inverse game

xxy

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Network Games of Linear-Quadratic

payoff:

$$U_i = a_i b_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in v} G_{ij} a_j$$

Taking the first-order derivative of the payoff u_i:

$$\frac{\partial U_i}{\partial a_i} = b_i - a_i + \beta (G\boldsymbol{a})_i$$

pure strategy Nash equilibrium action $\boldsymbol{a}$:

$$(I - \beta G) a = b$$

It is often easier to observe the individual actions a,

$$\begin{aligned} & \min_{\mathbf{G}, \mathbf{B}} f(\mathbf{G}, \mathbf{B}) \\ & = \|(\mathbf{I} - \beta \mathbf{G})\mathbf{A} - \mathbf{B}\|_F^2 + \theta_1 \|\mathbf{G}\|_F^2 + \theta_2 \|\mathbf{B}\|_F^2, \\ & \text{s.t. } G_{ij} = G_{ji},\ G_{ij} \geq 0,\ G_{ii} = 0 \text{ for } \forall i, j \in \mathcal{V}, \\ & \quad \|\mathbf{G}\|_1 = N, \end{aligned}$$

Compressed sensing

considers the model problem of recovering an input vector:

$$y = Af + e, \quad f \in \mathbb{R}^n, \quad A \in M_{m\times n}$$

if has full rank, then one can clearly recover the plaintext from Af

Problem formulation

Consider the parameter identification of the following stochastic system:

$$y_{k+1} = \theta^T \varphi_k + w_{k+1}, \quad k \geq 0 \tag{1}$$

- θ is the unknown r-dimensional parameter vector
- $\varphi_k \in \mathbb{R}^r$ is the regressor vector
- y_{k+1} w_{k+1} is the system output
- w_{k+1} is noise

Denote the family of σ-algebras $\{\mathcal{F}_k\}$ as:

A time - independent digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is called strongly connected if for any $i, j \in \mathcal{V}$, there exists a directed path from i to j.

A nonnegative stochastic matrix A is called doubly stochastic if $A\mathbf{1} = \mathbf{1}$ and $\mathbf{1}^T A = \mathbf{1}^T$.

In the paper we consider a network with N agents. The inter - action relationship among agents is described by a time - varying digraph $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$, where k is the time, $\mathcal{V} = \{1, \ldots, N\}$ is the agent set, and $\mathcal{E}(k) \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. By $(i, j) \in \mathcal{E}(k)$ we mean that agent j can receive information from agent i at time k. Assume $(i, i) \in \mathcal{E}(k)$ for all $k = 1, 2, \ldots$. Denote the neighbors of agent i at time k by $N_i(k) = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(k)\}$. The adjacency matrix associated with the graph is denoted by $W(k) = [w_{ij}(k)]_{i,j=1}^N$, where $w_{ij}(k) > 0$ if and only if $(j, i) \in \mathcal{E}(k)$, otherwise $w_{ij}(k) = 0$.

The dynamics of agent i, $i = 1, \ldots, N$ is given by

$$y_{i,k+1} = \boldsymbol{\phi}_{i,k}^T \boldsymbol{\theta}^* + d_{i,k+1} \tag{1}$$