VLIDORT Notes 3
Column Jacobian vs Profile Jacobian

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Abstract

This notes discuss the connection between columnar Jacobian and profile Jacobian.

1 Principles

Let x_c is the columnar quantity, and x_i its value on layer i. The columnar weighting function could be computed as the summation of profile Jacobian over the layers:

$$x_c \frac{\partial I}{\partial x_c} = \sum x_i \frac{\partial I}{\partial x_i} \tag{1}$$

2 Examples

2.1 Aerosol optical depth

Columnar AOD is summation of AOD at all layers,

$$\tau_{aer}^c = \sum \tau_{aer}^i \tag{2}$$

AOD of each layer are independent with each other. Thus,

$$\frac{\partial \tau_{aer}^i}{\partial \tau_{aer}^c} = 1 \tag{3}$$

The columnar weighting function is

$$\tau_{aer}^c \frac{\partial I}{\partial \tau_{aer}^c} = \sum \left(\tau_{aer}^i \frac{\partial I}{\partial \tau_{aer}^c} \right)$$

$$\begin{aligned}
&= \sum \left(\tau_{aer}^i \frac{\partial I}{\partial \tau_{aer}^i} \frac{\partial \tau_{aer}^i}{\partial \tau_{aer}^c} \right) \\
&= \sum \left(\tau_{aer}^i \frac{\partial I}{\partial \tau_{aer}^i} \right) \quad (4)
\end{aligned}$$

2.2 Aerosol single scattering albedo

Similar the columnar Jacobian with respect to AOD, the derivation for single scattering albedo is like:

$$\omega_{aer}^c = \frac{\sum \tau_{aer}^i \omega_{aer}^i}{\tau_{aer}^c} = \sum \frac{\tau_{aer}^i \omega_{aer}^i}{\tau_{aer}^c} \quad (5)$$

$$\frac{\partial \omega_{aer}^i}{\partial \omega_{aer}^c} = \frac{\tau_{aer}^c}{\tau_{aer}^i} \quad (6)$$

$$\begin{aligned}
\omega_{aer}^c \frac{\partial I}{\partial \omega_{aer}^c} &= \sum \left(\frac{\tau_{aer}^i \omega_{aer}^i}{\tau_{aer}^c} \frac{\partial I}{\partial \omega_{aer}^c} \right) \\
&= \sum \left(\frac{\tau_{aer}^i \omega_{aer}^i}{\tau_{aer}^c} \frac{\partial I}{\partial \omega_{aer}^i} \frac{\partial \omega_{aer}^i}{\partial \omega_{aer}^c} \right) \\
&= \sum \left(\frac{\tau_{aer}^i \omega_{aer}^i}{\tau_{aer}^c} \frac{\partial I}{\partial \omega_{aer}^i} \frac{\tau_{aer}^c}{\tau_{aer}^i} \right) \\
&= \sum \left(\omega_{aer}^i \frac{\partial I}{\partial \omega_{aer}^i} \right) \quad (7)
\end{aligned}$$

2.3 Aerosol microphysical parameters

To be continued ...