

1.

$$\text{forward SDE} \quad dx_t = f(x_t, t)dt + g(x_t, t)dW_t$$

$$\Rightarrow \text{Fokker-Planck} : \frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}(fP) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(g^2P)$$

$$\text{continuity equation} : \frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}(uP), \quad u = \frac{dx_t}{dt}$$

$$\Rightarrow -\frac{\partial}{\partial x}(uP) = -\frac{\partial}{\partial x}(fP) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(g^2P)$$

$$\Rightarrow uP = fP - \frac{1}{2}\frac{\partial}{\partial x}(g^2P)$$

$$u = f - \frac{1}{2P} \cdot P \frac{\partial g^2}{\partial x} - \frac{g^2}{2} \frac{\partial P}{\partial x} \frac{1}{P}$$

$$= f - \frac{1}{2} \frac{\partial g^2}{\partial x} - \frac{g^2}{2} \frac{\partial}{\partial x}(\log P)$$

$$\Rightarrow dx_t = \left[ f(x_t, t) - \frac{1}{2} \frac{\partial}{\partial x} g^2(x_t, t) - \frac{g^2(x_t, t)}{2} \frac{\partial}{\partial x} (\log P(x_t, t)) \right] dt$$