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1 Exercise Set 3

Due: **10:00 25 April 2022**

Discussion: **13:00 29 April 2022**

Online submission at via ILIAS in the directory Exercises / Übungen -> Submission of Exercises / Rückgabe des Übungsblätter

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3 1. Averaging spectral radio data [50 Points]

In this exercise you will work on a data set of artificial radio observation `radio-map.fits`. It is a map of 5x5 spectra with 201 measured frequency/velocity channels (channel width is unity). If you number all spectra from 1 to 25 we use the following scheme to assign spectra to positions on the map (spectrum 1 is the top left spectrum):

a. Compute the total "integrated-intensity" map of the observations, i.e. integrate the spectra over the full spectral range for all positions and plot the 5x5 map in a suitable way. **10 Points**

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
from astropy.io import fits
from astropy.utils.data import get_pkg_data_filename
```

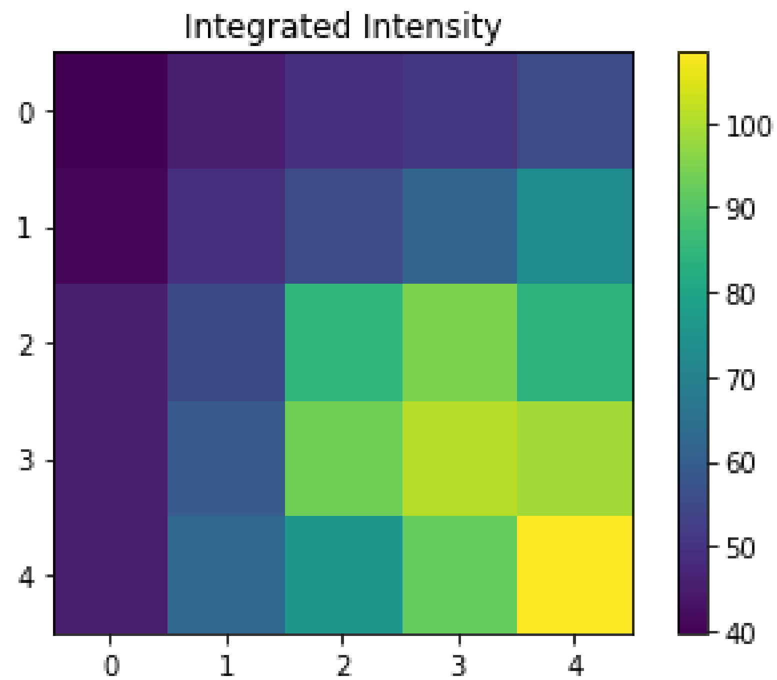
[2]:
```
data_file = get_pkg_data_filename('radio-mapfits.sec')
fits.info(data_file)
```

```
Filename: radio-mapfits.sec
No.    Name      Ver    Type      Cards   Dimensions   Format
  0  PRIMARY       1 PrimaryHDU      13   (201, 5, 5)   float64
```

[3]:
```
#extracting the data from the fits file
spectral_dataset = fits.getdata(data_file, ext = 0)
```

[4]:
```python
#integrated intensity by summing over intensities across all wavelength
integrated_intensity = np.sum(spectral_dataset, axis = 2)

plt.figure()
plt.imshow(integrated_intensity, cmap='viridis')
plt.colorbar()
plt.title('Integrated Intensity')
plt.show()
```



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labels?

We choose to plot a colormap because it helps us compare the total intensity of the 25 positions. From this we can easily identify that intensity increases from upper left corner to bottom right corner of the map.

b. Compute two channel maps by integrating over the frequency channels 50-100 and 100-150. Compare the two maps. **10 Points**

[5]:
```python
from mpl_toolkits.axes_grid1 import make_axes_locatable

channel50to100 = np.sum(spectral_dataset[:,:,50:100],axis = 2) #summing intensities over 50-100 range
channel100to150 = np.sum(spectral_dataset[:,:,100:150],axis = 2) #summing intensities over 100-150 range

#plotting the color maps
fig = plt.figure(figsize= (15,8))
```

```
ax1 = fig.add_subplot(121)
im1 = ax1.imshow(channel50to100, cmap='viridis', vmin=0, vmax=80)
divider = make_axes_locatable(ax1)
cax = divider.append_axes('right', size='5%', pad=0.05)
fig.colorbar(im1, cax=cax, orientation='vertical')
ax1.set_title('Frequency band 50-100')


ax2 =  fig.add_subplot(122)
im2 = ax2.imshow(channel100to150, cmap='viridis', vmin=0, vmax=80)
divider = make_axes_locatable(ax2)
cax = divider.append_axes('right', size='5%', pad=0.05)
fig.colorbar(im2, cax=cax, orientation='vertical')
ax2.set_title('Frequency band 100-150')

plt.show()
```

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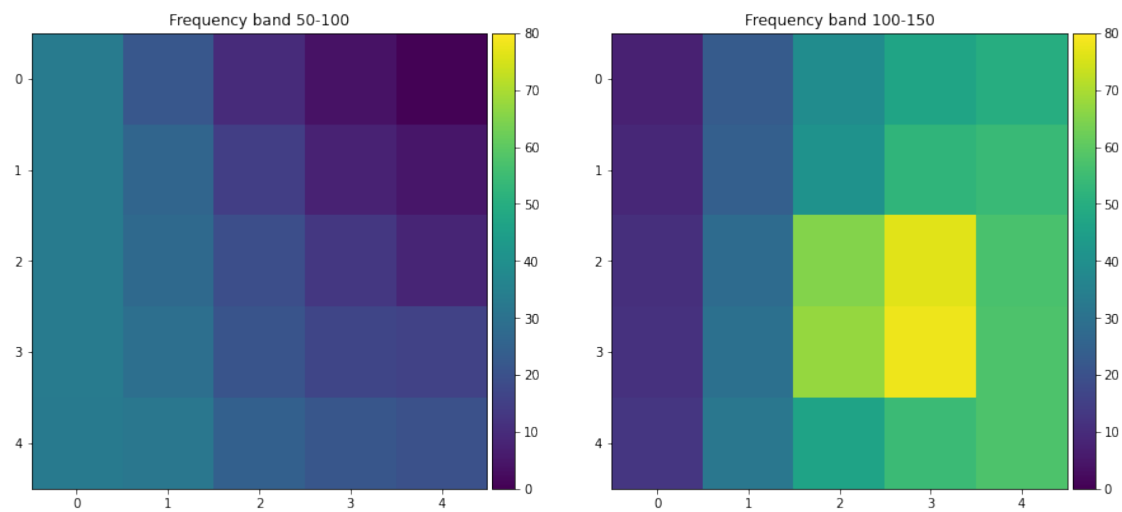


Labels?

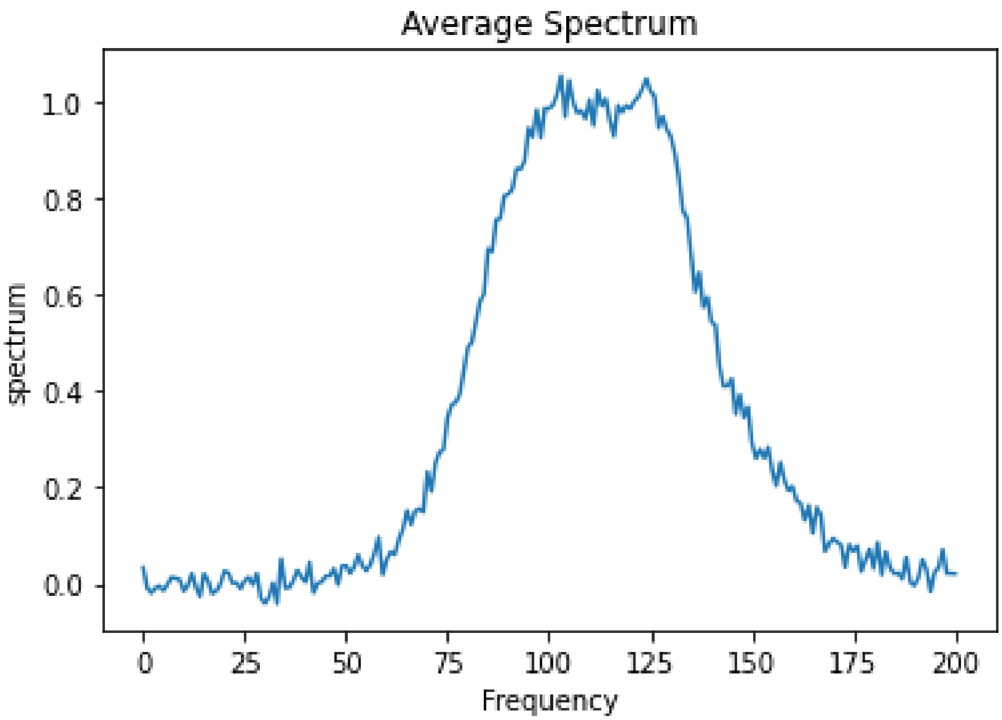
Comparing the two maps, we can clearly see from the color-bars that the intensity in the range 100-150 is higher (max intensity is ∼ 70) than the intensity in range 50 to 100 (max intensity is ∼ 30). We wished to make a common color bar for both the plots for easier comparison but could not figure our how to do that.

c. Compute the average spectrum, by averaging all 25 positions. **10 Points**

[6]:
```
#calculating average over 25 positions

sum1 = np.sum(spectral_dataset, axis = 0)
average = np.sum(sum1, axis = 0)/25
plt.plot(average)
plt.xlabel('Frequency')
```

```
plt.ylabel('spectrum')
plt.title('Average Spectrum')
plt.show()
```



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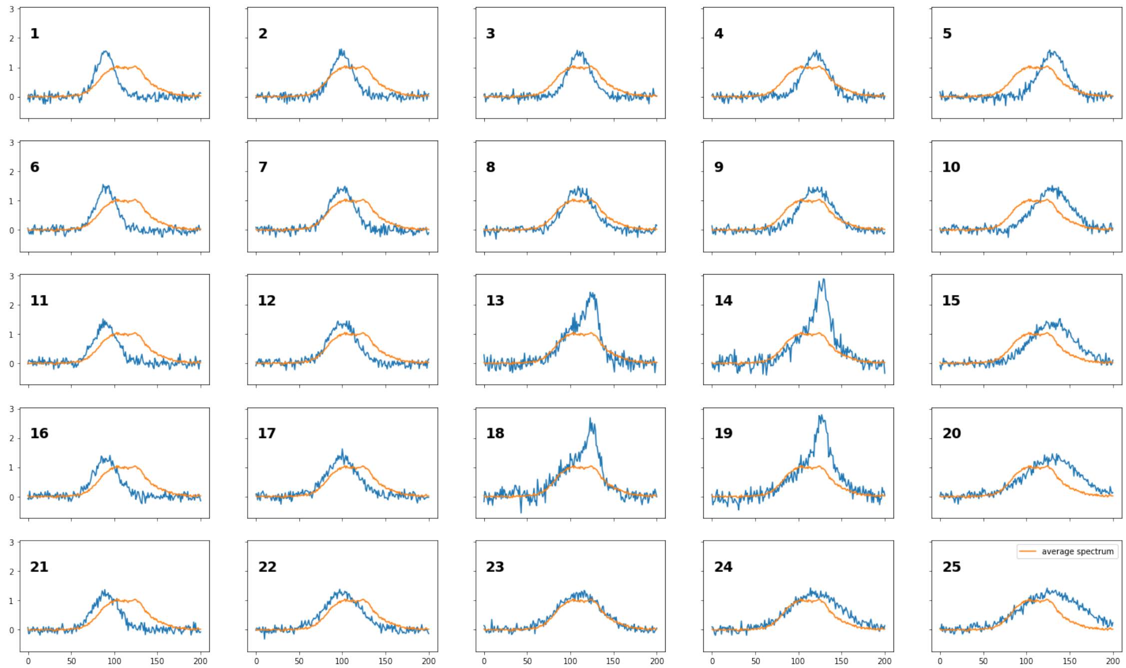
d. Plot every spectrum and overlay the average spectrum. Describe how the emission changes across the map. In particular how the line center position, the peak height and the line widths behave. **20 Points**

[7]:
```
figure, axes = plt.subplots(5,5,figsize = (25,15),sharex = True,sharey = True)
for i in range(5):
    for j in range(5):
        n = 5*(i)+ (j+1)
        axes[i][j].plot(spectral_dataset[i][j])
        axes[i][j].plot(average, label= 'average spectrum')
        axes[i][j].text(2.0,2.0, n, fontsize = 18, fontweight = 'semibold')

plt.legend()
```

[7]: <matplotlib.legend.Legend at 0x1143ced30>



labels?

We plot every spectrum separately but with common axes with average spectrum overlaid on top, this way we can see how the peak position, peak height and line width change as compared to average spectrum across the map.

Peak Position: As we go from left to right, the peak shifts towards higher frequencies. This behaviour is consistent throughout the map and also confirms what we see in part **b.**.

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Peak Height: The peak height is approximately similar throughout the map except for panels, 13, 14, 18 and 19 where it is relatively higher.

Line Width: The lines broaden as we go from top to bottom and left to right. On the upper left panels, the lines are narrower and towards the bottom right panels, we see broader lines. This can also be used to explain why we see a higher total intensity in bottom right in part **a.** since there is significant emission accross the entire frequency range.

4 2. Bayesian inference - conditional probabilities [50 Points]

Consider the proverbial bad penny, for which prior information has indicated that there is a probability of $P_r(B) = 99$ that it is unbiased ("ok"); or a probability of P(B)=0.01 that it is double-headed ("dh").

a. Draw a tree diagram and label and label the nodes with the corresponding events and the vertices with the corresponding probabilities. **5 Points**

```
                                |
                                |
[Fair coin]___0.99____________|____________0.01___[Unfair coin]
          |                                              |
```

format to be on one page

```
                                 |                                                        |
[heads]___0.50______|____0.50___[tails]           [heads]___1.00_____|_____0.00____[tails]
```

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also include symbols (A, $\bar{B}$, P(B|A), etc)

b. Suppose that the coin is fair, what is the probability P(A|B) of obtaining n heads in a row? **10 Points**

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```
=> P(A|B) = P(n heads in a row| fair coin) = (1/2)^n
```

c. Now suppose, the coins is unfair, i.e. double-headed, what is the probability P(A|B) of obtaining n heads in a row in this case? **5 Points**

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```
=> P(A|B) = P(n heads in a row | unfair coin) = 1
```

d. Give the probability P(A) = P(n heads in a row) given our initial priors. **10 Points**

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```
=> P(A) = P(n heads in a row | fair coin)P(fair coin) + P(n heads in a row | unfair coin)P
        = {(1/2)^n * 0.99} + {1*0.01}
```

e. Give the (Bayesian) posterior probability P(B|A) of obtaining n heads in a row. **10 Points**

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```
=> P(B|A) = P( unfair coin| n heads in a row) = P(n heads in a row|unfair coin)P(unfair coi
                in a row)
          = 0.01/((1/2)^n*0.99 +0.01)
```

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f. What are the odds on the penny being fair, i.e. what is the (Bayesian) posterior probability P(B|A) given this information, if we obtain 2 heads in a row? What is the probability if we obtain 7 heads in a row? In the last case (7 heads) how might we consider the fairness of the coin? Or of the experimenter who provided us with the prior information? **10 Points**

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```
=> P(B|A) = P(fair coin|n heads in a row) = P(n heads in a row|fair coin)P(fair coin)/P(n h
             row)
           = (1/2)^n/((1/2)^n*0.99 +0.01)
   a) n=2
        P(B|A) = (0.25*0.99)/(0.25*0.99 + 0.01) ~ 0.96
   b) n=7
        P(B|A) = (0.008*0.99)/(0.008*0.99 + 0.01) ~ 0.44

=> We see that in case we obtain  7 heads in a row, the probability of the coin being fair
=> This calculation is also indicative of the fact that the prior that was provided by the
```

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