Engineering Mathematics Spring 2022

Problem Set 3

Instructor: Yongsoo Song **Due on:** May 27, 2022

Please submit your answer through eTL. It should be a single PDF file (not jpg), either typed or scanned. **Please include your student ID and name (e.g. 2020-12345 YongsooSong.pdf).** You may discuss with other students the general approach to solve the problems, but the answers should be written in your own words. You should cite any reference that you used, and mention what you used it for.

Problem 1 (20 points)

Solve the following ODEs:

(a) $x^2y'' - 4xy' + 6y = 0$, $y(1) = 0.4$, $y'(1) = 0$

(b) $x^2y'' + 3xy' + y = 0$, $y(1) = 3.6$, $y'(1) = 0.4$

Problem 2 (30 points)

Assume that $y'' + p(x)y' + q(x)y = 0$ is a homogeneous linear ODE whose coefficients p, q are continuous on some open interval I, and let y_1, y_2 be solutions of the ODE. Prove Abel's formula:

$$W(y_1, y_2) = c \cdot \exp\left[-\int_{x_0}^{x} p(t)dt\right]$$

where $c = W(y_1(x_0), y_2(x_0))$.

Problem 3 (40 points)

Solve the following IVPs:

(a) $y'' + 4y' + 4y = e^{-2x}\sin 2x$, $y(0) = 1$, $y'(0) = -1.5$.

(b) $y'' + 9y = \sec 3x$.

(c) $y'' + 6y' + 9y = 16e^{-3x}/(x^2 + 1)$.

(d) $x^2y'' - 4xy' + 6y = 21x^{-4}$.

Problem 4 (40 points)

Let $y^{(n)} + a_{n-1}y^{(n-1)} + \cdots + a_1y' + a_0y = 0$ be an n-th order homogeneous linear ODE with constant coefficients. Find n solutions and show their linear independence.

Problem 5 (30 points)

Solve the following ODEs.

(a) $y''' + 2y'' - y' - 2y = 1 - 4x^3$

(b) $x^3y''' + xy' - y = x^2$, $y(1) = 1$, $y'(1) = 3$, $y''(1) = 14$

(c) $y''' - 2y'' - 9y' + 18y = e^{2x}$, $y(0) = 4.5$, $y'(0) = 8.8$, $y''(0) = 17.2$.

Problem 6 (40 points)

Draw the phase portraits of the following ODEs (programming allowed). Find solutions using initial conditions and highlight their trajectories. What are the types of critical points?

(a) $y_1' = 2y_1 + 2y_2, y_2' = 5y_1 - y_2$, $y_1(0) = 0, y_2(0) = 7$.

(b) $y_1' = 3y_1 + 2y_2, y_2' = 2y_1 + 3y_2, y_1(0) = 0.5, y_2(0) = -0.5$.