Homework # 1
Due date: Sept. 27 (Tuesday)

[Textbook]
Prob. 1.2

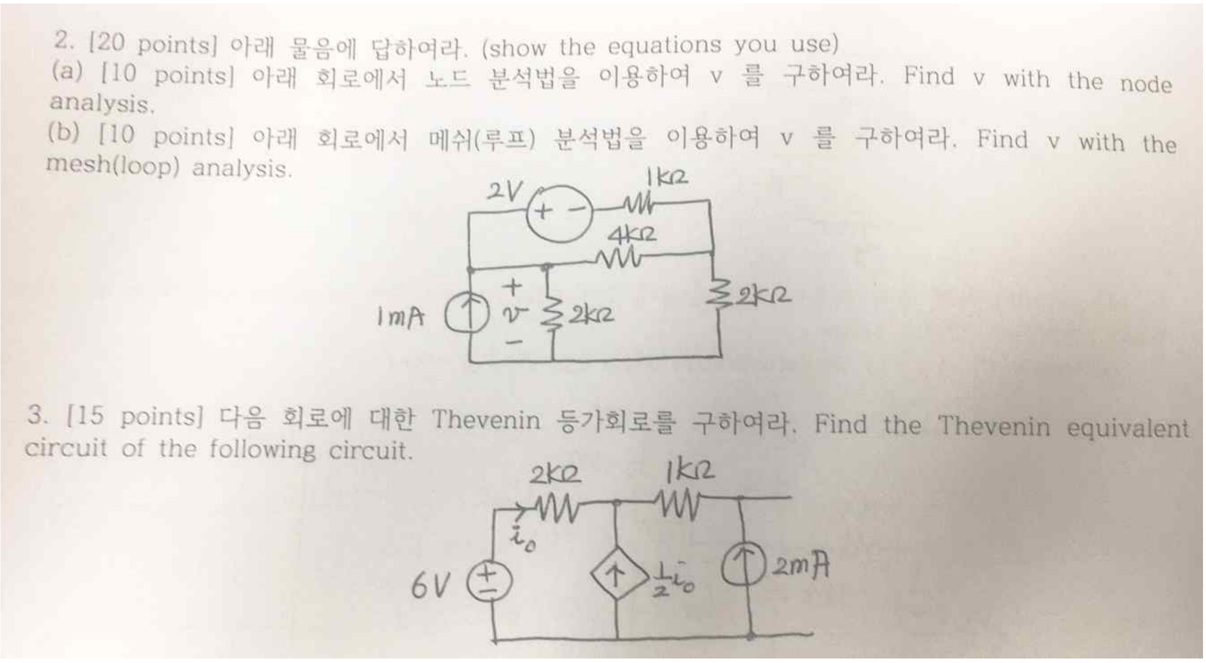
Exercise 2.3 (d), 2.8 (c)

Problem 2.9, 2.11 (b)

Exercise 3.10, 3.16, 3.25

Problem 3.3, 3.6

[Previous exams] 2019. $1^{st}$ exam



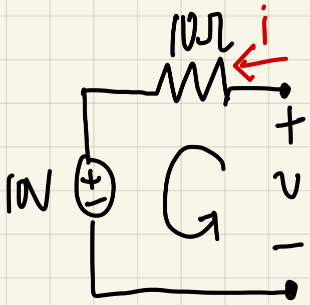
2. [20 points] 아래 물음에 답하여라. (show the equations you use)
(a) [10 points] 아래 회로에서 노드 분석법을 이용하여 v 를 구하여라. Find v with the node analysis.
(b) [10 points] 아래 회로에서 메쉬(루프) 분석법을 이용하여 v 를 구하여라. Find v with the mesh(loop) analysis.



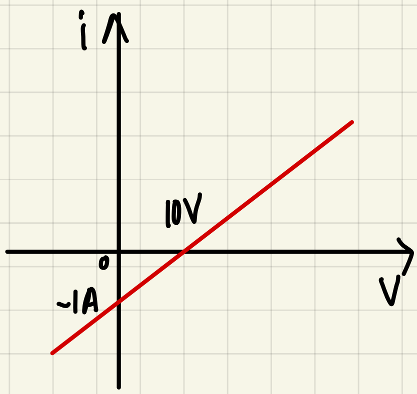
3. [15 points] 다음 회로에 대한 Thevenin 등가회로를 구하여라. Find the Thevenin equivalent circuit of the following circuit.

# Problem 1.2

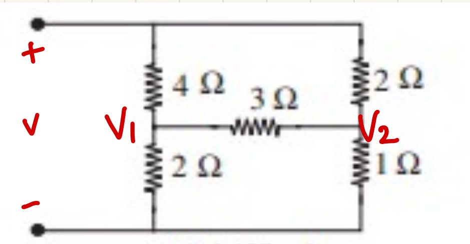
**PROBLEM 1.2** Sketch the $v$–$i$ characteristic of a battery rated at 10 V with an internal resistance of 10 Ohms.




$v = 10i + 10$




# Exercise 2.3 (d)



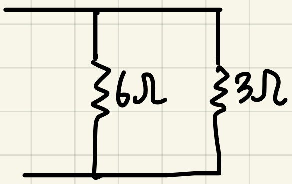
(d) Difficult

Node analysis.

$V_1: \frac{V_1 - V}{4} + \frac{V_1 - V_2}{3} + \frac{V_1}{2} = 0 \rightarrow 13V_1 - 3V - 4V_2 = 0$

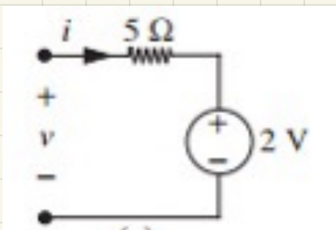
$V_2: \frac{V_2 - V}{2} + \frac{V_2 - V_1}{3} + \frac{V_2}{1} = 0 \rightarrow -2V_1 - 3V + 11V_2 = 0$

$V_1 = V_2 = \frac{1}{3}V \rightarrow$ No current flows through $3\Omega$.




$\therefore \frac{1}{\frac{1}{6} + \frac{1}{3}} = 2\Omega$

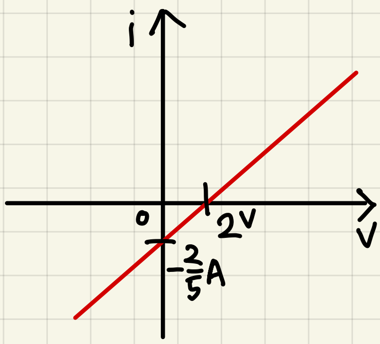
# Exercise 2.8 (c)



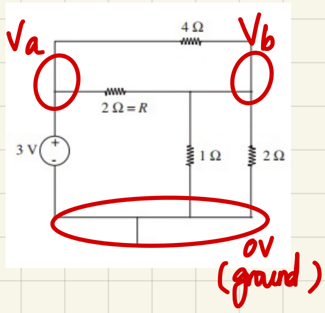
(c)

$v = 5i + 2$

$i = \frac{1}{5}v - \frac{2}{5}$



Problem 2.9




Sol② Node analysis

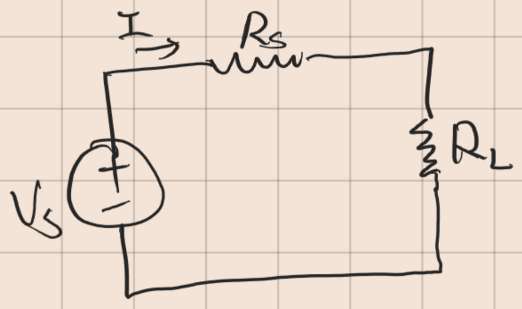
$V_a = 3V$

$\frac{V_b-3}{2} + \frac{V_b-3}{4} + \frac{V_b}{2} + \frac{V_b}{1} = 0 \rightarrow V_b = 1V$

Voltage at Resistor R : $V_a - V_b = 2V$

$P = \frac{V^2}{R} = \frac{4}{2} = 2W$

# Problem 2.11 (b)




Let $I$ current in this circuit, then

$$I = \frac{V_S}{R_S + R_L}$$

Let $P_L$ power dissipated in $R_L$, then

$$P_L = I^2 \cdot R_L$$

$$= \frac{V_S^2}{(R_S + R_L)^2} \cdot R_L$$

$$\frac{\partial P_L}{\partial R_L} = V_S^2 \left\{ \frac{1}{(R_S + R_L)^2} + \frac{-2R_L}{(R_S + R_L)^3} \right\}$$

$$= V_S^2 \cdot \frac{R_S - R_L}{(R_S + R_L)^3}$$

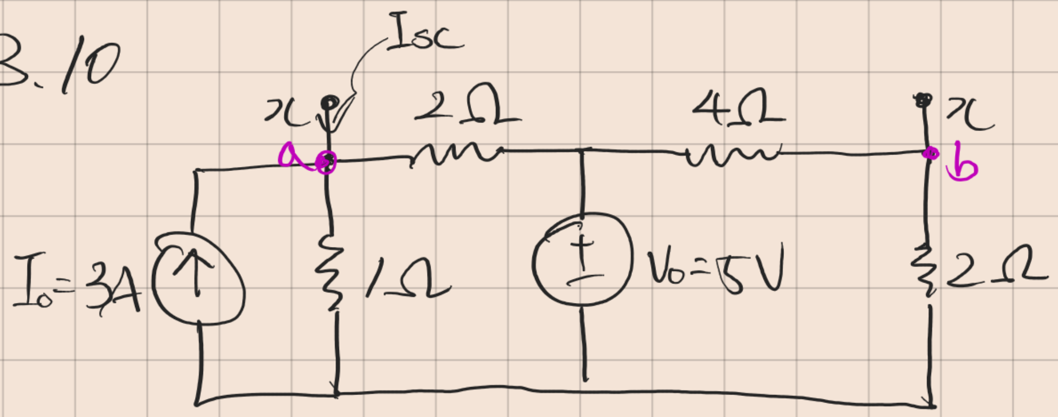
Let $\frac{\partial P_L}{\partial R_L} = 0$, $R_S = R_L$

If $R_S < R_L$, $\frac{\partial P_L}{\partial R_L} < 0$

If, $R_S > R_L$, $\frac{\partial P_L}{\partial R_L} > 0$

$\therefore$ $P_L$ is maximized when $R_S = R_L$ $\boxtimes$

Ex 3.10




Set $I_o = 0$, $V_o = 0$, then

$$R_{TH} = 1 // 2 + 4 // 2$$

$$= \frac{1 \cdot 2}{1+2} + \frac{4 \cdot 2}{4+2} = 2\,\Omega$$

Use node analysis, then

For a, $3 = \frac{V_a - 0}{1} + \frac{V_a - 5}{2}$

$\therefore V_a = \frac{11}{3}\,V$

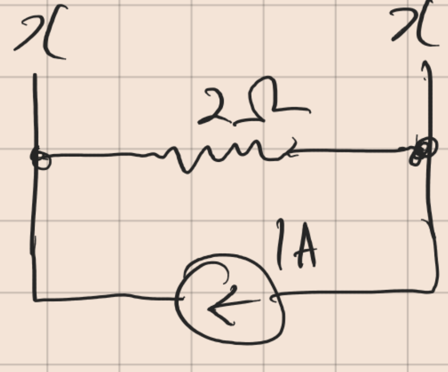
For b, $\frac{5 - V_b}{4} = \frac{V_b - 0}{2}$

$\therefore V_b = \frac{5}{3}\,V$

$$V_{oc} = V_a - V_b = \frac{11}{3} - \frac{5}{3} = 2V$$

$$I_{sc} = \frac{V_{oc}}{R_{TH}} = 1A$$

$\therefore$ Norton equivalent



Ex 3.16

1) set $V_1, V_2$ as 0

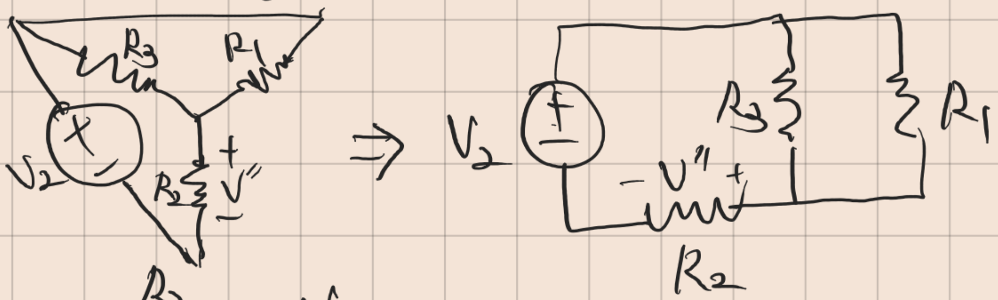
Let $V'$ voltage across $R_2$

No current through $R_2$


$\therefore V' = 0\,V$

2) Set $V_1, I$ as 0

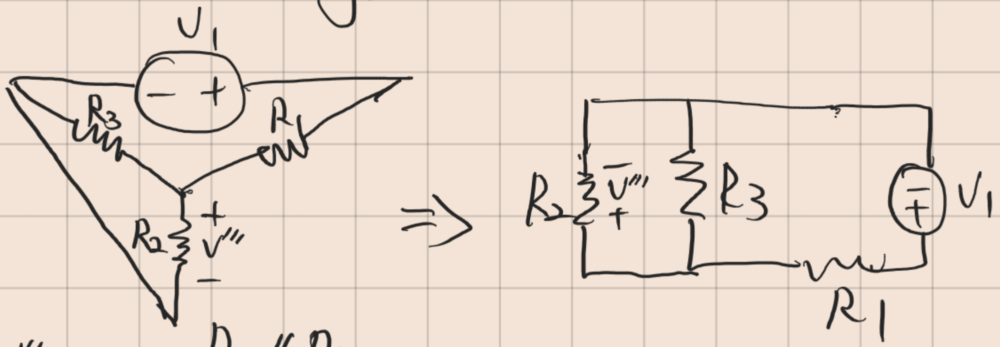
Let $V''$ voltage across $R_2$




$$V'' = \frac{R_2}{R_1 /\!/ R_3 + R_2} V_2$$

3) set $V_2, I$ as 0

Let $V'''$ voltage across $R_2$




$$V'' = \frac{R_2 /\!/ R_3}{R_1 + R_2 /\!/ R_3} \cdot V_1$$

$$\therefore V = V' + V'' + V'''$$

$$= \frac{R_2}{R_1 /\!/ R_3 + R_2} V_2 + \frac{R_2 /\!/ R_3}{R_1 + R_2 /\!/ R_3} V_1$$

EXERCISE 3.25 Find the node potential $E$ in Figure 3.128.

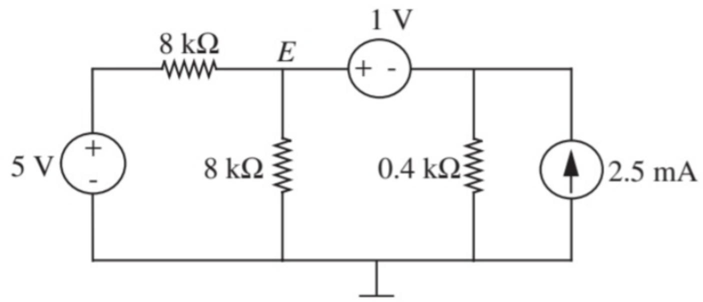
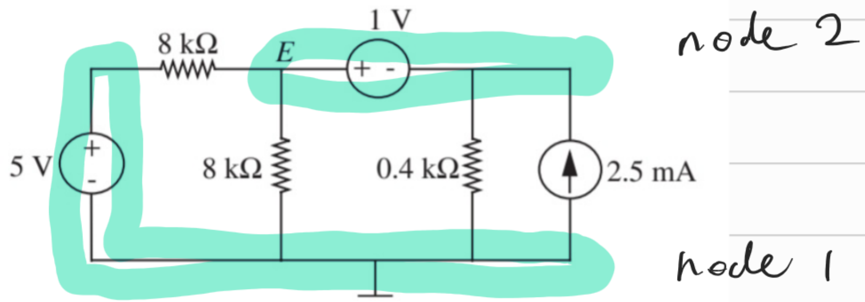



Figure 3.128

Label the nodes node 1 & node 2 as follows:




By the node analysis, we get the following equation:

$$\frac{E-5}{8} + \frac{E}{8} + \frac{E-1}{0.4} = 2.5$$

$$E - 5 + E + 20(E-1) = 20$$

$$22E = 45$$

$$\therefore E = \frac{45}{22} \text{ (V)}$$

**PROBLEM 3.3** Find $V_0$ in Figure 3.132. Solve by (1) node method, (2) superposition. All resistances are in ohms.

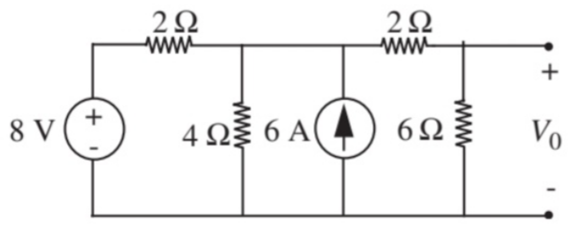
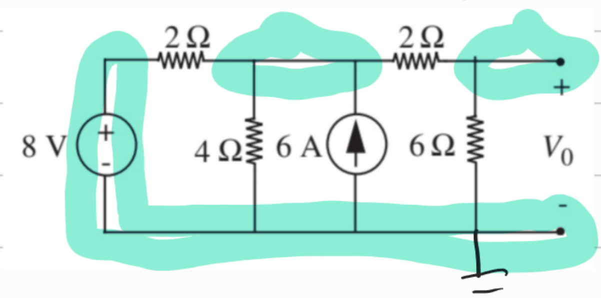



Figure 3.132

(1) node method

First, choose one of nodes in the circuit as a reference node.

And then label the nodes node 1, node 2, and node 3 as follows:




By the node analysis, we get the following two equations:

from node 1,

$$\frac{V_1 - 8}{2} + \frac{V_1}{4} + \frac{V_1 - V_0}{2} = 6 \quad \cdots ①$$

from node 2,

$$\frac{V_1 - V_0}{2} = \frac{V_0}{6} \quad \cdots ②$$

From eqn. ②, we get:

$$3V_1 - 3V_0 = V_0$$

$$\therefore V_1 = \frac{4}{3} V_0 \quad \cdots ③$$

Substituting $V_1$ in ① with ③, we get:

$$\frac{2V_0}{3} - 4 + \frac{V_0}{3} + \frac{V_0}{6} = 6$$

$$4V_0 - 24 + 2V_0 + V_0 = 36$$

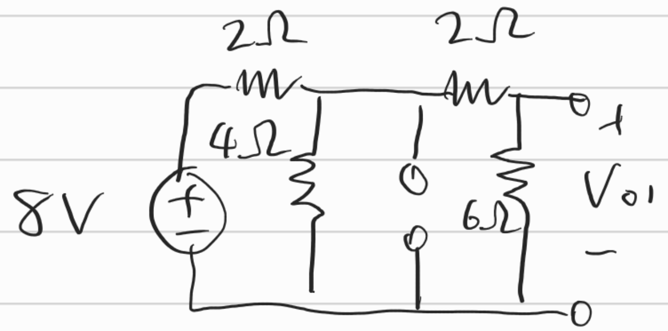
$$7V_0 = 60 \qquad \therefore V_0 = \frac{60}{7} \text{ (V)}$$

(2) superposition

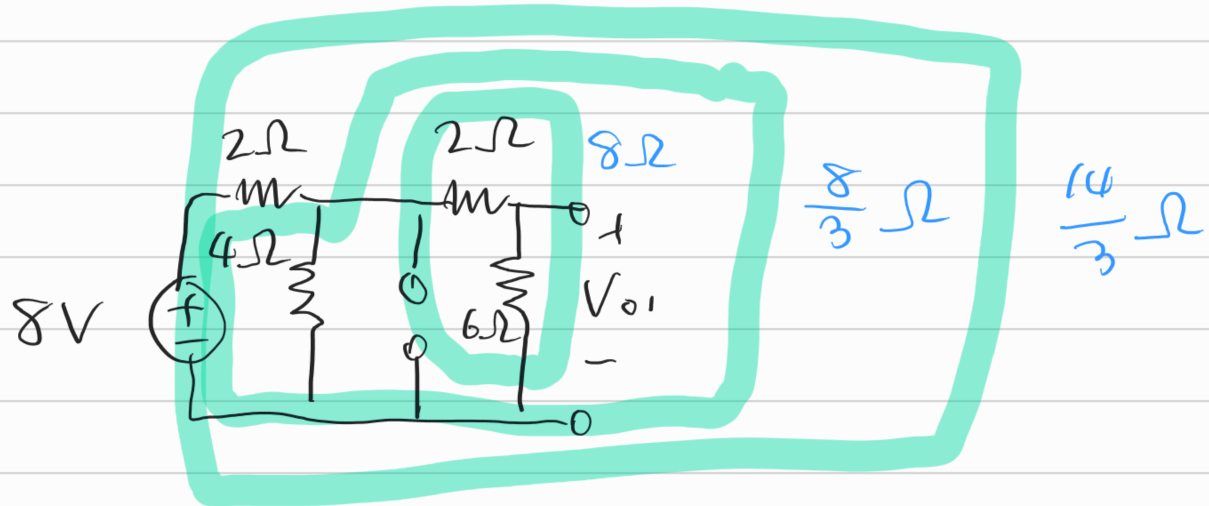
Since the given circuit is a linear circuit, we can compute the contribution of each voltage / current in $V_0$ by the superposition principle.

① contribution of the 8V voltage source ($V_{01}$)

To ignore the effect of the current source, substitute it with an open circuit



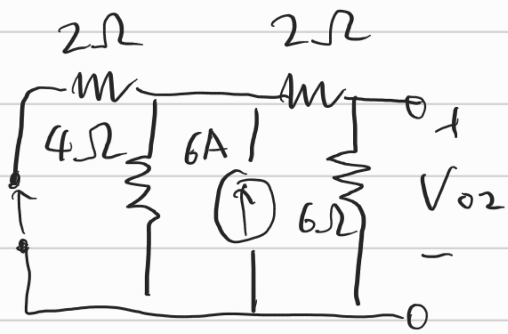

Now, we can find equivalent resistances for some groups of resistors as follows:



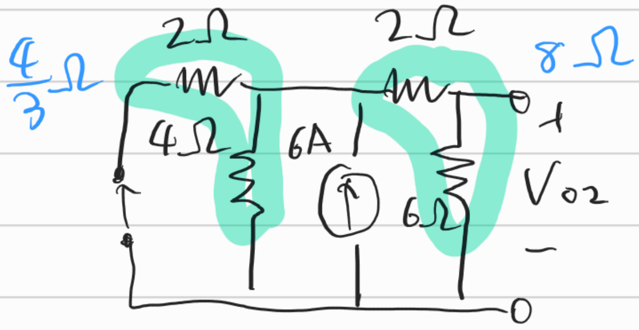

$$V_{o1} = 8 \times \frac{\frac{8}{3}}{\frac{14}{3}} \times \frac{6}{8} = \frac{24}{7}\,(V)$$

② contribution of the 6A current source ($V_{o2}$)

In a similar way to ①, we substitute the voltage source with a short circuit



Now, we can find equivalent resistance for groups of resistors as follows:




Then, we get:

$$V_{o2} = 6 \times \frac{\frac{4}{3}}{\frac{4}{3}+8} \times 6$$

$$= 36 \times \frac{1}{7} = \frac{36}{7} (V)$$

Finally, by the superposition principle, we get:

$$V_o = V_{o1} + V_{o2} = \frac{24}{7} + \frac{36}{7} = \frac{60}{7} (V)$$

**PROBLEM 3.6** Find $v_i$ for $I = 3$ A, $V = 2$ V in Figure 3.135. Strategy: To avoid numerical errors, derive expressions in literal form first, then check dimensions.

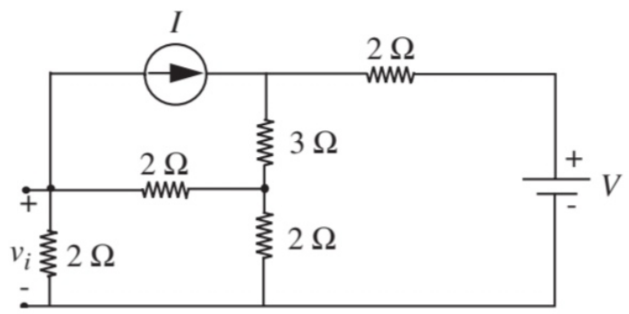


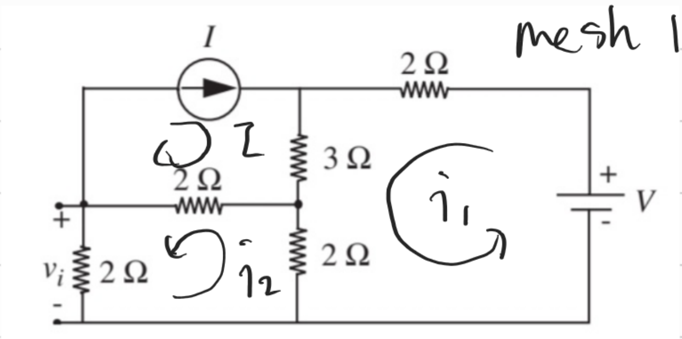

Figure 3.135

We can use the mesh analysis method

Label the meshes mesh 1, 2, and 3 as follows:

mesh 3

mesh 2




By the mesh analysis, we get the following 2 equations:

from mesh 1,

$$2i_1 + 3(i_1 + I) + 2(i_1 - i_2) = V$$

$$\Rightarrow 7i_1 - 2i_2 = V - 3I \quad \cdots ①$$

from mesh 2,

$$2(i_2 - i_1) + 2(i_2 + I) + 2i_2 = 0$$

$$\Rightarrow 3i_2 - i_1 = -I \quad \cdots ②$$

From ① and ②, we get

$$7(3i_2+I) - 2i_2 = 19i_2 + 7I = V - 3I$$

$$\therefore i_2 = \frac{V-10I}{19}$$

Since $v_i = 2i_2$, we finally get:

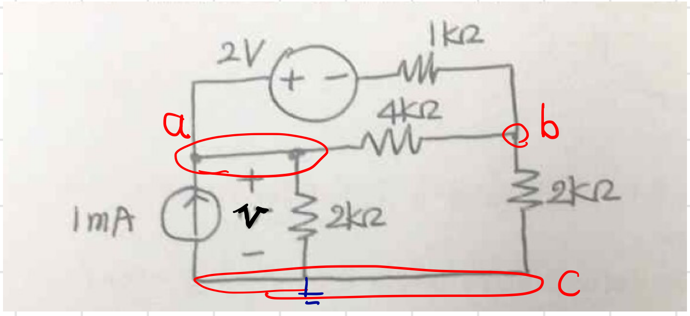
$$v_i = 2i_2 = \frac{2V-20I}{19} = \frac{2\cdot 2 - 20\cdot 3}{19}$$

$$= -\frac{56}{19} \ (V)$$

prev exam

2. (a) node analysis

Solution




Let C be a reference node

then, $V_c = 0$, $V_a = V$

① KCL at b.

$$\frac{(V-2)-V_b}{1} + \frac{V-V_b}{4} + \frac{0-V_b}{2} = 0$$

$$\Rightarrow 5V - 7V_b = 8$$

② KCL at c

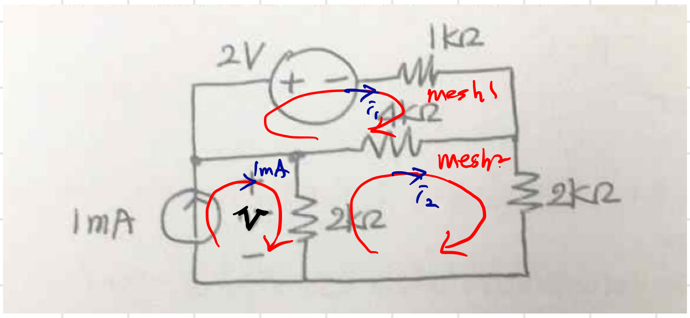
$$-1 + \frac{V}{2} + \frac{V_b}{2} = 0$$

$$\Rightarrow V + V_b = 2 \quad \Rightarrow 7V + 7V_b = 14$$

from ① and ② $\therefore V = \frac{11}{6}\,V$

2. (b) mesh (loop) analysis

Solution




① KVL at mesh1

$$2 + i_1 + 4(i_1 - i_2) = 0$$

$$\Rightarrow 5i_1 - 4i_2 = -2$$

② KVL at mesh2

$$2(i_2 - 1) + 4(i_2 - i_1) + 2i_2 = 0$$

$$\Rightarrow -4i_1 + 8i_2 = 2$$

from ① and ② $i_2 = \frac{1}{12}$

$V = 2(1 - i_2)$ $\therefore V = \frac{11}{6}\,V$

prev exam

3. Thevenin equivalent circuit

Solution 1

① KVL at loop 1

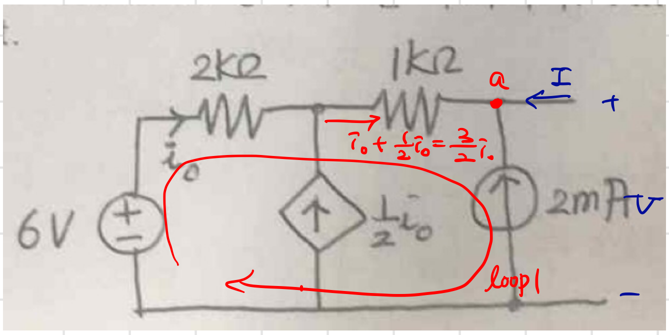
$-6 + 2i_o + \frac{3}{2}i_o + V = 0$

$\Rightarrow V = -\frac{7}{2}i_o + 6$

② KCL at a

$I + \frac{3}{2}i_o + 2 = 0$

$\Rightarrow i_o = -\frac{2}{3}(I+2)$



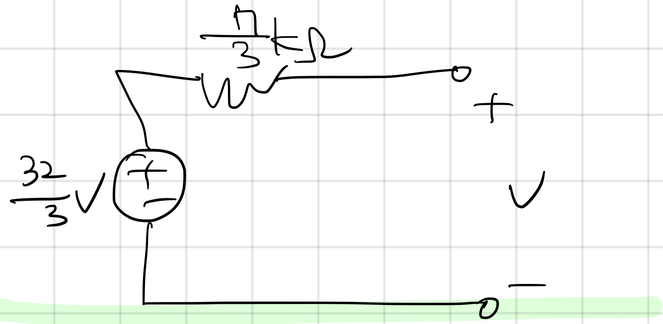

from ① and ②

$V = \left(-\frac{7}{2}\right)\cdot\left(-\frac{2}{3}\right)(I+2) + 6.$

$\Rightarrow V = \frac{7}{3}I + \frac{32}{3}$

$V = R_{TH} I + V_{TH}$

$\therefore V_{TH} = \frac{32}{3}V$ , $R_{TH} = \frac{7}{3}k\Omega$




Solution 2

To get $V_{th}$

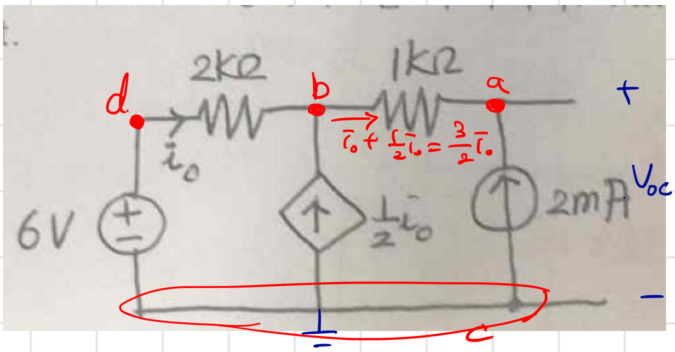
Let c be a reference node.

then $V_c = 0$ , $V_d = 6$ , $V_a = V_{oc}$

① KCL at a.

$\frac{V_b - V_{oc}}{1} + 2 = 0$

$\Rightarrow V_{oc} = V_b + 2$




② KCL at c

$i_o + \frac{1}{2}i_o + 2 = 0$

$\Rightarrow i_o = -\frac{4}{3}$

③ KCL at d.

$i_o + \frac{V_b - 6}{2} = 0$

$\Rightarrow V_b = -2i_o + 6 = (-2)\left(-\frac{4}{3}\right) + 6 = \frac{26}{3}$

from ① and ③

$\therefore V_{oc} = \frac{32}{3}V = V_{TH}$

To get $R_{th}$

① KVL at loop 1

$$-6 + 2I_o + \frac{3}{2}I_o = 0$$

$$\Rightarrow \frac{7}{2}I_o = 6 \Rightarrow I_o = \frac{12}{7}$$

② KCL at a

$$\frac{3}{2}I_o + 2 - I_{sc} = 0$$

from ① and ②

$$I_{sc} = \frac{3}{2}\cdot\frac{12}{7} + 2 = \frac{32}{7}$$

$$R_{th} = \frac{V_{th}}{I_{sc}} = \frac{\frac{32}{3}}{\frac{32}{7}}$$

$$\therefore R_{th} = \frac{7}{3}\ k\Omega$$