Computational Many-Body Physics

apl. Prof. Dr. R. Bulla

SS 2022

Exercise Sheet No. 3

Please upload your solutions to ILIAS until **Thursday, May 19, 16:00**.

Exercise 1: Metropolis algorithm for the two-dimensional Ising model

(8 points)

In this exercise, we consider the Ising model in a magnetic field

$$H = -J \sum_{<ij>} S_i^z S_j^z - h \sum_i S_i^z ,$$

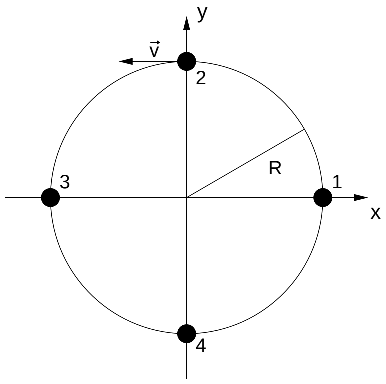
on a two-dimensional square lattice ($N \times N$) with coupling between nearest neighbours and periodic boundary conditions. The strength of the coupling can be set to $J = 1$ (the ferromagnetic case) and N can be set to $N = 64$.

a) Write a code which generates a Markov chain of 10000 spin configurations for a given temperature T, starting from a random spin configuration. In each iteration, the (possibly) new spin configuration is generated by flipping a *single* spin, as described in the script and in the lecture (Video No. 8). Plot the final spin configurations for temperatures $T = 0.1$, 0.5, 2.0, and 10.0 and magnetic field $h = 0$. (4 points)

b) Calculate the temperature dependence of the average magnetization for various values of h. (4 points)

Exercise 2: N-body gravitational systems in 2d

(13 points)



In this exercise, we consider a system of N bodies (with masses $m_i = m$, $i = 1, \ldots, N$), interacting via the gravitational force between the bodies. The bodies are moving in circular orbits with radius R as sketched in the above figure for $N = 4$. The velocities of the bodies have constant and equal magnitude: $|\vec{v}_i(t)| = v$.

a) Specify the orbits $\vec{r}_i(t)$, $i = 1, \ldots, N$, for arbitrary values of N. The figure shows the positions for time $t = 0$, with angles given by $\varphi_i = (i-1)2\pi/N$. (1 point)

b) Calculate (analytically) the total force $\vec{F}_1$ acting on body 1 at time $t = 0$. This gives

$$\vec{F}_1 = \begin{pmatrix} F_x \\ 0 \end{pmatrix} \ , \quad \text{mit} \ \ F_x = -\frac{Gm^2}{4R^2} \sum_{j=2}^{N} \frac{1}{\sin((j-1)\frac{\pi}{N})} \ .$$

(3 points)

c) From Newton's equation of motion, calculate the velocity v of the bodies. This results in:

$$v^2 = \frac{Gm}{4R} \sum_{j=2}^{N} \frac{1}{\sin((j-1)\frac{\pi}{N})} \ . \qquad (1)$$

(1 point)

d) Write a code which simulates the movement of the N bodies via a numerical solution of the set of coupled differential equations. Show that with the velocities given in eq. (1), the bodies indeed perform circular orbits as in part a). (6 points)

e) Check whether these orbits are stable or unstable with respect to small changes in the initial conditions, such as displacing one of the bodies at time $t = 0$ by a small amount. (2 points)