# COMS30015 Computational Neuroscience

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## 1 PART 1. Data Analysis

### 1.1 Question 1.

1. Coefficient of Variation(CV) of neuron A is: 1.2423490962154482
2. Fano Factor of neuron A with different time bins:
   - bin = 100 ms: 1.2269881697035632
   - bin = 300 ms: 1.5989561565522086
   - bin = 600 ms: 1.987523409106593
   - bin = 1 s: 3.046003535490889
3. CV of neuron A with trial_id = 0 is: 1.1344903381885438
   CV of neuron A with trial_id = 1 is: 1.0964245085584634
4. Fano Factor of neuron A with different trial_id:
   - bin = 100 ms: id_0 is: 1.2243819693974507 id_1 is: 1.1496323529411765
   - bin = 300 ms: id_0 is: 1.6535785329660964 id_1 is: 1.3738448942041508
   - bin = 600 ms: id_0 is: 2.1242824490466900 id_1 is: 1.6100958987376404
   - bin = 1 s: id_0 is: 3.1755985080984264 id_1 is: 2.1650735294117647

As shown above, there's no significant difference between Coefficient of Variation(CV) and Fano Factor under different trial ids, because both id_0 and id_1 are stimuli with random points, while CV and Fano Factor are the representation of covariance, so there is no excessive difference in the case of random.

However, for some of the minor differences, the bins of 300 and 600 may cause part of the trail to not be calculated, and 1000 cannot be divisible by them. These reasons will lead to difference in values.

## 1.2 Question 2.

The d' value is: 0.05250895879262016

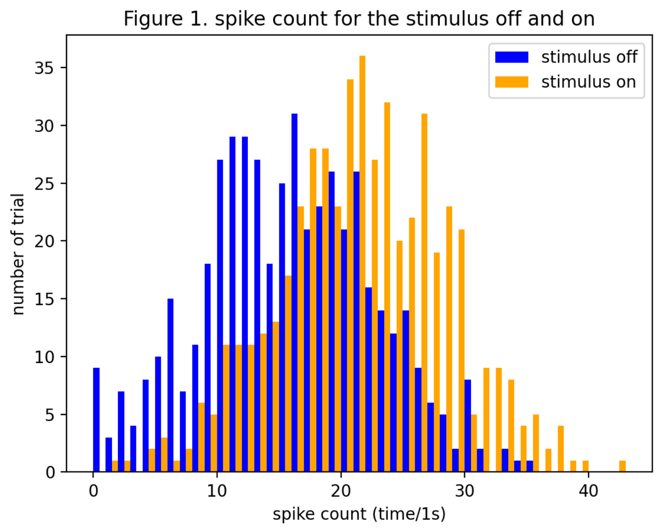



Figure 1: Spike count for the stimulus on and off on trials for neuron_A.

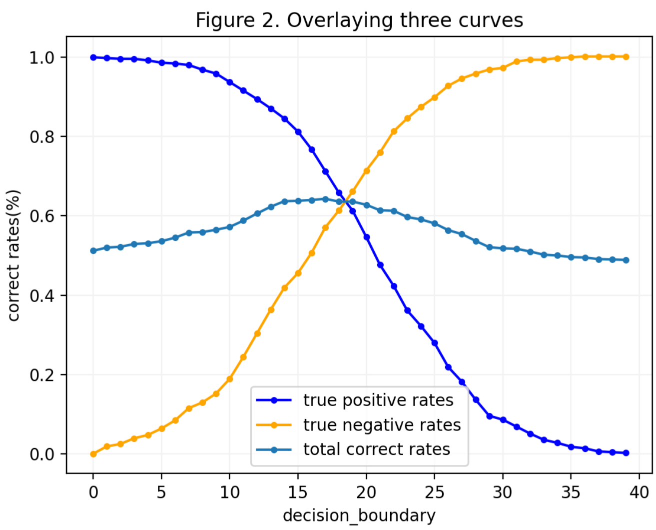



Figure 2: Overlaying three curves: true positive rate, true negative rate, total correct rate.

As shown in Figure 4, TPR represents a downward trend with the increase of the decision boundary, while FPR represents an upward trend. The total correct rates is kept at 50%. Those three lines intersect at a same point. To achieve max correct rates, the best decision boundary should be the intersection point, at about 18.5.

## 1.3 Question 3.

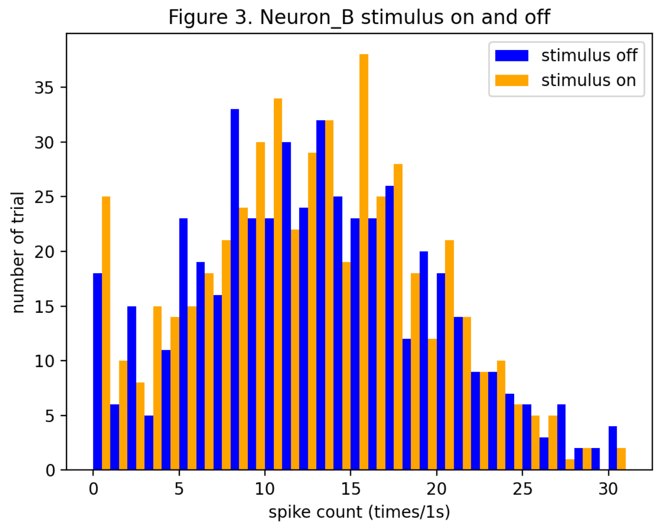



Figure 3: Spike count for the stimulus on and off on trials for neuron_B.

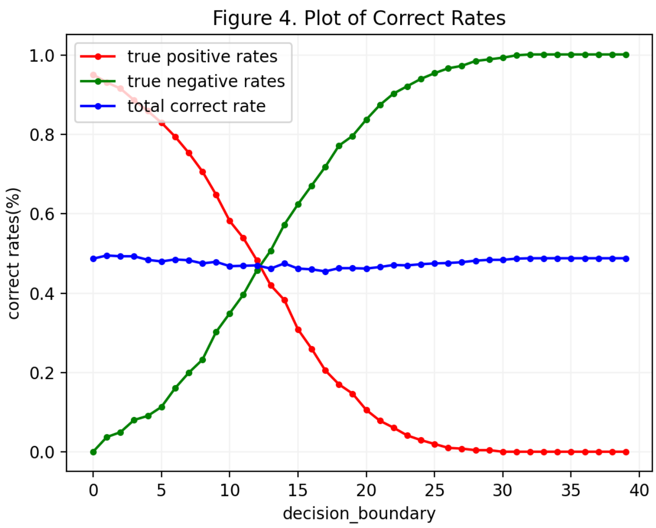



Figure 4: Plot of three curves: true positive rate, true negative rate, total correct rate.

The d' value is: 0.07793041688340883

- joint positive rate: 0.734375(to 6s.f)
- joint negative rate: 0.790554(to 6s.f)
- joint total rate: 0.761(to 3s.f)

The best decoding accuracy of Neuron_A is 0.625, while neuron_B is 0.465, and the joint rate is more accurate than both of them.

# 2 Auto-associative networks – Hopfield Networks

## 2.1 Question 1. MINIST dataset

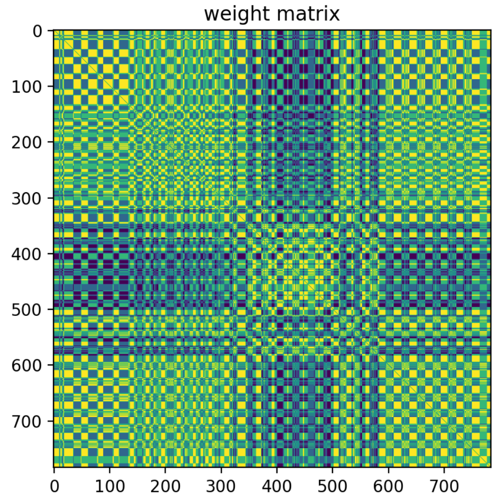
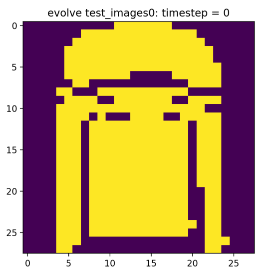
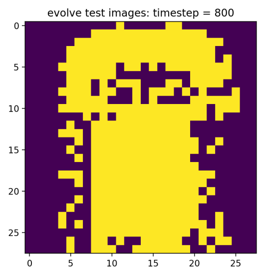



Figure 5: weight matrix

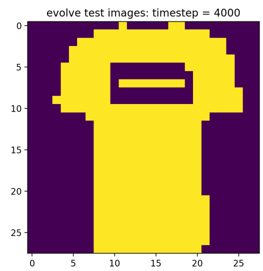
Test image 1:




(a) timestep=0



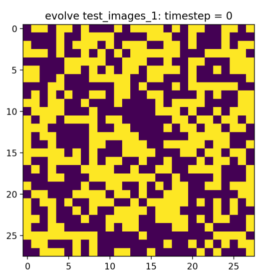

(b) timestep=800



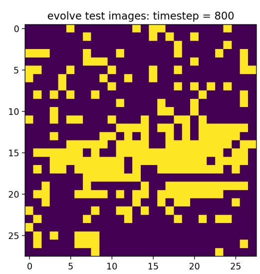

(c) timestep=4000

Figure 6: Network state at different time-steps for test image 1.

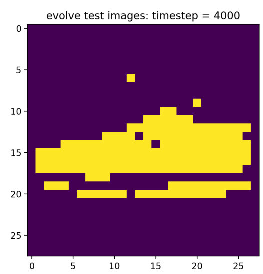
Test image 2:




(a) timestep=0




(b) timestep=800

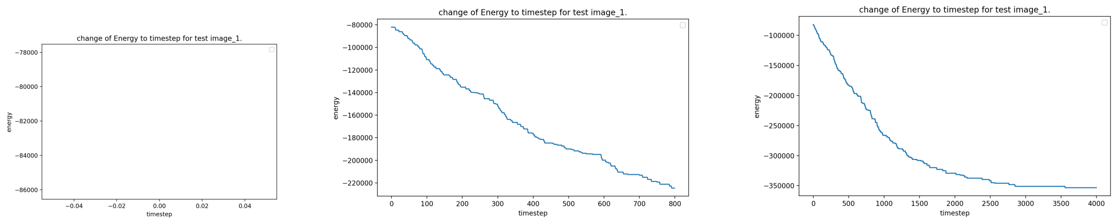



(c) timestep=4000

Figure 7: Network state at different time-steps for test image 2.

## 2.2 Question 2. Energy

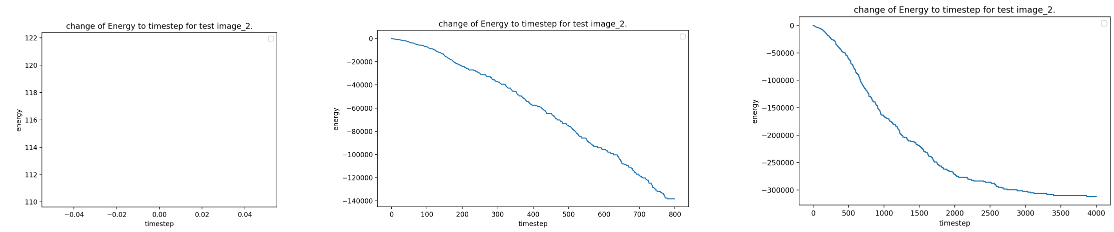
Test image 1:




(a) timestep=0 (b) timestep=800 (c) timestep=4000

Figure 8: Energy as a function of different timesteps for test image 1.

Test image 2:




(a) timestep=0 (b) timestep=800 (c) timestep=4000

Figure 9: Energy as a function of different timesteps for test image 2.

1. When time-step is 0, both of the energy figures are blank because the neurons are under the same state, so there is no change of energy.

2. For time-step = 1000 and 3000, the figures all show a downward trend. For Hopfield network, energy function is introduced as follows:

$$E = -\frac{1}{2}\sum\sum_{i\neq j} W_{i,j}y_iy_j$$

The energy change casue by $\delta y$ is:

$$\Delta E = -\frac{1}{2}\Delta y_i \sum_{j\neq i} W_{i,j}y_j$$

$$\begin{cases} W_{i,j}y_j > 0, & \delta y_i = 1 \\ W_{i,j}y_j < 0, & \delta y_i = -1 \end{cases} \tag{1}$$

It shows that $\delta E \leq 0$ is always true, the energy function of the network is a decreasing function. According to the defined update rules, decreases gradually with the increase of time until the network reaches a steady state. The steady state the network eventually reaches is called the Attractor. Points of attraction are a series of local minima of the energy function.

The state of each neuron starts from a random value, and then updates the state value every time to reduce the overall energy of the network until the state does not change. At this time, the energy value of the network reaches the minimum, which can be used as the termination judgment condition of Hopfield network training.

## 2.3 Question 3.

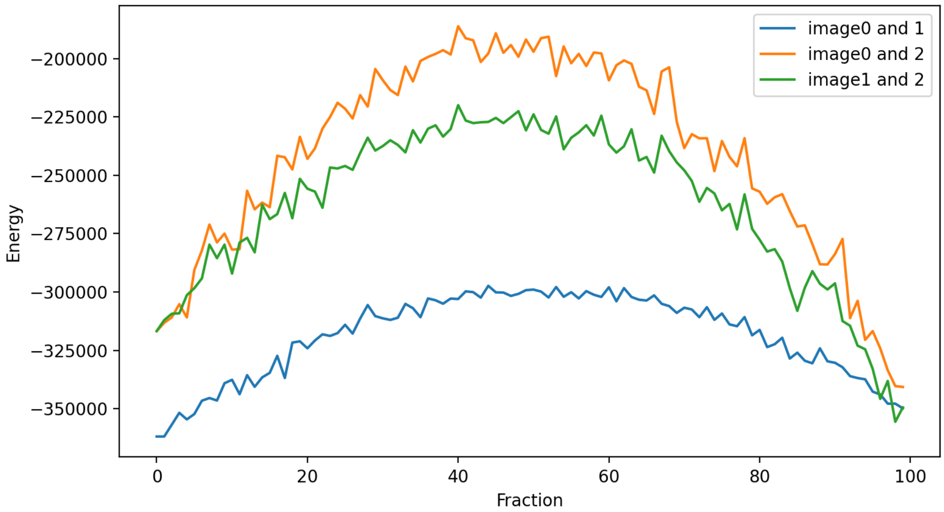



Figure 10: The energy function of the mixing fraction x.

As shown in the above figure, all three curves are parabolic. When the value of proportion X occupies a minimum/maximum value, in this case, mixed picture has a specific image has a very large proportion, and the change of energy is usually only related to this particular image. The change in energy is smaller when the proportion are similar, because based on model dynamics, two different network points(in this case, images) will constrain each other to reach a configuration point.

# 3 Integrate-and-fire neurons

## 3.1 Question 1.

According to the differential equation:

$$\tau_m \frac{dV_m}{dt} = (V_{rest} - V_m) + R_m I_{ext}, \quad V_m(t=0) = V_{rest}$$

where $I_{ext}$ is a constant input. The result of 1 second simulation is shown in Figure.11.

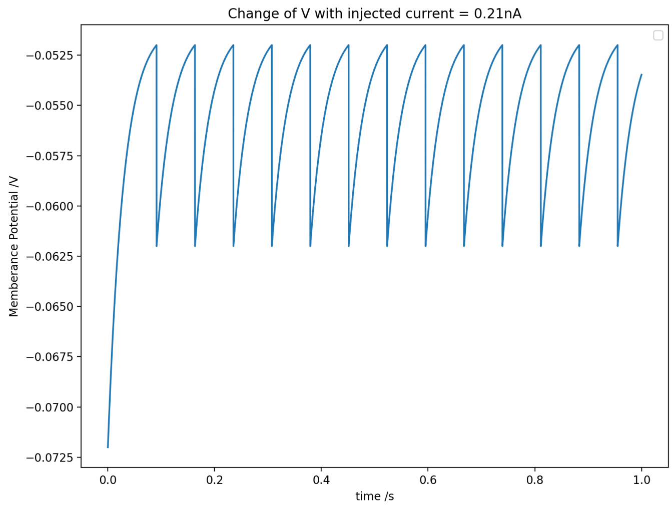



Figure 11: 1-sec simulation for the voltage as a function of time.

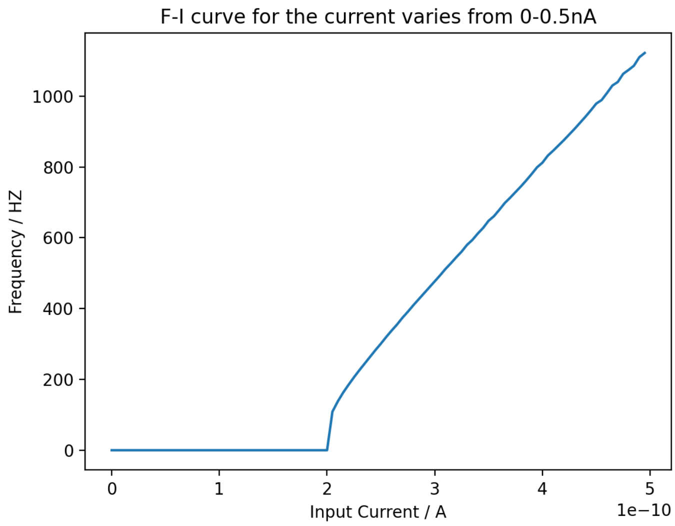



Figure 12: The spike frequency as a function of the input current(F-I curve).

The threshold input current = 0.2nA.

## 3.2 Question 2.

Figure 13 and Figure 14 show the simulation of neuron's voltage under two groups separately. Figure 15 shows the neuron's voltage with 100 synapses together.

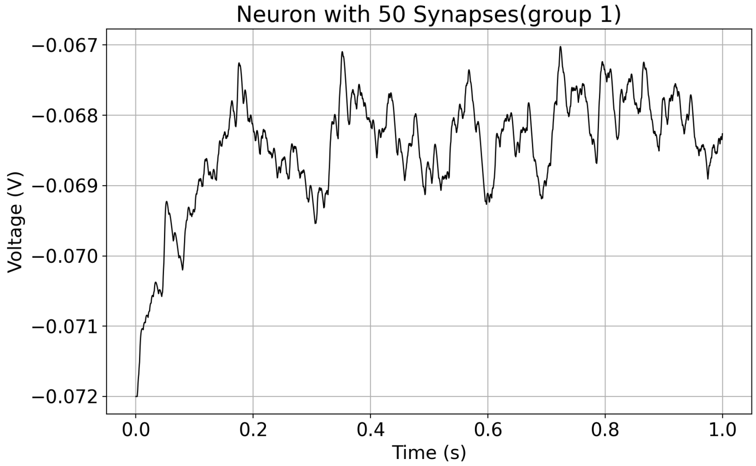



Figure 13: Neuron's voltage for 1s of Group 1.

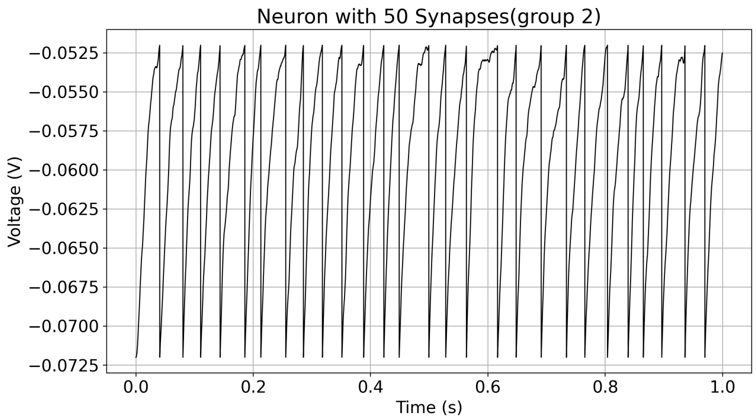



Figure 14: Neuron's voltage for 1s of Group 2.

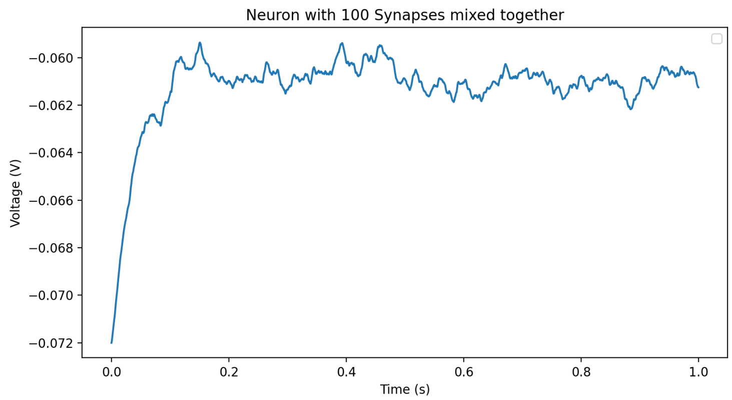



Figure 15: Neuron's voltage for 1s of Group 2.

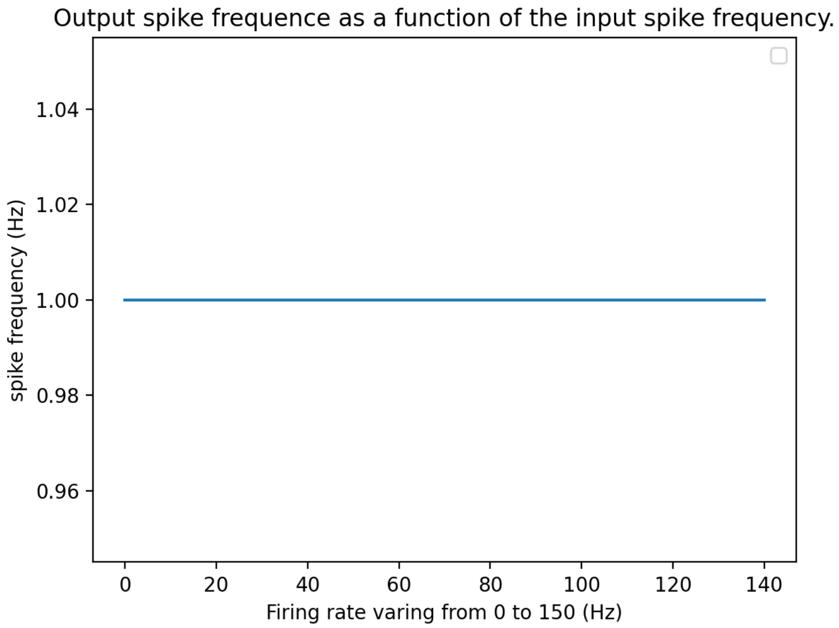



Figure 16: r1 fixed at 10Hz, r2 varying from 0 to 150Hz.

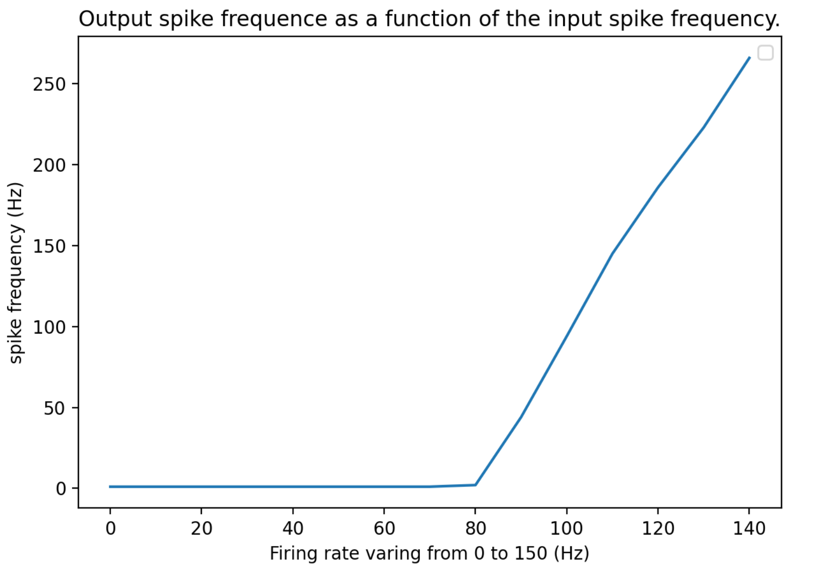



Figure 17: r2 fixed at 100Hz, r1 varying from 0 to 150Hz.

As shown in figure 16, when r1 is fixed at 10Hz and r2 is varying from 0 to 150 Hz, the plot is a straight line. This is because r1 is too small, the voltage created cannot reach the threshold value, and therefore no spike is generated.

Figure 17 shows that from the input of 80Hz, there is an output spike frequency and gradually rises. After 10 seconds, it rises to 266. The ascending process is [1, 2, 44, 94, 145, 186, 223, 266].This shape is because when the input is 0 to 80Hz, the voltage does not exceed the threshold voltage, resulting in no output spike at this time. However, when the input frequency surpass 80Hz, more and more voltage are created, and the output spike frequency becomes larger and larger with the input spike frequency.

## 3.3 Question 3.

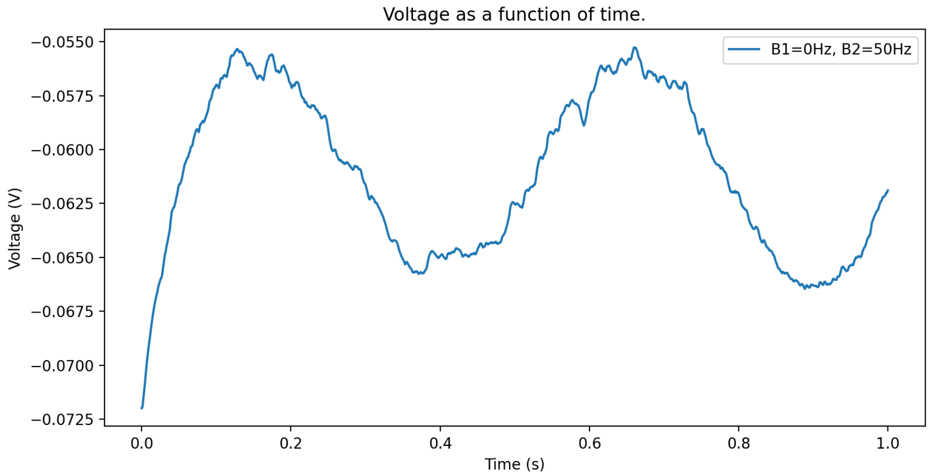



Figure 18: B1=0Hz, B2=50Hz

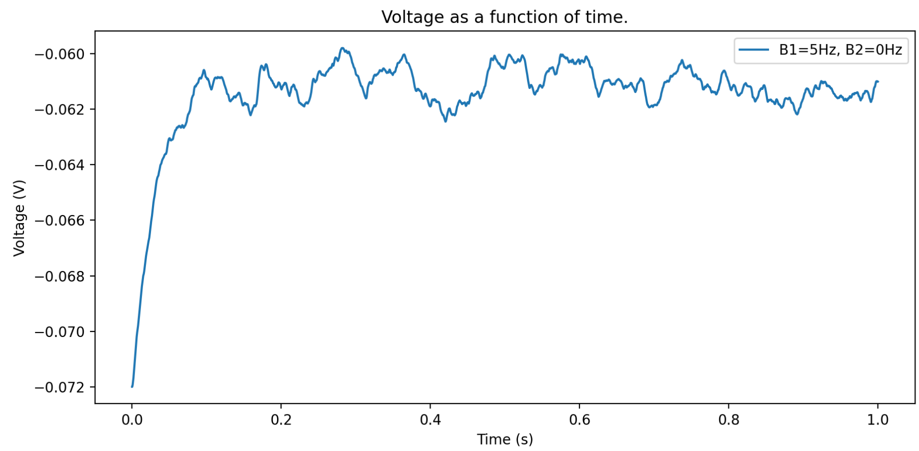



Figure 19: B1=5Hz,B2=0Hz

As shown in figure 18, compared to figure 19, the frequency of voltage oscillation is slower and the amplitude is larger, because the value of firing rate 2 changes between 50 150Hz, while the value of firing rate 1 changes between 5 15Hz, which leads to a rapid change in amplitude.

- Case 1: B1 = 0Hz, B2 = 50Hz
  The mean input firing rate of neuron under this condition is: 5510Hz.
- Case 2: B1 = 5Hz, B2 = 0Hz
  The mean input firing rate of neuron under this condition is: 5490Hz.

It can be seen from Question 2. that the required input frequency is above 80Hz, with one group is 100Hz and the other group is above 80Hz to generate a voltage exceeding the threshold value. Although their average value is the same, the peak value is different, which leads to the mean firing rate of different output.

## 3.4 Question 4.

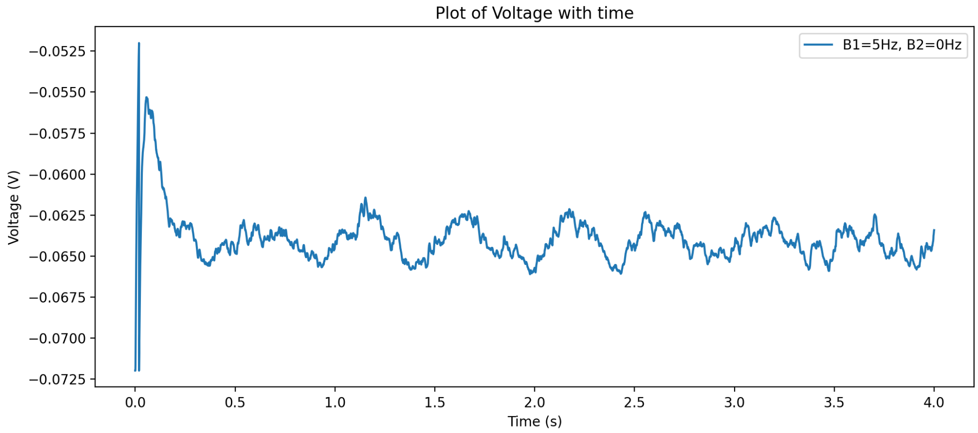



Figure 20: B1=5Hz,B2=0Hz

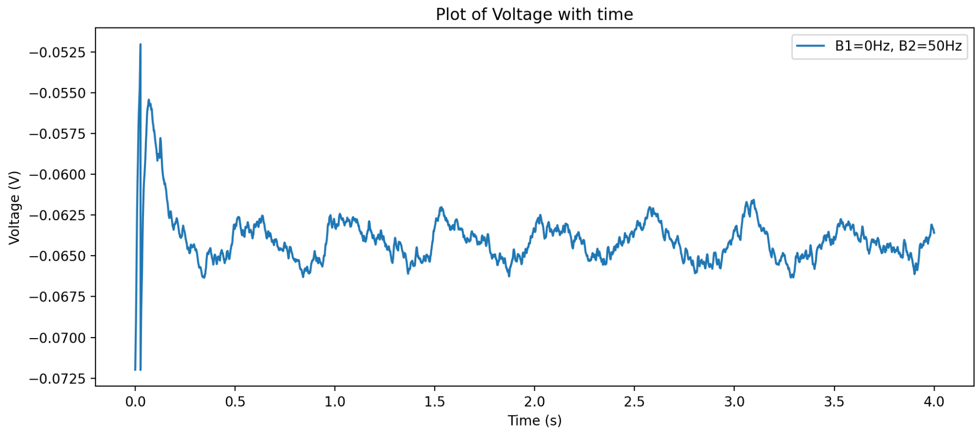



Figure 21: B1=0Hz,B2=50Hz

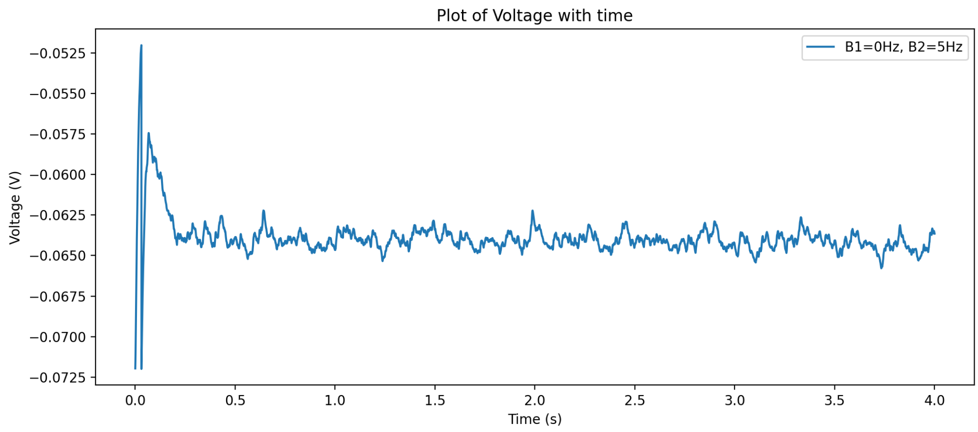



Figure 22: B1=0Hz,B2=5Hz

Figure 20, figure 21 and figure 22 show the voltage change over time in 4 seconds. It can be clearly seen that the voltage has changes drastically in the first few milliseconds. However, with the synaptic depression on, the voltage slowly dropped. As the neuron reaches the steady-state, under the conditions of figure 20 and figure 21, the voltage fluctuate between -66mV and -62mV, and the amplitude becomes smaller with the time.

The obvious difference between figure 22 and the previous two figures is that the amplitude of the waves has become smaller, the voltage changes over -62.5mV and -65mV. This is due to the firing rate in the second group is 100Hz, with the value of the amplitude of the oscillation parameter B is only 5Hz, which results in a smaller amplitude of the change compared with the previous two plots.