1、

$$E_0(f) = \frac{h_0^3}{3} f''(\xi) = \frac{(b-a)^3}{24} f''(\xi)$$

$$E_1 = -\frac{h_1^3}{12} f''(\xi) = -\frac{(b-a)^3}{12} f''(\xi)$$

$$\Rightarrow |E_1(f)| = \left|\frac{(b-a)^3}{12} f''(\xi)\right| = 2\left|\frac{(b-a)^3}{24} f''(\xi)\right|$$

$$= 2|E_0(f)|$$

3.(a)

If $f=1$, then $I_2(f) = \frac{2}{3}(3) = 2$

and $\int_{-1}^{1} f(x)dx = \int_{-1}^{1} 1 dx = 2$

If $f=x$, $I_2(f) = \frac{2}{3}(-1+1) = 0$

$\int_{-1}^{1} x\,dx = 0 = I_2(f)$

If $f=x^2$, $I_2(f) = \frac{2}{3}(\frac{1}{2}+\frac{1}{2}) = \frac{2}{3}$

$\int_{-1}^{1} x^2 dx = \frac{2}{3} = I_2(f)$

If $f=x^3$, $I_2(f) = \frac{2}{3}(0) = 0$

$\int_{-1}^{1} x^3 dx = 0 = I_2(f)$

If $f=x^4$, $I_2(f) = \frac{2}{3}(\frac{1}{4}+\frac{1}{4}) = \frac{1}{3}$

$\int_{-1}^{1} x^4 dx = \frac{2}{5} \neq I_2(f)$

thus degree of exactness is 3

$p = r+2 = 5$

3.(b)

If $f=1$, then $I_4(f) = \frac{1}{4}(8) = 2$

$\int_{-1}^{1} 1 dx = 2 = I_4(f)$

If $f=x$, $I_4(f) = 0 = \int_{-1}^{1} x dx$

If $f=x^2$, $I_4(f) = \frac{2}{3} = \int_{-1}^{1} x^2 dx$

If $f=x^3$, $I_4(f) = 0 = \int_{-1}^{1} x^3 dx$

If $f=x^4$, $I_4(f) = \frac{1}{4}(\frac{56}{27}) = \frac{14}{27} \neq \int_{-1}^{1} x^4 dx$

thus degree of exactness is 3

$p = r+2 = 5$

5

for $f=1$

$$I_w(f)=\int_0^1 \sqrt{x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^1 = \frac{2}{3} = a$$

for $f=x$

$$I_w(f)=\int_0^1 x^{\frac{3}{2}}\,dx = \frac{2}{5} = \frac{2}{3}x_1 \Rightarrow x_1 = \frac{3}{5}$$

for $f=x^2$

$$I_w(f)=\int_0^1 x^{\frac{5}{2}}\,dx = \frac{2}{7} \neq \frac{2}{3}\cdot\left(\frac{3}{5}\right)^2 \simeq a f(x_1)$$

thus $r=1$, $a=\frac{2}{3}$, $x_1=\frac{3}{5}$

6.

for $f=1$

$$I(f)=\int_0^1 dx = 1 = \alpha_1+\alpha_2$$

for $f=x$

$$I(f)=\int_0^1 x\,dx = \frac{1}{2} = \alpha_2+\alpha_3 \Rightarrow \begin{array}{l}\alpha_1=\frac{2}{3}\\ \alpha_2=\frac{1}{3}\\ \alpha_3=\frac{1}{6}\end{array}$$

for $f=x^2$

$$I(f)=\int_0^1 x^2\,dx = \frac{1}{3} = \alpha_2$$