1.

$$h(x_1,x_2) = \sigma(b + w_1x_1 + w_2x_2) = \sigma(z)$$

$$\frac{\partial h}{\partial b} = \sigma'(z),\ \frac{\partial h}{\partial w_1} = \sigma'(z)\cdot x_1,\ \frac{\partial h}{\partial w_2} = \sigma'(z)x_2$$

$$\nabla_\theta L = \left(\frac{\partial L}{\partial b}, \frac{\partial L}{\partial w_1}, \frac{\partial L}{\partial w_2}\right) = \left((h-y)\frac{\partial h}{\partial b}, (h-y)\frac{\partial h}{\partial w_1}, (h-y)\frac{\partial h}{\partial w_2}\right)$$

$$= \left((\sigma(z)-3)\sigma'(z), (\sigma(z)-3)\sigma'(z)x_1, (\sigma(z)-3)\sigma'(z)x_2\right)$$

$$\Rightarrow \theta' = \theta_0 + \alpha\nabla_\theta L,\ \alpha \in \mathbb{R}$$

$$= \theta_0\left((\sigma(z)-3)\sigma'(z), (\sigma(z)-3)\sigma'(z)x_1, (\sigma(z)-3)\sigma'(z)x_2\right),\ \sigma'(z) = \sigma(z)(1-\sigma(z))$$

2.(a)

$$\sigma(x) = \frac{1}{1+e^{-x}} \Rightarrow \sigma'(x) = \frac{e^{-x}}{(1+e^{-x})^2} = \underline{\sigma(x)(1-\sigma(x))}$$

$$\Rightarrow \sigma''(x) = \frac{d}{dx}(\sigma(x)(1-\sigma(x))) = \sigma'(x)(1-\sigma(x)) - \sigma(x)\sigma'(x)$$

$$= \underline{\sigma'(x)(1-2\sigma(x))}$$

$$\Rightarrow \sigma'''(x) = \underline{\sigma''(x)(1-2\sigma(x)) - 2(\sigma'(x))^2}$$

(b)

$$\sigma(x) = \frac{1}{1+e^{-x}} = \frac{e^{\frac{x}{2}}e^{-\frac{x}{2}}}{(e^{\frac{x}{2}}+e^{-\frac{x}{2}})e^{-\frac{x}{2}}} = \frac{1}{2}\frac{2e^{\frac{x}{2}}+e^{-\frac{x}{2}}-e^{-\frac{x}{2}}}{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}$$

$$= \frac{1}{2}\left(1 + \frac{e^{\frac{x}{2}}-e^{-\frac{x}{2}}}{e^{\frac{x}{2}}+e^{-\frac{x}{2}}}\right) = \underline{\frac{1}{2}\left(1+\tanh\frac{x}{2}\right)}$$