**Subject: Bayesian Optimization for Hyperparameter Tuning**

Hi Nick,

I hope this email finds you well. In this email, I want to show you the hyperparameter tuning task we discussed before. As you know, finding the optimal hyperparameters in both supervised and unsupervised learning models is important. Now, our task is to tune two hyperparameters: k and t, where k is the number of clusters and t is the score threshold, to see how k and t will impact the model performance.

When the objective function f is cheap, we could perform a grid search and random search. However, if f is expensive, we should switch to other sophisticated methods, such as Bayesian Optimization. More than that, for example, the grid search is inefficient because it evaluates the model performance for every combination of hyperparameters. It does not choose the next hyperparameter based on the previous result. Therefore, it will cost time on evaluating unoptimized hyperparameters.

Bayesian Optimization techniques are useful because they attempt to find the global optimum in a minimum number of steps. Does not like grid search, Bayesian Optimization uses a probabilistic model to select the next hyperparameter to evaluate based on the highest expected improvement called EI(x), where x is the location of the sample. And it can converge to the optimal hyperparameters more efficiently.

How does Bayesian Optimization work? It takes in the prior result of f and updates the prior with samples gathered from f to make the next a better approximation.

I will show you how to use Bayesian Optimization to tune the hyperparameters:

1. Define a probabilistic model like a surrogate model of the objective function over hyperparameters.
2. Evaluate the model for a set of initial hyperparameters.
3. Update the previous model based on the observed performance.
4. Repeat steps 2-3 until we meet the criteria (e.g., reach the maximum number of iterations).

There are several libraries you can use to implement Bayesian Optimization for tuning hyperparameters like it could use scikit-learn for Gaussian process implementation, a popular surrogate model for Bayesian Optimization.

I hope this email gives you a brief overview of Bayesian Optimization, and please let me know if you have any questions. Good luck with the task!

Best Regards,

Jingyi