Chapter 1. Financial markets, prices and risk.

① Statistical techniques for analysing prices

② Prices, returns and volatilities — volatility clusters 波动集群, fat tails 肥尾分布, nonlinear dependence

< Prices, returns and Stock indices >

< Stock indices >

- stock market index 股票市场指数: how a representative portfolio of stock prices 股价组合. changes over time.
- price-weighted index: weights stocks based on their prices.
- value-weighted index: weights stocks according to the total market value of outstanding shares.
  - └ impact of change in stock price $\boxed{\text{proportional}}$ to overall market value. 股价变动 ∝ 整体市值.

eg. SP500 (US). FTSE100 (UK). TOPIX (JP). Dow Jones (US), Nikkei 225 (JP)

- total returns: include inflation.
- Bonds and Equities 债券和资产.

① The very long-run returns on equities are much higher than those on bonds.

② Bonds can have a very good short and medium-term performance.

③ Especially if inflation, → hence interest rates is falling. (通货膨胀↓, 利息↓)

< prices and returns >

P_t: asset price, t= frequency (yearly, weekly, monthly). — $P_{time,\ asset}$.

- Returns 回报: The relative change in the price of a financial asset over a given time interval.
  often expressed as a percentage.

① Simple returns: % changes in prices: $R_t = \dfrac{P_t - P_{t-1} + d}{P_{t-1}}$. ← dividents 股息.

└ used for accounting purpose, and investors.

② Continuously compounded returns 持续复利: The logarithm of gross return. ⇒ symmetric.

$$Y_t = \log(1+R_t) = \log\left(\frac{P_t}{P_{t-1}}\right) = \log(P_t) - \log(P_{t-1})$$

$$\log\left(\frac{1000}{200}\right) = -\log\left(\frac{200}{1000}\right)$$

$$P_{t+1} = P_t e^{R_t}.$$

└ easier math. used in derivatives pricing (Black-Scholes model)

$\lim_{\Delta t \to 0} Y_t = R_t$:
$$\begin{cases} \log(1000) - \log(990) = 0.01005, & \approx \frac{1000}{990} - 1 = 0.0101 \\ \log(1000) - \log(800) = 0.223, & \neq \frac{1000}{800} - 1 = 0.25. \end{cases}$$

- Portfolios: R_t, portfolios:

weighted sum of returns of K individual assets:

$$R_{t,p} = \sum_{k=1}^{K} W_k R_{t,k} = W' R_t.$$

$$Y_{t,p} = \log\left(\frac{P_{t,p}}{P_{t-1,p}}\right) \neq \sum_{k=1}^{K} W_k \log\left(\frac{P_{t,k}}{P_{t-1,k}}\right) \Leftarrow \log\text{无法加减.}$$

✓ < 1.2 coding > SP500 stats. — GitHub.

- 3 stylized factors: volatility clusters + fat tails + non-linear dependence.

< Voliatility > 易变性.

① unconditional voliatility is over an entire time period (σ).
② conditional vo.. is in a given time period, condition on before (σ_t).

σ_t: $t \to$ on a particular time period.

- Daily Volatility: $\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (y_i - \mu)^2}$. $\Leftarrow$ the s.d of returns.

Annualised: $\sqrt{250} \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (y_i-\mu)^2}$

not 365, 250 is the typical open market days.

- Volatility cluster: 波动性聚类 — 观察到价格的大变化往往伴随大的变化，小的同理.

↳ comes in many cycles, both long-term and short-term.

Engle (1982): ARCH model (自回归条件异方差)

- Autocorrelations: 自相关. (Graphical Methods)

∗ correlations measure how 2 variables (x, y) move together:

$$\text{Corr}(x,y) = \frac{\sum_{t=1}^{T} (x_t - \mu_x)(y_t - \mu_y)}{\sigma_x \sigma_y}$$

* Autocorrelations measure how a single variable is correlated with itself.

往前推N个 (previous).

1 lag: $\hat{\beta}_1 = Corr(X_t, X_{t-1})$

i lags: $\hat{\beta}_i = Corr(X_t, X_{t-i})$

The coef of an ACF (autocorr func): observations and lags.

<The LB test for autocorrelations> 检验自相关系数在N个 lags 上的联合显著性.

Ljung-Box: 对于 randomness 的检验. 对于时间序列是否存在滞后相关

assume the data is Normally Distributed: $\sim \chi^2$.

<Fat Tails & Nonnormality> 肥尾.

- A random variable is said to have fat tails if it exhibits more extreme outcomes than a normally distributed random variable with the same mean and variance.

(The mean-variance model assumes normality)

<The Student t-distribution>

The degrees of freedom, (v): $\begin{cases} v = \infty \to \text{normal} \\ v < 2 \to \text{superfat tails} \end{cases}$

(for typical stock $3 < v < 5$.

- t分布: 根据小样本来估计母体呈 Normal D 且 s.d 未知的 expectation.

若母体 s.d 已知. 当 sample size 趋向大时. 根据中央极限定理. 渐进为 Normal D.

① Peak is higher than normal

② Sides are lower than normal

③ Tails are much thicker (fatter) than normal.

<Identification of fat tails>: statistical tests and graphical model.

① Jarque-Bera (JB) (偏态. 峰度)

test for normality and may points to fat tails rejected.

$\frac{k}{6} Skewness^2 + \frac{T}{24}(Kurtosis - 3)^2 \sim \chi^2(2)$

Kolmogrov-Smirnov (KS)

Based on minimum distribution estimation.

comparing sample with a reference distribution. (normal eg.)

② QQ plot. Quantile-Quantile Plots. 通过比较两个 prob. distri 的分位数对这两个 prob. distri 进行比较

< Non-linear dependence >

└ the dependence between different return series changes according to market conditions.

- Correlation: linear: $y = \alpha x + \varepsilon$.

  └ normal distribution, dependence is linear:

  $E(Y|X)$ is a linear function of X.

- NLD:

  └ different returns are relatively independent during normal times,

  but highly dependent during crises. (危机时则高度相关)

* ① Assumption of normality may lead to a gross underestimation of risk.

② Volatility is the correct measure of risk $\boxed{\text{iff}}$ the returns are normal.

If they follow the Student-t or any fats, then volatility will only be partical correct.

<Random Numbers and Monte-Carlo >

- The fundamental input in MC analysis is a long sequence of random numbers.
- It's impossible to obtain pure random numbers:

  { there's no natural phenomena that is purely random.
    computers are deterministic by definition.

- Simulate a random walk.

  $R_t \sim N(\mu, \sigma^2)$. ⇒ simple returns

  $\bar{P}_{t+1} = P_t \cdot R_{t+1}$ ⇒ simulated tomorrow price.

< Copulas > 耦合. (关联结构)

- create a multivariate distribution with a range of types of dependence.

⇒处理随机变量相关性. 由一组随机变量的边际分布来确定它们的联合分布.

marginal returns ⟶ uniform distribution by prob. integral transformation.

一个关联结构(dependence structure)可以表达为一个基于平均分布之上的联合分布.

而关联结构(copula)即是边缘均匀随机变量之上的联合分布.

<The Gaussian Copula>

$$C(u,v) = \Phi_p(\Phi^{-1}(u), \Phi^{-1}(v))$$ Φ_p: bivariate normal.

Φ: normal distribution. $\Phi^{-1}(\cdot)$ inverse normal, $u, v \in [0,1]$ uniform Random Variable.

• The probability integral transformation.

① Let a random variable X have a continous distribution F. and define a new random variable U as:

└ 任何连续分布均可转化成统一变量.

$$U = F(X)$$

Then, regardless of original distribution F, $U \sim \text{Uniform}(0,1)$.

② Let F be the distribution of X, G the distribution of Y and H the joint distribution of (X, Y). Assume that F and G are continous.

Then there exists a unique copula C such that:

$$H(X,Y) = C(F(X), G(Y))$$

Sklar's theorem to decompose joint density:

$$h(X,Y) = f(X) \cdot g(Y) \cdot c(F(X), G(Y)).$$

<Exceedance Correlation>

└ Show the correlations of (standardized) stock returns X and Y as being conditional on exceeding some threshold:

$$\tilde{\rho}(p) = \begin{cases} \text{Corr}[X,Y \mid X \le Q_X(p) \text{ and } Y \le Q_X(p)], & \text{for } p \le 0.5 \\ \text{Corr}[X,Y \mid X > Q_X(p) \text{ and } Y > Q_Y(p)], & \text{for } p > 0.5. \end{cases}$$

Where $Q_X(p)$ and $Q_Y(p)$ are the p-th quantiles of X and Y given a distribution assumption.