

# inverse game

xyy

tju

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# Network Games of Linear-Quadratic

payoff:

$$U_i = a_i b_i - \frac{1}{2} a_i^2 + \beta a_i \sum_{j \in v} G_{ij} a_j$$

Taking the first-order derivative of the payoff  $u_i$ :

$$\frac{\partial U_i}{\partial a_i} = b_i - a_i + \beta (Ga)_i$$

pure strategy Nash equilibrium action  $\mathbf{a}$ :

$$(I - \beta G)\mathbf{a} = \mathbf{b}$$

It is often easier to observe the individual actions  $\mathbf{a}$ ,

$$\begin{aligned} & \min_{\mathbf{G}, \mathbf{B}} f(\mathbf{G}, \mathbf{B}) \\ &= \|(\mathbf{I} - \beta \mathbf{G})\mathbf{A} - \mathbf{B}\|_F^2 + \theta_1 \|\mathbf{G}\|_F^2 + \theta_2 \|\mathbf{B}\|_F^2, \\ & \text{s.t. } G_{ij} = G_{ji}, G_{ij} \geq 0, G_{ii} = 0 \text{ for } \forall i, j \in \mathcal{V}, \\ & \|\mathbf{G}\|_1 = N, \end{aligned}$$

# Compressed sensing

considers the model problem of recovering an input vector:

$$y = Af + e, \quad f \in \mathbb{R}^n, \quad A \in M_{m \times n}$$

if  $A$  has full rank, then one can clearly recover the plaintext from  $Af$

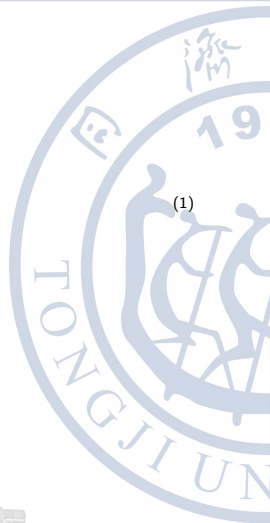
# Problem formulation

Consider the parameter identification of the following stochastic system:

$$y_{k+1} = \theta^T \varphi_k + w_{k+1}, \quad k \geq 0 \quad (1)$$

- $\theta$  is the unknown  $r$ -dimensional parameter vector
- $\varphi_k \in \mathbb{R}^r$  is the regressor vector
- $y_{k+1}$  is the system output
- $w_{k+1}$  is noise

Denote the family of  $\sigma$ -algebras  $\{\mathcal{F}_k\}$  as:



A time - independent digraph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  is called strongly connected if for any  $i, j \in \mathcal{V}$ , there exists a directed path from  $i$  to  $j$ .

A nonnegative stochastic matrix  $A$  is called doubly stochastic if  $A\mathbf{1} = \mathbf{1}$  and  $\mathbf{1}^T A = \mathbf{1}^T$ .

In the paper we consider a network with  $N$  agents. The interaction relationship among agents is described by a time - varying digraph  $\mathcal{G}(k) = (\mathcal{V}, \mathcal{E}(k))$ , where  $k$  is the time,  $\mathcal{V} = \{1, \dots, N\}$  is the agent set, and  $\mathcal{E}(k) \subseteq \mathcal{V} \times \mathcal{V}$  is the edge set. By  $(i, j) \in \mathcal{E}(k)$  we mean that agent  $j$  can receive information from agent  $i$  at time  $k$ . Assume  $(i, i) \in \mathcal{E}(k)$  for all  $k = 1, 2, \dots$ . Denote the neighbors of agent  $i$  at time  $k$  by  $N_i(k) = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}(k)\}$ . The adjacency matrix associated with the graph is denoted by  $W(k) = [w_{ij}(k)]_{i,j=1}^N$ , where  $w_{ij}(k) > 0$  if and only if  $(j, i) \in \mathcal{E}(k)$ , otherwise  $w_{ij}(k) = 0$ .

The dynamics of agent  $i$ ,  $i = 1, \dots, N$  is given by

$$y_{i,k+1} = \phi_{i,k}^T \theta^* + d_{i,k+1} \quad (1)$$