A Dynamic Exemplars Selection-based Differential Evolution Algorithm for Constrained Multi-objective Optimization

***Abstract*—Constrained multi-objective optimization problems (CMOPs) pose a significant challenge due to the presence of multiple conflicting objective functions requiring optimization, alongside numerous complex constraints that must be satisfied. When employing evolutionary algorithms (EAs) to address such problems, it is imperative to simultaneously consider the diversity, convergence, and feasibility of the population. To address this concern, this paper introduces a differential evolution algorithm based on dynamic learning exemplars, named DESDE, wherein the selection of learning exemplars for each individual is determined through evolutionary algebra. Specifically, to generate offspring with diversity, individuals have a large number of learning exemplars to choose from during the early stage of evolution, thus more promising regions can be explored. Moreover, to expedite population convergence in the later stage, only individuals with superior performance are chosen as learning exemplars.**

***Index Terms*—Constrained multi-objective optimization, Differential evolution, Learning exemplars**

I. Algorithm Description

A. Procedure

The framework and procedural flow of DESDE are shown in Fig. 1 and Algorithm 1 respectively. Initially, a population comprising NP individuals is randomly generated. Subsequently, the objective function values and constraint violation degrees for these individuals are computed. The fitness values of the individuals are determined through the utilization of the improved ϵ constrained method. The improved ϵ constrained method, as proposed by Qiao *et al.* [1], integrates the conventional ϵ constrained method with a multi-objective approach. To be specific, it employs the multi-objective method to arrange individuals within the ϵ constraint boundary defined by the conventional ϵ constrained method. This strategy enhances the uniformity of the distribution of individuals within the ϵ constraint boundary, thereby augmenting the diversity of the population. Hence, in this study, the improved ϵ constrained method is adopted. Subsequently, in the absence of meeting the termination condition, the iterative process commences. Initially, the differential evolution algorithm, incorporating dynamic exemplars selection, is employed to generate a set of NP offspring individuals. Subsequently, the objective function values and constraint violation degrees of these offspring individuals are computed. Following this, a fusion occurs between the parent population and the offspring individuals. The fitness values for all individuals within this combined population are then computed utilizing the improved ϵ constrained method. Finally, based on the computed fitness values, the top-performing NP individuals are selected for progression to the subsequent generation. This iterative process continues until the termination condition is met, at which point the population Pop is returned.

Algorithm 1: The Procedure of DESDE

Input : NP (population size)
$MAXFES$ (maximum number of function evaluations)
Output: Pop
$POP \leftarrow$ Initialize the population;
Calculate the individuals' objective function values and constraint violation degree in the population;
$FES = NP$;
$Fit \leftarrow$ The fitness value of each individual is calculated based on the improved ϵ constrained method;
while $FES \leq MAXFES$ **do**
    $Off \leftarrow$ Generate NP offspring individuals using differential evolution algorithm based on dynamic exemplars selection;
    Evaluate objective function values and constraint violation degree of individuals in the Off;
    $FES = FES + NP$;
    $Pop \leftarrow Pop \cup Off$;
    $Fit \leftarrow$ Calculate fitness value for each individual in the mixed population Pop based on the improved ϵ constrained method;
    $Pop \leftarrow$ The former NP individuals with better fitness values are selected to enter the next generation;
end
Return Pop;

B. Differential evolution algorithm based on dynamic exemplars selection

To enhance the quality of offspring individuals, a strategy for generating offspring based on dynamic exemplars selection is devised. This strategy aims to preserve population diversity in the early stage of evolution while promoting population convergence in the later stage. The chosen tool for offspring generation is the differential evolution algorithm, renowned for its robust search capabilities. The procedural details are outlined in Algorithm 2.

Initially, the dynamic exemplars selection parameter, denoted as p, undergoes an update to determine the number of learning exemplars within the population. The updated formula is expressed as follows:

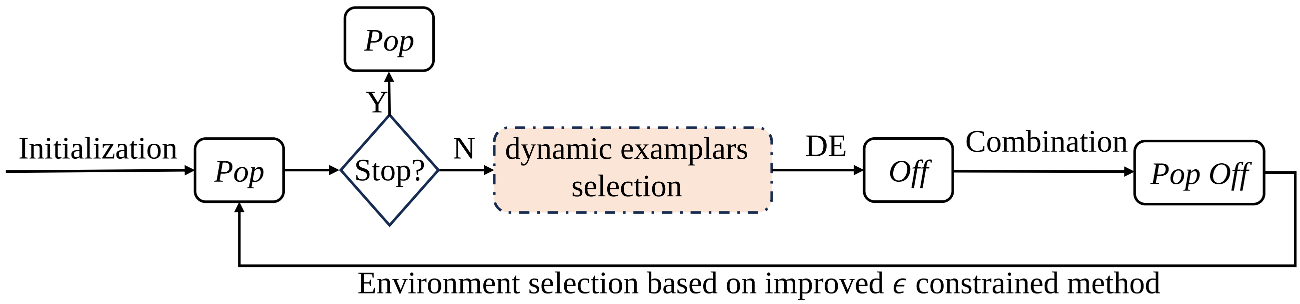

Fig. 1. The framework of DESDE.

Algorithm 2: The Procedure of Differential evolution algorithm based on dynamic exemplars selection

Input : NP (population size)
F (scale factor)
p (a parameter for dynamic learning exemplars selection)
Output: Off (offspring population)
$p \leftarrow$ Update parameter p according to Eq. (4);
$\{exem\} \leftarrow$ The former $NP * p$ individuals with better fitness values are selected as exemplar individuals;
for *i = 1 : NP* **do**
  $\{\vec{x}_a, \vec{x}_b\} \leftarrow$ Two individuals are randomly selected from population Pop;
  $\vec{x}_{exem} \leftarrow$ Randomly select an individual from the exemplar individuals as the learning exemplar of this individual;
  $\vec{v}_i = \vec{x}_i + F * (\vec{x}_{exem} - \vec{x}_i) + F * (\vec{x}_a - \vec{x}_b)$;
  $Off = off \cup \vec{v}_i$;
end
Return Off;

$$p = 1 - \frac{0.99 * FES}{MAXFES} \tag{1}$$

where FES represents the current number of evaluations, $MAXFES$ represents the maximum number of evaluations. From Eq. (4), the initial value of p is 1 and the minimum value is 0.01, in order to ensure that there is at least one model for individuals to learn.

Subsequently, the top $NP * p$ individuals with superior fitness in the population are chosen as exemplar individuals. For each individual in the population, the following operation is executed: two distinct individuals, denoted as $\vec{x}_a$ and $\vec{x}_b$, are randomly selected from the population. Additionally, an individual is randomly chosen from the exemplar individual pool $exem$ as the learning exemplar for the current individual. Subsequently, the offspring individual is generated using the ensuing operations:

$$\vec{v}_i = \vec{x}_i + F * (\vec{x}_{exem} - \vec{x}_i) + F * (\vec{x}_a - \vec{x}_b) \tag{2}$$

where $\vec{v}i$ represents the offspring generated by the current individual, and F denotes the scaling factor, randomly chosen from the scaling pool $\{0.6, 0.8, 1.0\}$. Extensive experimentation has demonstrated that the variability of F contributes to the generation of superior offspring [2] [3] [4]. $\vec{x}_{exem}$ is an individual randomly selected from the learning exemplars pool. Each individual in the population generates an offspring following the aforementioned process, ultimately yielding the offspring population Off.

C. Analysis

The central mechanism of DESDE revolves around the dynamic selection of learning exemplars. The utilization of diverse learning exemplars aids in generating offspring with varied functions, thus facilitating the evolution of the population in different directions.

During the initial stage of evolution, the parameter p is initialized to 1, allowing every individual in the population an opportunity to be selected as a learning exemplar. In this phase, individual learning is akin to random learning, fostering increased evolutionary directions within the population. This randomness contributes to broadening the population distribution and enhancing overall diversity. Consequently, unexplored areas of the solution space can be systematically investigated by the evolving population.

As the evolutionary process unfolds, the parameter p undergoes gradual reduction, leading to a corresponding decrease in the number of individuals in the learning exemplars pool. Specifically, only those individuals exhibiting the highest performance are retained in this pool. Consequently, the diversity of the population experiences a gradual decline, while convergence gradually increases. In the middle stage of evolution, a balance is achieved between diversity and convergence.

In the later stage of evolution, the parameter p assumes a small value, signifying that only individuals with the highest fitness ranking are eligible to enter the learning exemplars pool. This strategy ensures a heightened level of convergence within the population, facilitating exploration towards the CPF.

References

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