EXERCISE 12.2 For each of the circuits in Figure 12.66, find and sketch the indicated zero-input response corresponding to the indicated initial conditions

a) In Figure 12.66, find v_2, assuming $v_1(0) = 1$ V, $v_2(0) = 0$

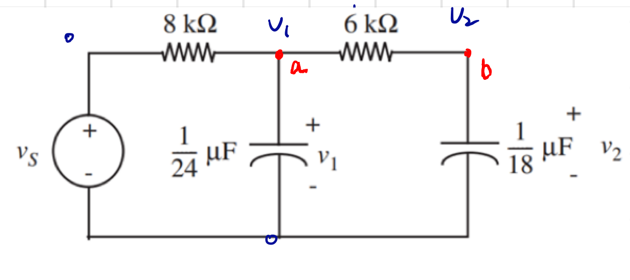


FIGURE 12.66

① KCL at a.

$$\frac{V_1}{8} + \frac{V_1 - V_2}{6} + \frac{1}{24}\frac{dV_1}{dt} = 0 \Rightarrow 7V_1 - 4V_2 + \frac{dV_1}{dt} = 0 \quad \ldots ①$$

② KCL at b.

$$\frac{V_1 - V_2}{6} = \frac{1}{18}\frac{dV_2}{dt} \Rightarrow V_1 = V_2 + \frac{1}{3}\frac{dV_2}{dt} \quad \ldots ②$$

from ① & ②

$$\frac{d^2V_2}{dt^2} + 10\frac{dV_2}{dt} + 9V_2 = 0$$

$$V_2(t) = A_1 e^{-9t} + A_2 e^{-t} \quad \ldots ③$$

$$V_2(0) = A_1 + A_2 = 0 \quad \ldots ④$$

from ② & ③

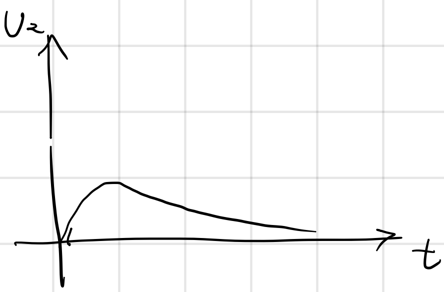
$$V_1(t) = A_1 e^{-9t} + A_2 e^{-t} + \frac{1}{3}(-9A_1 e^{-9t} - A_2 e^{-t})$$

$$V_1(0) = A_1 + A_2 - 3A_1 - \frac{1}{3}A_2 = 1 \Rightarrow -6A_1 + 2A_2 = 3 \quad \ldots ⑤$$

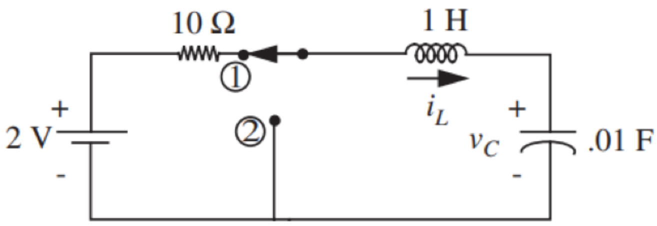
from ④ & ⑤

$$A_1 = -\frac{3}{8} \qquad A_2 = \frac{3}{8}$$

$$\therefore V_2(t) = -\frac{3}{8}e^{-9t} + \frac{3}{8}e^{-t} = \frac{3}{8}e^{-t}(1 - e^{-8t})$$



PROBLEM 12.6 In the circuit in Figure 12.77, the switch has been in position 1 for all $t < 0$. At $t = 0$, the switch is moved to position 2 (and remains there for $t > 0$). Find and sketch $v_C(t)$ and $i_L(t)$ for $t > 0$.



$V_c(0^-) = V_c(0^+) = 2$, $\quad i_L(0^-) = i_L(0^+) = 0$

when $t > 0$, $\quad i_L = C\frac{dV_c}{dt} \quad \cdots ①$

$V_c + L\frac{di_L}{dt} = 0 \quad \cdots ②$

from ① & ② $\quad i_L = -LC\frac{d^2 i_L}{dt^2} \Rightarrow \frac{d^2 i_L}{dt^2} + \frac{1}{LC} i_L = 0 \Rightarrow \frac{d^2 i_L}{dt^2} + 100\, i_L = 0$.

$i_L = A_1 e^{10jt} + A_2 e^{-10jt} \quad \cdots ③$

$i_L(0^+) = A_1 + A_2 = 0 \quad \cdots ④$

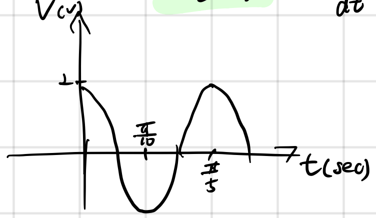
from ② & ③ $\quad V_c = -\left(10jA_1 e^{10jt} - 10jA_2 e^{-10jt}\right)$

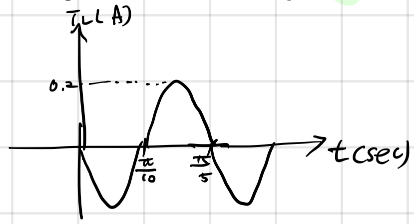
$V_c(0^+) = -10jA_1 + 10jA_2 = 2 \Rightarrow A_1 - A_2 = -0.2j$

from ④ & ⑤ $\quad A_1 = -0.1j \quad A_2 = 0.1j$

$\therefore V_c(t) = e^{10jt} + e^{-10jt} = 2\cos 10t$

$\therefore i_L(t) = C\frac{dV_c}{dt} = 0.01 \times (-2 \times 10 \times \sin 10t) = -0.2 \sin 10t$





EXERCISE 13.13 In the network shown in Figure 13.72,

$R = 1\text{ k}\Omega \quad C_1 = 20\ \mu\text{F} \quad C_2 = 20\ \mu\text{F}.$

a) Determine the magnitude and phase of $H(j\omega)$, the transfer function relating V_0/V_i.

b) Given $v_i(t) = \cos(100t) + \cos(10000t)$, determine the sinusoidal steady state output voltage, $v_0(t)$.

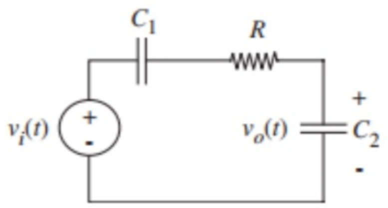
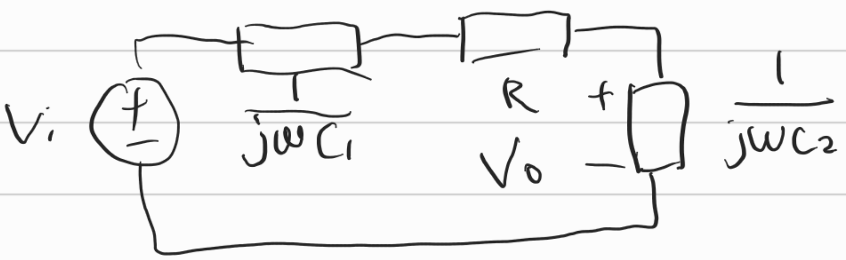


FIGURE 13.72

(a) Determine the magnitude and phase of $H(j\omega)$, the transfer function relating V_0/V_i.

The impedance circuit is as follows:



By the impedance method,

$$\frac{V_o}{V_i} = \frac{\frac{1}{j\omega C_2}}{R + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2}} = \frac{\frac{1}{20\mu j\omega}}{1k + \frac{1}{20\mu j\omega} + \frac{1}{20\mu j\omega}}$$

$$= \frac{1}{2 + \frac{j\omega}{50}} = \frac{1}{2} \times \frac{1}{1 + \frac{j\omega}{100}}$$

Magnitude: $\left|\frac{V_o}{V_i}\right| = \frac{1}{2\sqrt{1 + \frac{\omega^2}{100^2}}}$

Phase: $\angle\left|\frac{V_o}{V_i}\right| = \phi = \tan^{-1}\left(-\frac{\omega}{100}\right)$

(b) Given $V_i(t) = \cos(100t) + \cos(10000t)$, determine the sinusoidal steady state output voltage, $V_o(t)$

From the conculusion of (a), we get

$$V_o(t) = \frac{1}{2\sqrt{1+\frac{\omega^2}{100^2}}} \cos(\omega t + \phi), \text{ where } \tan\phi = -\frac{\omega}{100}$$

By the superposition principle, we can calculate the contributions of $\cos(100t)$ and $\cos(10000t)$ separately

① $\cos(100t) \rightarrow \omega = 100$

$\tan\phi = -1 \Rightarrow \phi = -\frac{\pi}{4}$

$$V_{o1}(t) = \frac{1}{2\sqrt{2}} \cos\left(100t - \frac{\pi}{4}\right)$$

② $\cos(10000t) \rightarrow \omega = 10000$

$\tan\phi = -100$

$$V_{o2}(t) = \frac{1}{2\sqrt{10001}} \cos\left(10000t - \tan^{-1}(-100)\right)$$

By ①&②,

$$V_o(t) = \frac{1}{2\sqrt{2}} \cos\left(100t - \frac{\pi}{4}\right) + \frac{1}{2\sqrt{10001}} \cos\left(10000t - \tan^{-1}(-100)\right)$$

PROBLEM 13.4 Refer to Figure 13.80 for this problem. Assume $R_1 = 1\ k\Omega$ and $L_1 = 10\ mH$.

a) Find the transfer function $H(j\omega) = V_1/V_o$.

b) Find R so that the DC gain is 1/10.

c) Find a value of L so that the response at high frequencies is equal to response at DC.

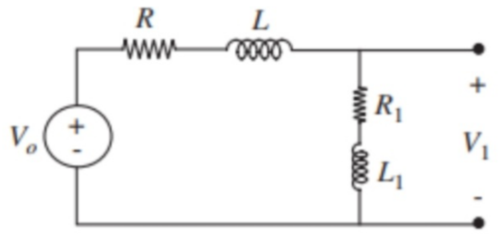


FIGURE 13.80

(a) Find the transfer function $H(j\omega) = V_1/V_o$

$$\frac{V_1}{V_o} = \frac{R_1 + j\omega L_1}{R + j\omega L + R_1 + j\omega L_1}$$

(b) Find R so that the DC gain is 1/10

We can get the DC gain by substituting ω with $\omega = 0$

$$\text{DC gain} = \frac{R_1}{R + R_1} = \frac{1}{R+1} = \frac{1}{10}$$

$\Rightarrow R = 9\ k\Omega$

(c) Find a value of L so that the response at high frequencies is equal to response at DC

High frequencies : $\omega \to \infty$

Note that as $\omega \to \infty$, $L_1/L_0 \to \dfrac{L_1}{L + L_1} = \dfrac{10}{L + 10}$

$$\frac{10}{L+10} = \frac{1}{10} \quad \Rightarrow L = 90\ mH$$

EXERCISE 15.12 Find and label clearly the Thévenin equivalent for the network in Figure 15.53.

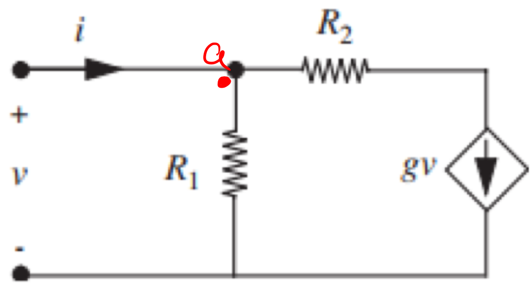


FIGURE 15.53

- To get V_{th}. assume $i=0$.

  then $R_1 g V_{th} = V_{th}$

  In order to satisfy the equation regardless R_1 and g,

  V_{th} should be zero.

$\therefore V_{th} = 0$

- To get R_{th}

  apply KCL at a, then $i = gv + \frac{v}{R_1} = v\left(g+\frac{1}{R_1}\right)$

$\therefore R_{th} = \frac{v}{i} = \frac{v}{v\left(g+\frac{1}{R_1}\right)} = \frac{R_1}{1+R_1 g}$

Exercise 15.13 Find i in terms of v for the linear network in Figure 15.18. Assume an idealized operational amplifier.

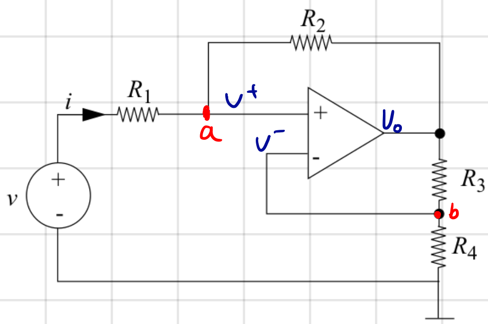


Figure 15.18:

$\bar{i} = \frac{V - V^+}{R_1}$, we need to find V^+

• KCL at a, $\frac{V - V^+}{R_1} = \frac{V^+ - V_o}{R_2}$ ... ①

• at b, $V^- = \frac{R_4}{R_3 + R_4} V_o$

Because $V^+ = V^-$ at an idealized op Amp, $V^+ = \frac{R_4}{R_3 + R_4} V_o$

$\Rightarrow \frac{R_3 + R_4}{R_4} V^+ = V^o$ ... ②

from ① & ②, $V^+ = \left(\frac{R_2 R_4}{R_2 R_4 - R_1 R_3}\right) \cdot V$

$\therefore \bar{i} = \frac{V - V^+}{R_1} = \frac{V}{R_1}\left(\frac{\cancel{R_2 R_4} - R_1 R_3 - \cancel{R_2 R_4}}{R_2 R_4 - R_1 R_3}\right) = \frac{-R_3}{R_2 R_4 - R_1 R_3} \cdot V = \frac{R_3}{R_1 R_3 - R_2 R_4} \cdot V$

EXERCISE 15.24 An operational amplifier is connected as shown in Figure 15.65.

a) What is the gain of the amplifier for $\omega = 0$?.

b) Find the expression for $V_o(j\omega)/V_i(j\omega)$.

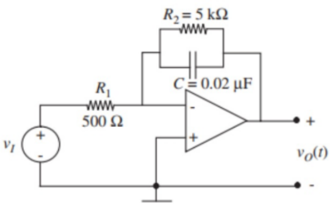


FIGURE 15.65

$$\frac{V_I}{R_1} = -\frac{V_O}{R_2} - \frac{V_O}{\frac{1}{j\omega C}} = -\left(\frac{1}{R_2} + j\omega C\right)V_O$$

$$\therefore \frac{V_O}{V_I} = -\frac{1}{R_1}\cdot\frac{1}{\frac{1}{R_2}+j\omega C}$$

$$= -\frac{1}{R_1}\cdot\frac{R_2}{1+j\omega C R_2}$$

(a) $\omega = 0$, gain $= \dfrac{V_O}{V_I} = -\dfrac{R_2}{R_1} = \dfrac{-5\cdot 10^3}{5\cdot 10^2} = \boxed{-10}$

(b) $$\frac{V_O(j\omega)}{V_I(j\omega)} = -\frac{1}{R_1}\cdot\frac{R_2}{1+j\omega C R_2}$$

$$= \boxed{-\frac{10}{1+j\omega\cdot 0.1}}$$

$\left\{\begin{array}{ll} \text{unit: } R_1, R_2 & (k\Omega) \\ C & (\mu F) \\ \omega & (\text{rad/msec}) \end{array}\right.$

PROBLEM 15.34 The circuit shown in Figure 15.105 behaves like an RLC circuit.

a) Find the transfer function V_4/V_1. (You may assume that the Op Amp is ideal, that is $V^+ = V^-$ to simplify your calculations.)

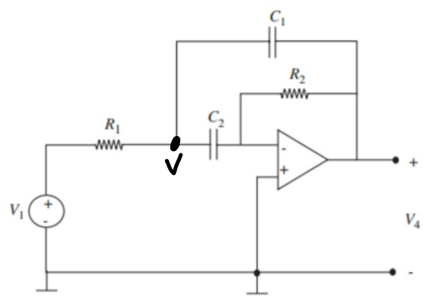


FIGURE 15.105

c) This circuit is known as an RC active filter. Is it a low-pass, high-pass, or band-pass filter? What is the expression for bandwidth in terms at R_1, C_1, etc.? That is, $B = \omega_2 - \omega_1$ where ω_1 and ω_2 are the half power frequencies.

① $$\frac{V_1 - V}{R_1} = \frac{V - V_4}{\frac{1}{j\omega C_1}} + \frac{V}{\frac{1}{j\omega C_2}}$$

$$\Rightarrow \frac{V_1 - V}{R_1} = (j\omega C_1 + j\omega C_2)V - j\omega C_1 V_4$$

$$\Rightarrow V_1 = (j\omega R_1 C_1 + j\omega R_1 C_2 + 1)V - j\omega R_1 C_1 V_4$$

② $$\frac{V}{\frac{1}{j\omega C_2}} = -\frac{V_4}{R_2}$$

$$\Rightarrow j\omega C_2 V = -\frac{V_4}{R_2} \Rightarrow V = -\frac{V_4}{j\omega R_2 C_2}$$

a) Replace V in ① with V in ②,

$$V_1 = -V_4\left(\frac{j\omega R_1 C_1 + j\omega R_1 C_2 + 1 - \omega^2 R_1 R_2 C_1 C_2}{j\omega R_2 C_2}\right)$$

$$\frac{V_4}{V_1} = -\frac{j\omega R_2 C_2}{j\omega R_1 C_1 + j\omega R_1 C_2 + 1 - \omega^2 R_1 R_2 C_1 C_2}$$

$$\boxed{= -\frac{1}{R_1\left(\frac{C_1 + C_2}{R_2 C_2}\right) + j\left(R_1 C_1 \omega - \frac{1}{\omega R_2 C_2}\right)}}$$

c) $$\left|\frac{V_4}{V_1}\right| = \frac{1}{\sqrt{\left\{R_1\left(\frac{C_1+C_2}{R_2C_2}\right)\right\}^2 + \left(R_1C_1\omega - \frac{1}{\omega R_2C_2}\right)^2}}$$

$\left|\frac{V_4}{V_1}\right|$ goes to 0 when $\begin{cases} \omega \to 0 \\ \omega \to \infty \end{cases}$

$\therefore$ Band pass filter

To find bandwith with half-power frequencies,

$$\omega R_1C_1 - \frac{1}{\omega R_2C_2} = \pm R_1 \frac{C_1+C_2}{R_2C_2}$$

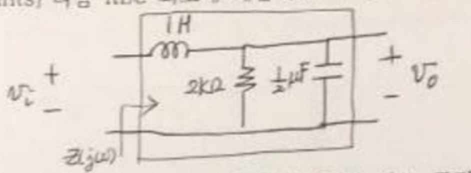
$$\Rightarrow \omega^2 \pm \frac{C_1+C_2}{R_2C_1C_2}\omega - \frac{1}{R_1R_2C_1C_2} = 0$$

Let $A = \frac{C_1+C_2}{R_2C_1C_2}$, $B = -\frac{1}{R_1R_2C_1C_2}$

$$\omega_1, \omega_2 = \frac{1}{2}\left(\pm A + \sqrt{A^2-4B}\right) \quad \left(\begin{matrix}\omega_1, \omega_2 > 0 \\ \sqrt{A^2-4B} > A\end{matrix}\right.$$

$$\therefore \boxed{\omega_2 - \omega_1 = A = \frac{C_1+C_2}{R_2C_1C_2}} \ (\mathrm{rad/msec})$$

1. [20 points] 다음 RLC 회로에 대한 물음에 답하여라.



(a) [10 points] 등가 임피던스가 실수가 되도록 하는 공진 주파수 ω 를 구하여라. Find the resonance frequency (ω) to make the equivalent impedance be real.

$$Z(jw) = jwL + \left(2 // \frac{1}{jwC}\right) \Rightarrow L = 1,\ C = \frac{1}{2}$$

$$= jw + \frac{2}{1+jw} = jw + \frac{2(1-jw)}{1+w^2} = jw + \frac{2}{1+w^2} - \frac{2jw}{1+w^2}$$

$$= \left(w - \frac{2w}{1+w^2}\right)j + \frac{2}{1+w^2}$$

$$w = \frac{2w}{1+w^2} \rightarrow w^3 = w \rightarrow w = 1\ (\text{rad/ms})$$

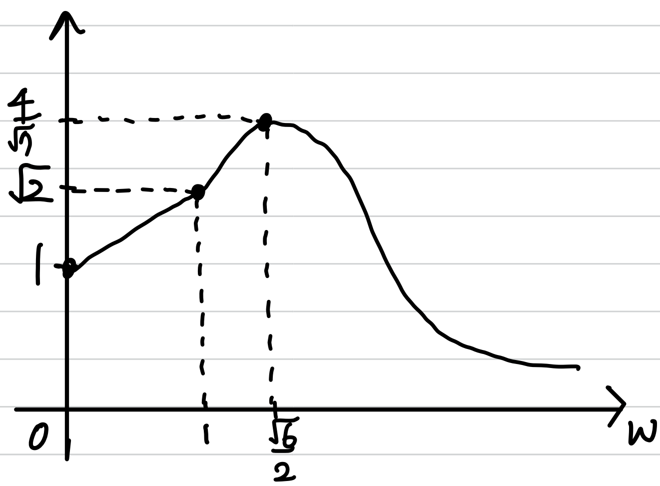
(b) [10 points] 이 회로의 주파수 응답 $H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}$ 을 구하고 크기$|H(j\omega)|$ 를 근사적으로 도시하여라. Find the frequency response of the circuit, $H(j\omega)=\frac{V_o(j\omega)}{V_i(j\omega)}$ and sketch the magnitude plot $|H(j\omega)|$. Hint: Find the values at ($\omega=0, \infty$, resonance frequency)

$$H(jw) = \frac{2 // \frac{1}{jwC}}{jwL + \left(2 // \frac{1}{jwC}\right)} = \frac{\frac{2}{1+jw}}{jw + \frac{2}{1+jw}} = \frac{2}{2 + jw(1+jw)}$$

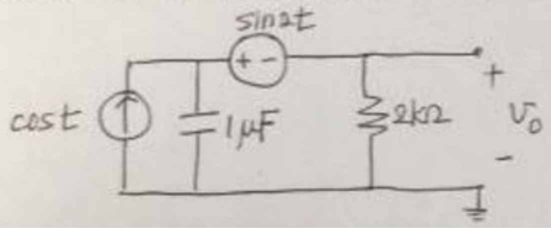
$$= \frac{2}{2 - w^2 + jw}$$

$$\therefore |H(jw)| = \frac{2}{\sqrt{(2-w^2)^2 + w^2}}$$

$$\therefore \begin{cases} w \to 0 & : 1 \\ w \to \infty & : 0 \\ w \to 1 & : \sqrt{2} \\ w \to \frac{\sqrt{6}}{2} & : \frac{4}{\sqrt{7}} \end{cases}$$

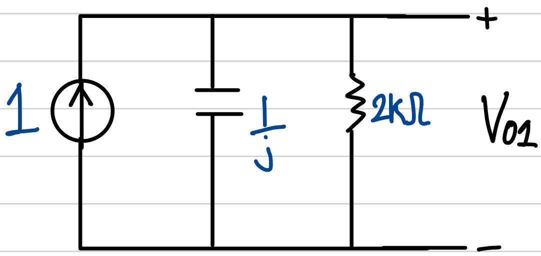


2. [10 points] 다음 회로에서 sinusoidal steady state 에서의 출력 전압을 구하여라.



Use superposition.

① $\cos t = Re\left(e^{jt}\right)$

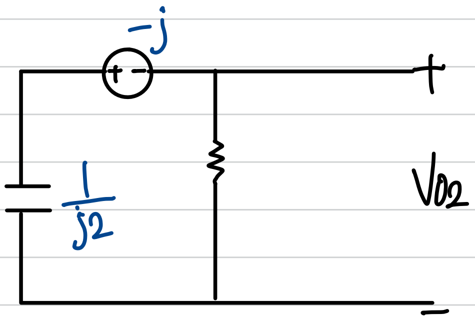


$$V_{o1} = \left(\frac{1}{j} // 2\right)\cdot 1 = \frac{2}{1+j2}$$

$$\therefore V_{o1}(t) = \frac{2}{\sqrt{5}}\cos(t-\theta_1),\quad \tan\theta_1 = 2$$

② sin2t

$$\sin 2t = \cos\left(2t-\frac{\pi}{2}\right) = Re\left(e^{j\left(2t-\frac{\pi}{2}\right)}\right) \Rightarrow -j$$



$$V_{o2} = \frac{2}{2+\frac{1}{2j}}\cdot j = \frac{2j}{2+\frac{1}{2j}} = \frac{-4}{4j+1} = \frac{4}{-1-4j}$$

$$\therefore V_{o2}(t) = \frac{4}{\sqrt{17}}\cos(2t-\theta_2),\quad \tan\theta_2 = 4$$

$$\therefore V_o(t) = V_{o1}(t) + V_{o2}(t) = \frac{2}{\sqrt{5}}\cdot\cos(t-\theta_1) + \frac{4}{\sqrt{17}}\cos(2t-\theta_2),\quad \tan(\theta_1)=2,\ \tan(\theta_2)=4$$