

1.

$$\text{forward SDE} \quad dX_t = f(X_t, t)dt + g(X_t, t)dW_t$$

$$\Rightarrow \text{Fokker-Planck} : \frac{\partial P}{\partial t} = -\frac{\partial}{\partial X}(fP) + \frac{1}{2} \frac{\partial^2}{\partial X^2}(g^2 P)$$

$$\text{continuity equation} : \frac{\partial P}{\partial t} = -\frac{\partial}{\partial X}(uP), \quad u = \frac{dX_t}{dt}$$

$$\Rightarrow -\frac{\partial}{\partial X}(uP) = -\frac{\partial}{\partial X}(fP) + \frac{1}{2} \frac{\partial^2}{\partial X^2}(g^2 P)$$

$$\Rightarrow uP = fP - \frac{1}{2} \frac{\partial}{\partial X}(g^2 P)$$

$$u = f - \frac{1}{2P} \cdot P \frac{\partial g^2}{\partial X} - \frac{g^2}{2} \frac{\partial P}{\partial X} \frac{1}{P}$$

$$= f - \frac{1}{2} \frac{\partial g^2}{\partial X} - \frac{g^2}{2} \frac{\partial}{\partial X}(\log P)$$

$$\Rightarrow dX_t = \left[f(X_t, t) - \frac{1}{2} \frac{\partial}{\partial X} g^2(X_t, t) - \frac{g^2(X_t, t)}{2} \frac{\partial}{\partial X}(\log P(X_t, t)) \right] dt$$