CSE 4190.101: Discrete Mathematics	Fall 2021
Problem Set 1	
Instructor: Yongsoo Song	**Due on:** Oct 04, 2021

Please submit your answer through eTL. It should be a single PDF file (not jpg), either typed or scanned. **Please include your student ID and name (e.g. 2020-12345 YongsooSong.pdf).** You may discuss with other students the general approach to solve the problems, but the answers should be written in your own words. You should cite any reference that you used, and mention what you used it for. You should follow the academic integrity rules that are described in the course information.

Problem 1 (5 points)

Let the following statements be given:

(p) "You can install the program"

(q) "Your computer has **less** than 2GB of RAM"

(r) "Your computer has 20GB of free disk space"

1. Translate the following statement into symbols of formal logic:

   "In order to install the program you need at least 2GB of RAM and 20GB of free disk space"

2. Give the *converse* of this statement in the symbols of formal logic.
3. Give the *converse* in English
4. Give the *contrapositive* (of the original statement) in the symbols of formal logic
5. Give the *contrapositive* in English

Problem 2 (5 points)

Determine if the same is true for the implication operation ($\rightarrow$) i.e., is the expression $(P \rightarrow Q) \rightarrow R$ logically equivalent to $P \rightarrow (Q \rightarrow R)$? Prove or disprove using the truth table method.

Problem 3 (5 points)

Construct a compound proposition that asserts that every cell of a 9×9 Sudoku puzzle contains at least one number (using the propositions $p(i, j, n)$ defined in the lecture note).

Problem 4 (10 points)

You are on a treasure island, and find a note with the following hints:

1. If this house is next to a lake, then the treasure is not in the kitchen
2. If the tree in the front yard is an elm, then the treasure is in the kitchen

3. This house is next to a lake
4. Either the tree in the front yard is an elm or the treasure is not buried under the flagpole (or both. This "or" is inclusive.)
5. The treasure is either under the flagpole or in the garage. (but not both. This is an exclusive or, there is only one treasure!)

In order to find where the treasure is, do the following:

(a) Identify 5 atomic statements that can be used to express the above statements in logical form. (Atomic statements should **not** contain any word corresponding to a logical connective.) List the 5 statements in the order in which they appear in the above messages, and call them L, K, E, P, G. Express each of the above compound statements (1-5) using L, K, E, P, G and logical connectives.

(b) Determine where the treasure is, and justify your answer.

Problem 5 (10 points)

Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide a counterexample for the statements that are false. In all statements, the variables x, y range over the set D.

1. $\forall x$, if x is odd then $x > 0$.
2. $\forall x$, if $x < 0$ then x is even
3. $\forall x$, $\exists y$ such that $y > x$
4. $\forall x$, (x is even or $\exists y, y > x$)
5. $\forall x$, (x is odd or $\exists y, y > x$)

Problem 6 (5 points)

Prove the following statement by contraposition:

"For any integer n, if $(n^2 + n + 1)$ is even, then n is odd."

Problem 7 (10 points)

Suppose that a, b and c are odd integers. Assume that a real number x satisfies the equation $ax^2+bx+c = 0$. Prove by contradiction that x is irrational.

Problem 8 (10 points)

Prove or disprove that you can tile a 10×10 checkerboard using straight tetrominoes.

Problem 9 (10 points)

For each of the following statements, determine if it is true or false. If true, prove it. Otherwise, disprove it by giving a counterexample.

(a) For any sets A, B, C, if $A \cup C \subseteq B \cup C$, then $A \subseteq B$.

(b) For any sets A, B, C, if $A \cup C \subseteq B \cup C$ and $A \cap C \subseteq B \cap C$, then $A \subseteq B$.

Problem 10 (10 points)

Let F be the set of all nonempty finite sets of integers, and let $f : F \to \mathbb{Z}$ be the function $f(S) = \sum_{x \in S} x$ mapping each set to the sum of its elements. So, for example, $f(\{1, -7, 9\}) = 3$.

(a) Determine if f is surjective, and prove your answer.

(b) Determine if f is injective, and prove your answer.

Problem 11 (10 points)

Let $f : \mathbb{N}^2 \to \mathbb{Z}$ be the function $f(x, y) = x - y$, where $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ is the set of positive integers.

(a) Show that $f : \mathbb{N}^2 \to \mathbb{Z}$ is not a bijection.

(b) Give a subset $S \subseteq \mathbb{N}^2$ such that $f : S \to \mathbb{Z}$ is a bijection, and prove the correctness of your answer.

Problem 12 (10 points)

Show that if S is a set, then there does not exist an onto function f from S to $\mathcal{P}(S)$, the power set of S. Conclude that $|S| < |\mathcal{P}(S)|$. [Hint: Suppose such a function f existed and consider the set $T = \{s \in S | s \notin f(s)\}$.]