Data Analysis in Astronomy and Physics (SoSe22)

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Regression - Fitting a paraboloid [data exercise]

In this problem we will fit a linear function $f(x, y) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 y^2$ to the data from datafile: `paraboloid_data.dat`.
Note: although $f(x, y)$ is technically quadratic in x and y, it is still linear in terms of the fitting parameters β_i
a) Perform the linear regression to fit a linear function $f(x, y) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 y^2$ and plot $f(x)$ together with the data points.
b) Plot the residuals and compute R^2.

Vibrations on a string

Consider a simplified 1-D example of standing waves on a vibrating string. The string is $1m$ in length, and the measured displacement is given in `string.dat`. We will want to analyse the amplitudes of the fundamental (dominant) mode and the overtones, which can be done by performing linear regression on the residuals.
a) Perform a linear regression to find the amplitude of the dominant mode. Plot the fit along with the measurements provided. What is R^2?

> *Hint:* Use the equation for an oscillating string, $f(x) = \sum_{n=1}^{N} \beta_n \sin(\frac{n\pi}{L}x)$, for $(N-1)$ overtones on the fundamental mode.

b) Successively repeat **a)** by successively including the next five overtones in your model. Summarise the results and the corresponding R^2 in a table.
c) Can you calculate more overtones? If so, how many and what are they? Are there any benefits or issues in calculating more overtones? Which is the model of best-fit?