1. TASEP model with extended particles

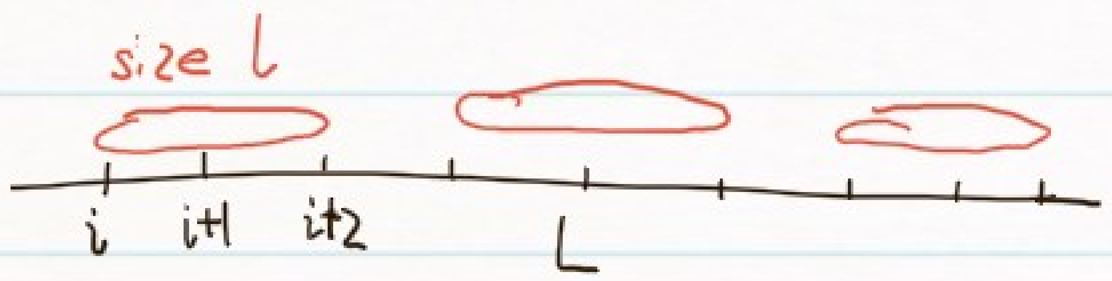
a) in the stationary state

$$\frac{d}{dt}(P(c,t)) = \sum_i r_i(c \leftarrow c')P(c') - \sum_i r_i(c' \leftarrow c)P(c) = 0$$

$$\sum_i r_i(c \leftarrow c')P(c') = \sum_i r_i(c' \leftarrow c)P(c)$$

$$\Rightarrow P(c') = P(c) = P^*(c)$$

b)



Total number of particles plus holes $= \underset{\text{particles}}{n} + \underbrace{L-nl}_{\text{holes}} = L-(l-1)n$

the # of ways that particles can be chosen from the object

$$= \binom{L-(l-1)n}{n} = Z(n,L) \Rightarrow P^*(c) = \binom{L-(l-1)n}{n}^{-1}$$

total # of particles plus holes

c) $J = \sum_i J_i = w \cdot \mathrm{Prob}[n_i = n_{i+1} = \cdots = n_{i+l-1} = 1, n_{i+l} = 0] \times (L-(l-1)n)$

$$= \frac{\binom{L-(l-1)n-2}{n-1}}{\binom{L-(l-1)n}{n}} = \frac{\frac{(L-(l-1)n-2)!}{(n-1)!(L-nl-1)!} \times (L-(l-1)n)}{\frac{(L-(l-1)n)!}{n!(L-ln)!}} = \frac{1}{\frac{(L-(l-1)n-1)}{n \cdot (L-ln)}} = \cdots$$

$$\cdots = \frac{n(L-nl)}{(L-(l-1)n-1)} \qquad \rho = \frac{n}{L}$$

$$= \frac{(n/L)\cdot[1-l\cdot(n/L)]}{(1-(l-1)(n/L)-1/L)}$$

$$= \frac{\rho\cdot(1-l\rho)}{(1-(l-1)\rho)}$$ in the limit of large L, $\frac{1}{L} \to 0$

d) extremal current principle

$$J = \begin{cases} \max\limits_{\rho_l \ge \rho_i \ge \rho_r}[J(\rho_i)] & \rho_l > \rho_r \\ \min\limits_{\rho_l \le \rho_i \le \rho_r}[J(\rho_i)] & \rho_l < \rho_r \end{cases}$$

$$J(\rho) = \frac{1-\rho l}{1-\rho(l-1)} + \rho\left(\frac{1-\rho l}{1-\rho(l-1)}\right)'$$

$$= \sim + \rho\left(\frac{-l(1-\rho(l-1)) + (1-\rho l)(l-1)}{(1-\rho(l-1))^2}\right) = \sim - \rho \cdot \frac{1}{(1-\rho(l-1))^2} = \frac{(1-\rho l)(1-\rho(l-1)) - \rho}{(1-\rho(l-1))^2} \cdots$$

$$\cdots = \frac{1-\rho l - \rho(l-1) + \rho^2 l(l-1) - \rho}{(1-\rho(l-1))^2} = \frac{\rho^2 l(l-1) - 2\rho l + 1}{(1-\rho(l-1))^2}$$

when $J'(\rho) = 0 \Rightarrow \rho^2 l(l-1) - 2\rho l + 1 = 0 \Rightarrow \rho^2 l^2 - 2\rho l + 1 = \rho^2 l$

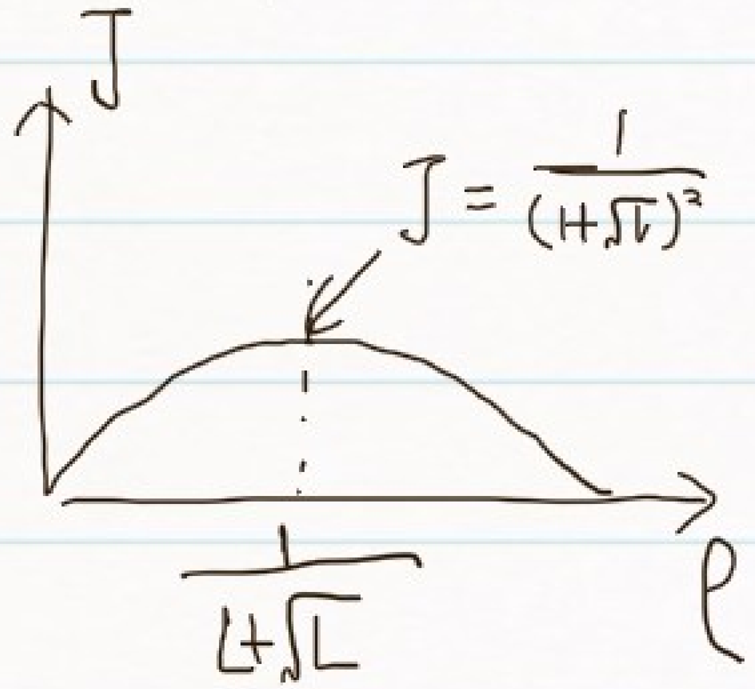
$\Rightarrow (\rho l - 1)^2 = \rho^2 l \Rightarrow 1 - \rho l = \sqrt{l}\rho \Rightarrow \rho = \frac{1}{l+\sqrt{l}}$

$J_{max}\left(\rho = \frac{1}{l+\sqrt{l}}\right) = (1+\sqrt{l})^{-2}$

$$J = \begin{cases} \max\limits_{\rho_l > \rho_i > \rho_r} [J(\rho_i)] & \rho_l > \rho_r \\ \min\limits_{\rho_l < \rho_i < \rho_r} [J(\rho_i)] & \rho_l < \rho_r \end{cases} \qquad J = \rho \frac{1-\rho l}{1-\rho(l-1)}$$

$$\rho_l = \frac{\alpha}{1+(l-1)\alpha} \qquad \rho_r = \frac{1-\beta}{l} \qquad \rho = \frac{1}{l+\sqrt{l}}$$

$$J(\rho_l) = J(\alpha) = \frac{\alpha(1-\alpha)}{1+(l-1)\alpha} \qquad J(\rho_r) = \frac{\beta(1-\beta)}{1+(l-1)\beta} \qquad J(\rho) = (1+\sqrt{l})^{-2}$$



$\rho_l > \rho_r$: $\alpha+\beta+\alpha\beta(l-1) > 1$

case 1: $\rho_r < \rho_l < \rho \Rightarrow \frac{1-\beta}{l} < \frac{\alpha}{1+(l-1)\alpha} < \frac{1}{l+\sqrt{l}} \Rightarrow \beta > \frac{1}{\sqrt{l}+1},\ \alpha < \frac{1}{\sqrt{l}+1}$

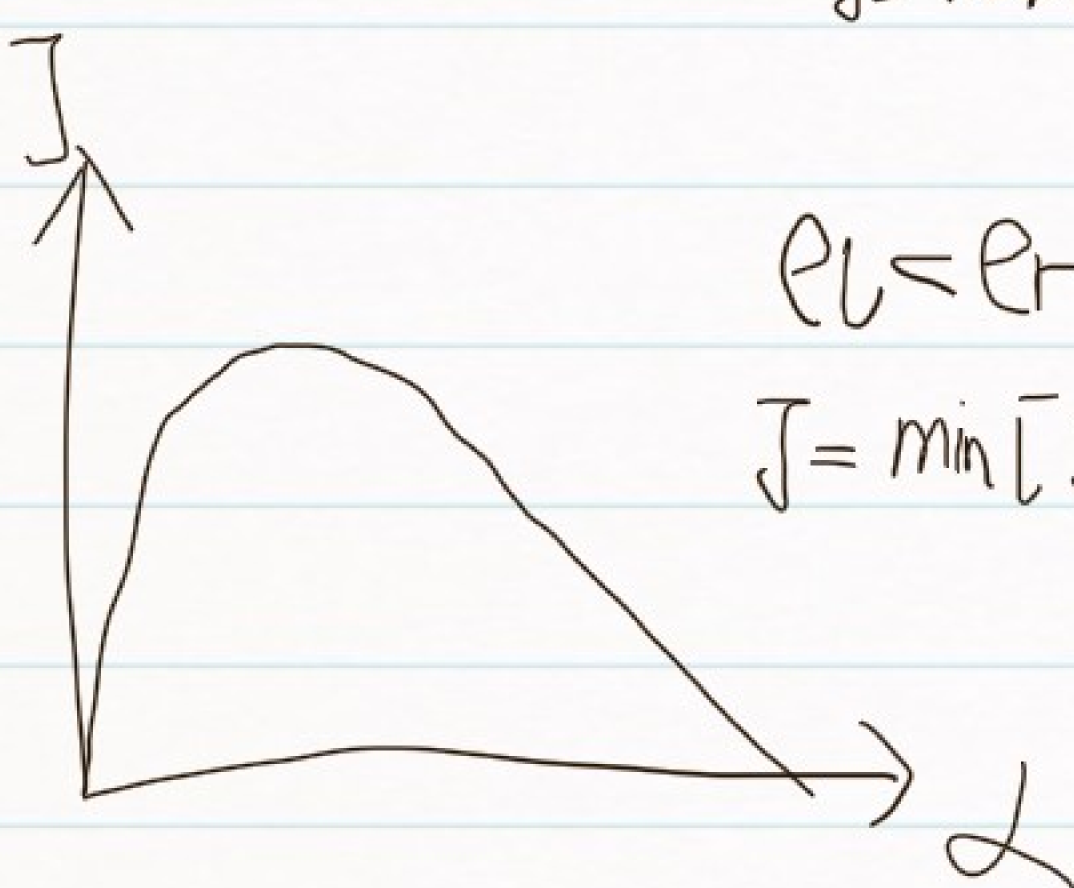
$J = \max[J(\rho_i)] = J(\rho_l) = J(\alpha) \qquad \bar{\rho} = \rho_l$

case 2: $\rho_r < \rho < \rho_l \Rightarrow \frac{1-\beta}{l} < \frac{1}{l+\sqrt{l}} < \frac{\alpha}{1+(l-1)\alpha} \Rightarrow \beta > \frac{1}{\sqrt{l}+1},\ \alpha > \frac{1}{\sqrt{l}+1}$

$J = \max[J(\rho_i)] = J(\rho) \qquad \bar{\rho} = \rho$

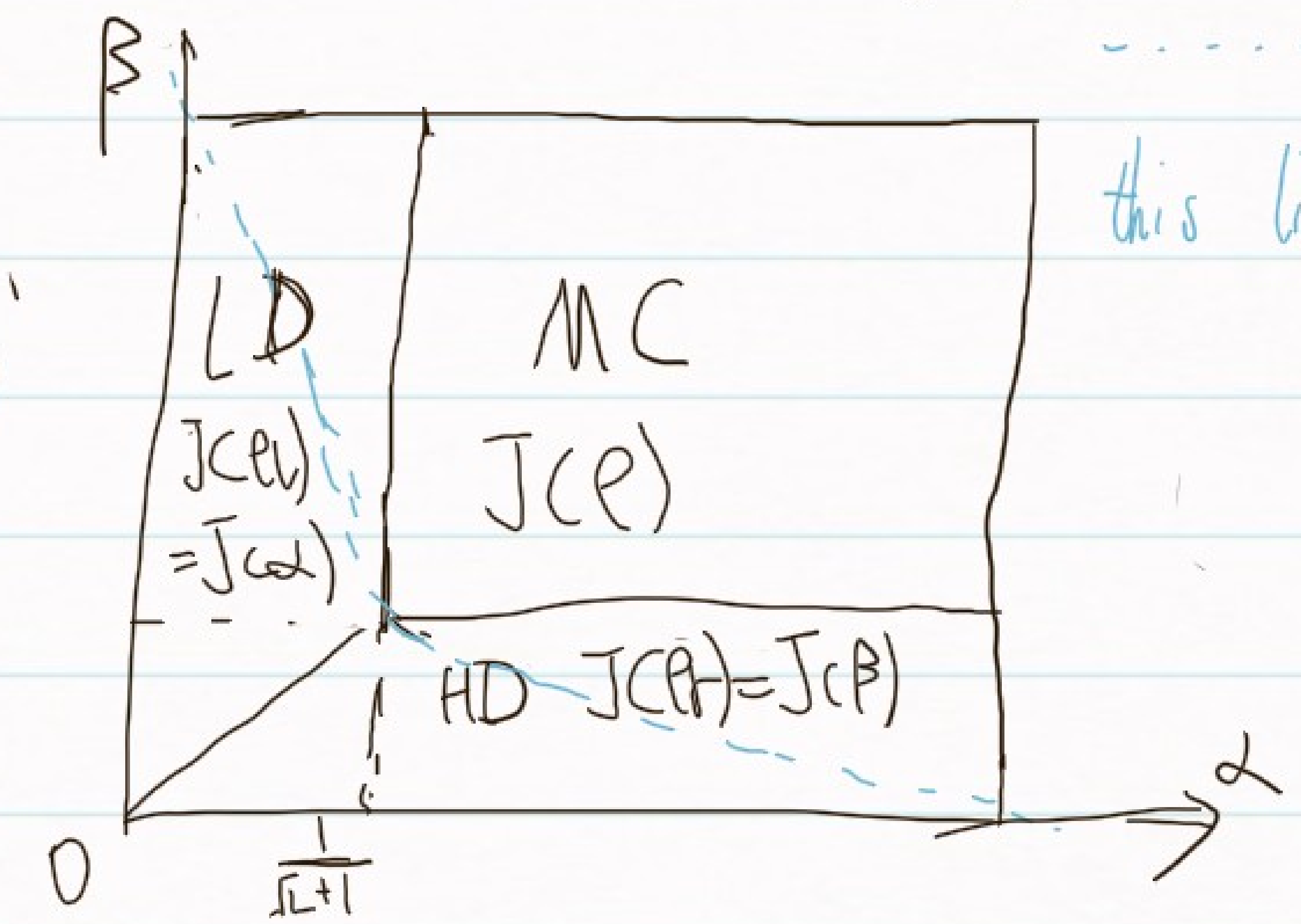
case 3: $\rho < \rho_r < \rho_l \qquad \frac{1}{l+\sqrt{l}} < \frac{1-\beta}{l} < \frac{\alpha}{1+(l-1)\alpha} \Rightarrow \beta < \frac{1}{\sqrt{l}+1},\ \alpha > \frac{1}{\sqrt{l}+1}$

$J = \max[J(\rho_i)] = J(\rho_r) = J(\beta) \qquad \bar{\rho} = \rho_r$



$\rho_l < \rho_r$: $\alpha+\beta+\alpha\beta(l-1) < 1$

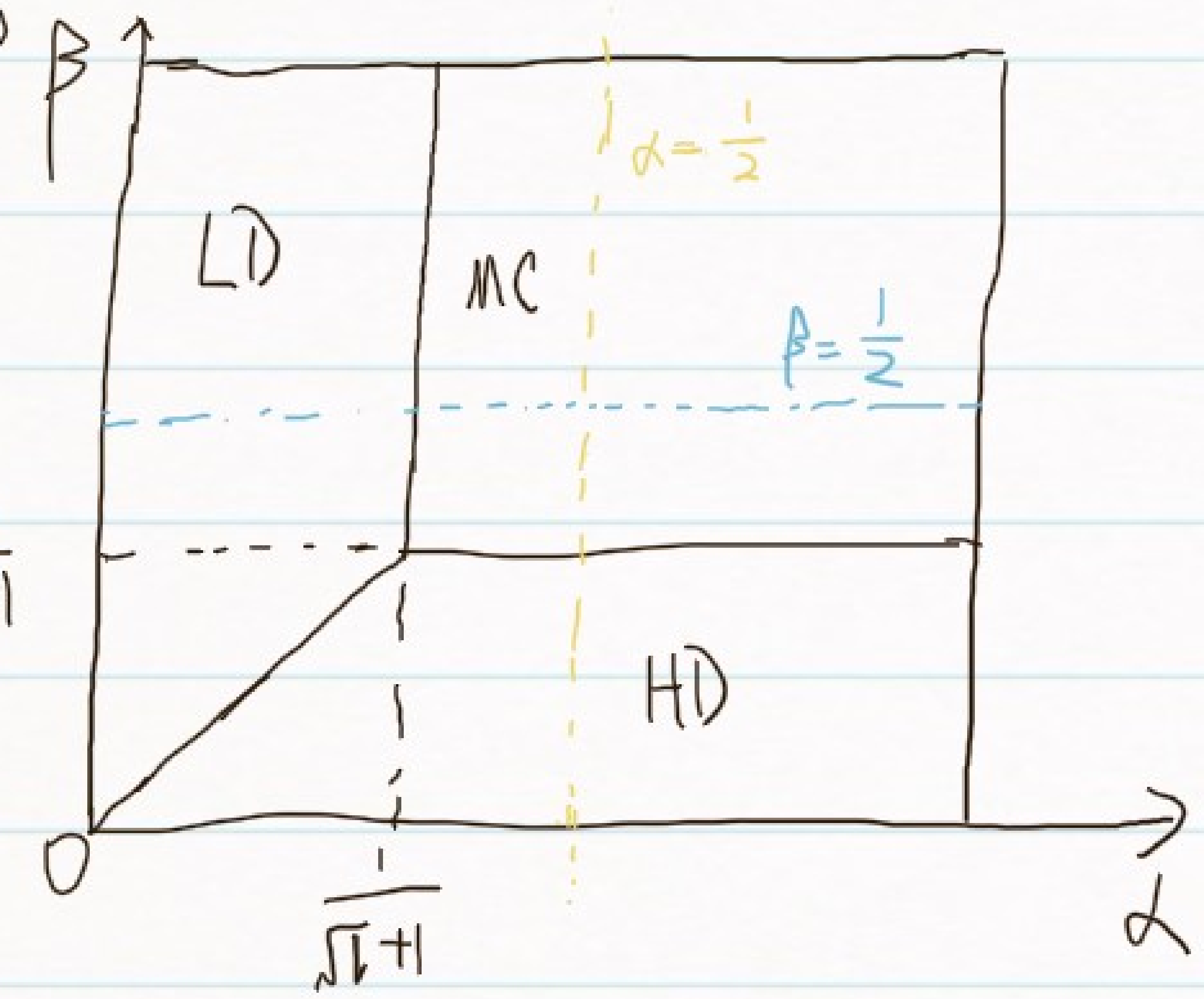
$$J = \min[J(\rho_i)] = \begin{cases} J(\rho_l) = J(\alpha) & (\alpha < \beta) \\ J(\rho_r) = J(\beta) & (\alpha > \beta) \end{cases}$$



----- $\alpha+\beta+\alpha\beta(l-1) = 1$

this line go across the point $\left(\frac{1}{\sqrt{l}+1}, \frac{1}{\sqrt{l}+1}\right)$

(e)

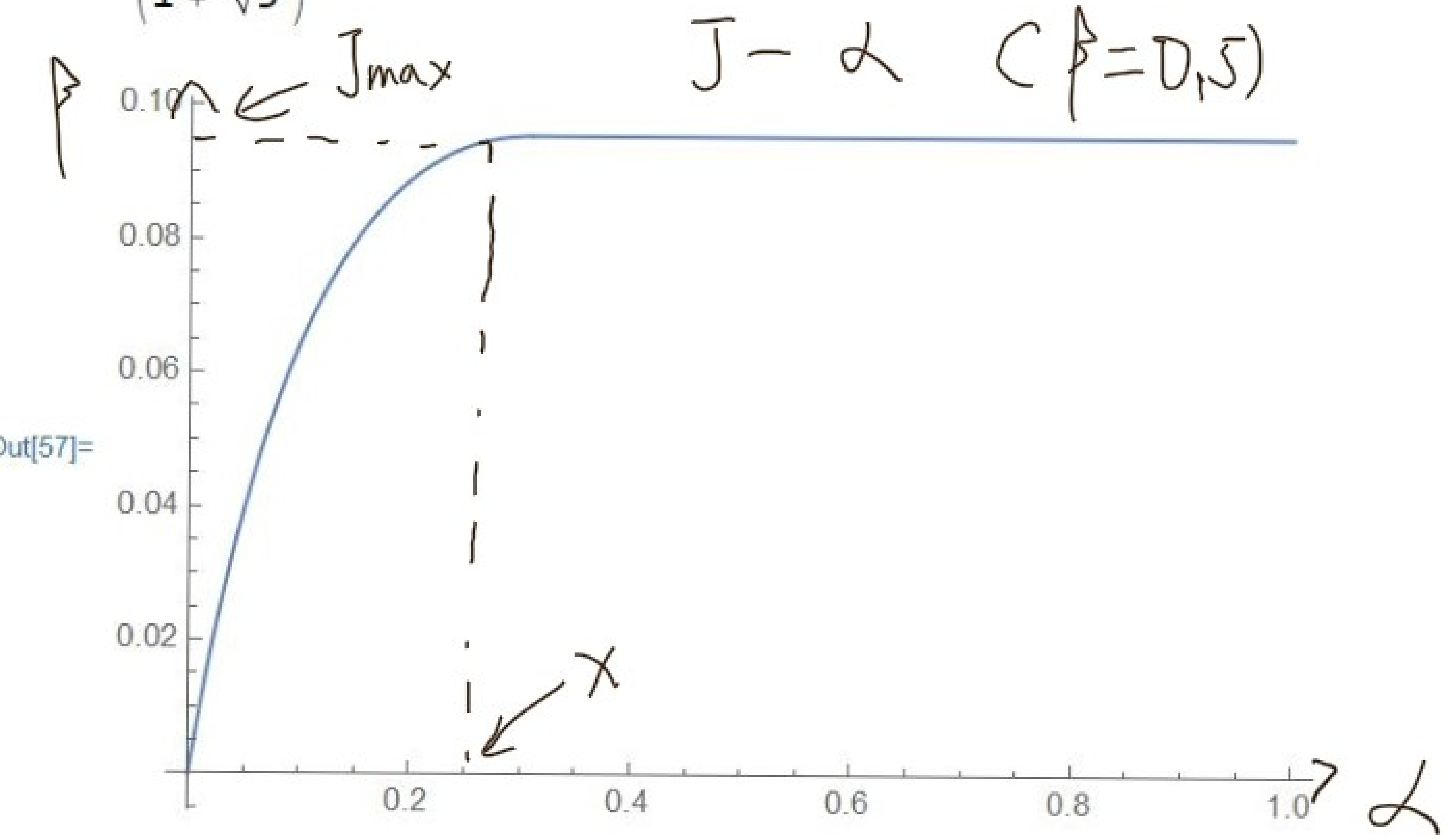



```
In[55]:= x = 1 / (Sqrt[5] + 1)
                 平方根
         Jmax = (1 + Sqrt[5]) ^ (-2)
                     平方根
         Plot[
         绘图
          Piecewise[{{alpha * (1 - alpha) / (1 + 4 * alpha), 0 <= alpha <= x},
          分段函数
            {Jmax, x <= alpha <= 1}}], {alpha, 0, 1}, PlotRange → Full]
                                                     绘制范围        全范围
```

$$\text{Out[55]=}\ \frac{1}{1+\sqrt{5}}$$

$$\text{Out[56]=}\ \frac{1}{\left(1+\sqrt{5}\right)^2}$$

Out[57]=

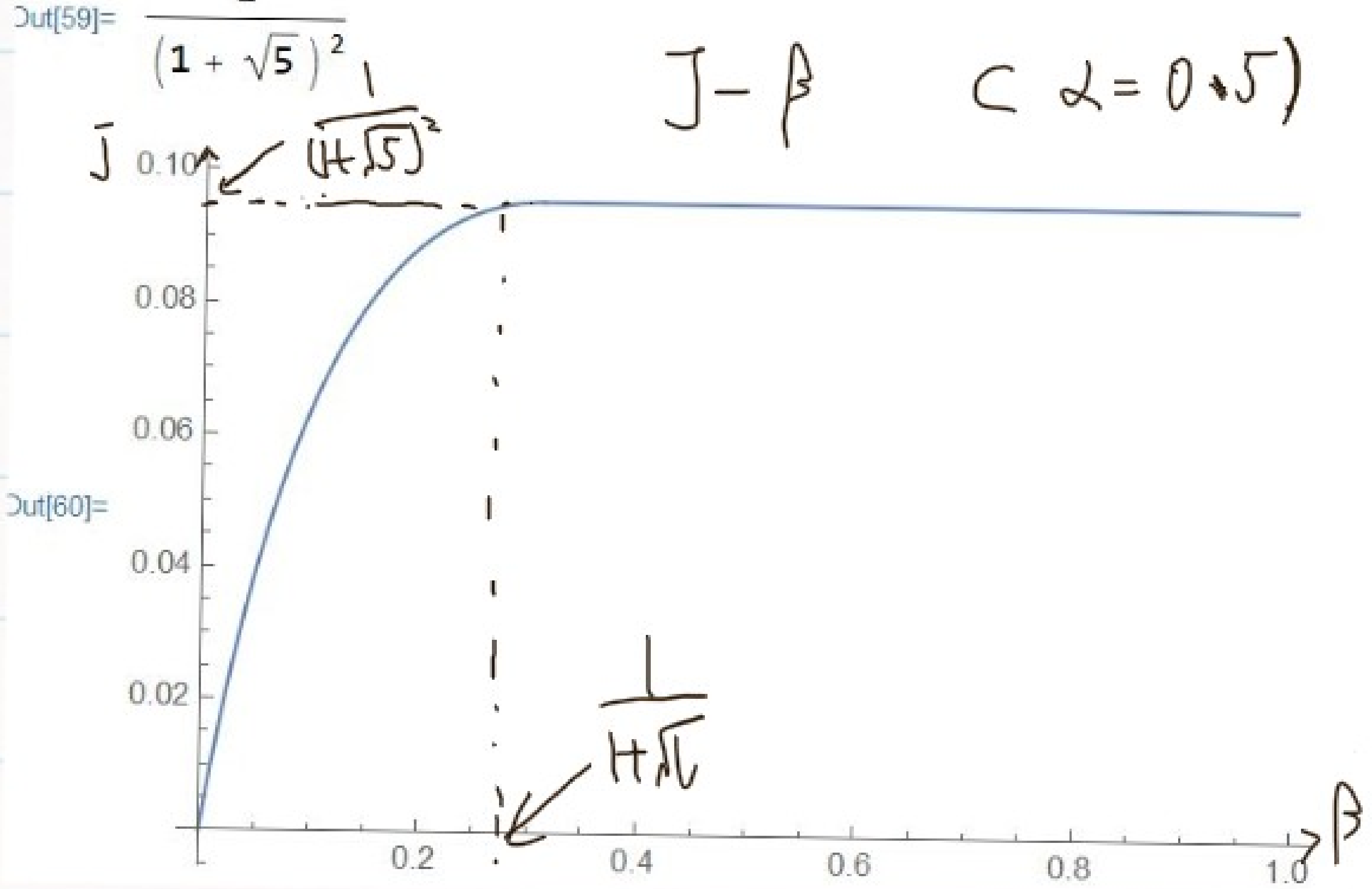



```
In[58]:= x = 1 / (Sqrt[5] + 1)
                 平方根
         Jmax = (1 + Sqrt[5]) ^ (-2)
                     平方根
         Plot[
         绘图
          Piecewise[{{beta * (1 - beta) / (1 + 4 * beta), 0 <= beta <= x},
          分段函数
            {Jmax, x <= beta <= 1}}], {beta, 0, 1}, PlotRange → Full]
                                                  绘制范围        全范围
```

$$\text{Out[58]=}\ \frac{1}{1+\sqrt{5}}$$

$$\text{Out[59]=}\ \frac{1}{\left(1+\sqrt{5}\right)^2}$$

Out[60]=



(f)

$\alpha = 0.8$, $\beta = 0.9$ $\quad \alpha, \beta > \frac{1}{\sqrt{l}+1}$

So J in maximum current

$$J_{MC} = \frac{1}{(1+\sqrt{l})^2}$$

In[68]:=
```
J[l_] = (1 + Sqrt[l])^(-2)
          平方根

DiscretePlot[J[l], {l, 1, 10}, PlotRange → All]
离散图                         绘制范围    全部
```

Out[68]= $\frac{1}{(1+\sqrt{l})^2}$

Out[69]=