Engineering Mathematics 2
Seoul National University

Homework 1

Exercise 1.4

We are playing a tournament in which we stop as soon as one of us wins n games. We are evenly matched, so each of us wins any game with probability $1/2$, independently of other games. What is the probability that the loser has won k games when the match is over?

Exercise 1.6

Consider the following balls-and-bin game. We start with one black ball and one white ball in a bin. We repeatedly do the following: choose one ball from the bin uniformly at random, and then put the ball back in the bin with another ball of the same color. We repeat until there are n balls in the bin. Show that the number of white balls is equally likely to be any number between 1 and $n-1$.

Exercise 1.8

I choose a number uniformly at random from the range $[1, 1\,000\,000]$. Using the inclusion-exclusion principle, determine the probability that the number chosen is divisible by one or more of 4, 6, and 9.

Exercise 1.15

Suppose that we roll ten standard six-sided dice. What is the probability that their sum will be divisible by 6, assuming that the rolls are independent? (*Hint:* Use the principle of deferred decisions, and consider the situation after rolling all but one of the dice.)

Exercise 1.18

We have a function $F : \{0, \ldots, n-1\} \to \{0, \ldots, m-1\}$. We know that, for $0 \le x, y \le n-1$, $F((x + y) \bmod n) = (F(x) + F(y)) \bmod m$. The only way we have for evaluating F is to use a lookup table that stores the values of F. Unfortunately, an Evil Adversary has changed the value of $1/5$ of the table entries when we were not looking.

Describe a simple randomized algorithm that, given an input z, outputs a value that equals $F(z)$ with probability at least $1/2$. Your algorithm should work for every value of z, regardless of what values the Adversary changed. Your algorithm should use as few lookups and as little computation as possible.

Suppose I allow you to repeat your initial algorithm three times. What should you do in this case, and what is the probability that your enhanced algorithm returns the correct answer?

Exercise 1.23

There may be several different min-cut sets in a graph. Using the analysis of the randomized min-cut algorithm, argue that there can be at most $n(n-1)/2$ distinct min-cut sets.