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Biological Physics II

Problem Set 7

Please hand in your solutions before 12:00 noon on Wednesday, July 21, 2021.

1. TASEP model with extended particles *2+4+5+6+8+5= 30 pts*

Consider a generalized TASEP, which we call l-TASEP, where each particle covers $l \geq 1$ sites[1]. Each particle moves one site to the right at rate 1 if the site is empty, and does not move otherwise. 1-TASEP describes the usual TASEP model.

a) For an l-TASEP with n particles on a ring of L sites, prove that all configurations have the same probability in the stationary state.

b) Now consider the problem on an open chain. Prove that the total number of possible configurations of n particles of size l on a chain of L sites is

$$Z(n, L) = \binom{L-(l-1)n}{n}, \tag{1.1}$$

for $L \geq ln$. Hint: What is the total number of objects, i.e particles plus holes? In how many ways can the requisite number of holes be chosen from these objects?

c) In the limit of large L, use (1.1) and the assumption that all configurations are equally likely to show that the steady state current is

$$J = \rho \frac{1-\rho l}{1-\rho(l-1)} \tag{1.2}$$

where the particle density $\rho = n/L$. Hint: The probability that a site to the right of a particle is empty can be assumed to be the same as the probability that the first site of a large lattice with particle density ρ is empty. Why?

[1] This model is treated in [1]*Lakatos, Greg, and Tom Chou. "Totally asymmetric exclusion processes with particles of arbitrary size." Journal of Physics A: Mathematical and General 36.8 (2003): 2027*, [2] *Shaw, Leah B., R. K. P. Zia, and Kelvin H. Lee. "Totally asymmetric exclusion process with extended objects: a model for protein synthesis." Physical Review E 68.2 (2003): 021910* and other papers.

d) Now consider the l-TASEP on the open chain with entry rate α and exit rate β. The particle enters in a single step, i.e it hops on to the lattice at rate α provided the first l sites are empty. The exit is sequential, i.e when the particle occupies the last l sites, it keeps moving to the right one site at a time at a rate 1, until it occupies only the last site, which it exits at rate β. Assume that the left and right boundary densities are[2]

$$\rho_l = \frac{\alpha}{1 + (l-1)\alpha}; \quad \rho_r = \frac{1-\beta}{l}. \tag{1.3}$$

Using the extremal current principle explained in the lectures and the relation (1.2), derive the phase diagram of the system from the relation. Plot the phase boundaries on the $\alpha - \beta$ plane and mark the various phases.

e) Write a code to numerically simulate the l-TASEP on the open chain as described in the previous part[3]. Use a system size of at least $L = 500$. For the 5-TASEP, compute and plot the steady-state current and density as a function of β for $0 < \beta < 1$, with $\alpha = 0.5$; similarly, compute and plot the current and density as a function of α for $0 < \alpha < 1$, with $\beta = 0.5$. Analyze the plots to check if they agree with the phase diagram obtained through the approximate analytical result in the previous part.

f) For $\alpha = 0.8$ and $\beta = 0.9$, plot the steady state current as a function of l for $l = 1, 2, \ldots, 10$. Check whether the relevant analytical results from part d) match with your numerical result.

[2] It is easy to obtain ρ_R by balancing currents at the last site. Obtaining ρ_L is more involved; see Refs [1] and [2] in the previous footnote.

[3] While computing the current, make sure that you correctly measure the time intervals (which are, of course, distinct from the number of simulation steps in general).