcal{T}_Q : S_1, \dots, S_m . S corresponds to R_i in \mathcal{P}_Q . Then, θ_S is defined in (20) where $\bowtie_{j=k}^{i+1} R_j$ is a shorthand for $R_k \bowtie \dots \bowtie R_{i+1}$ in \mathcal{P}_Q and $\bowtie_{q=2}^p \tilde{S}_{q-1}$ is a shorthand for $\tilde{S}_1 \bowtie \dots \bowtie \tilde{S}_{p-1}$.

$$\frac{\sum_{p=1}^m |S \bowtie (\bowtie_{j=k}^{i+1} R_j) \bowtie (\bowtie_{q=2}^p \tilde{S}_{q-1}) \bowtie (S_p \overline{\bowtie} \tilde{S}_p)|}{|S|} \quad (20)$$

M BOUND ADDITIONAL DANGLING INTERMEDIATE RESULTS PRODUCED BY TTJ^{bj-} COMPARED WITH TTJ^{bj+} IN § 5.4.3

THEOREM M.1. TTJ^{bj-} produces $(n-2) \sum_{j=0}^{k-3} (n-1)^j = O(kn^k) = \mathcal{O}(n^k)$ more dangling intermediate results than TTJ^{bj+} for Query (5) when $k \geq 3$ and $n \geq 2$.

PROOF. Fix an arbitrary $k \geq 3$ and $n \geq 2$. Compared with TTJ^{bj+} , R_{k-1} generates $n-2$ additional dangling intermediate results because $(k, k+1)$ is part of the final join result, first copy of $(k-1, k)$ as dangling intermediate results is generated by both TTJ^{bj-} and TTJ^{bj+} . The rest $n-2$ copies of $(k-1, k)$ is generated by TTJ^{bj-} but avoided by TTJ^{bj+} due to backjumping. For R_{k-2} , TTJ^{bj-} generates $(n-2)(n-1)$ more dangling intermediate results than TTJ^{bj+} because $n-2$ copies of $(k-2, k-1)$ will be selected by TTJ^{bj-} during join and each of those selected tuples will further join with every $n-1$ copies of $(k-1, k)$ in R_{k-1} . Applying the same reasoning, TTJ^{bj-} on R_{k-3} will generate $(n-2)(n-1)^2$ more dangling intermediate results than TTJ^{bj+} and so on. Summing up all the additional dangling intermediate results generated by TTJ^{bj-} on each relation from R_2 till R_{k-1} , we have $(n-2) \sum_{j=0}^{k-3} (n-1)^j \leq \sum_{j=1}^k n^j$. The result follows. \square