Propositional Logic

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Readings: AIMA 7.1∼7.5

Outline

Generic Knowledge-Based Agent

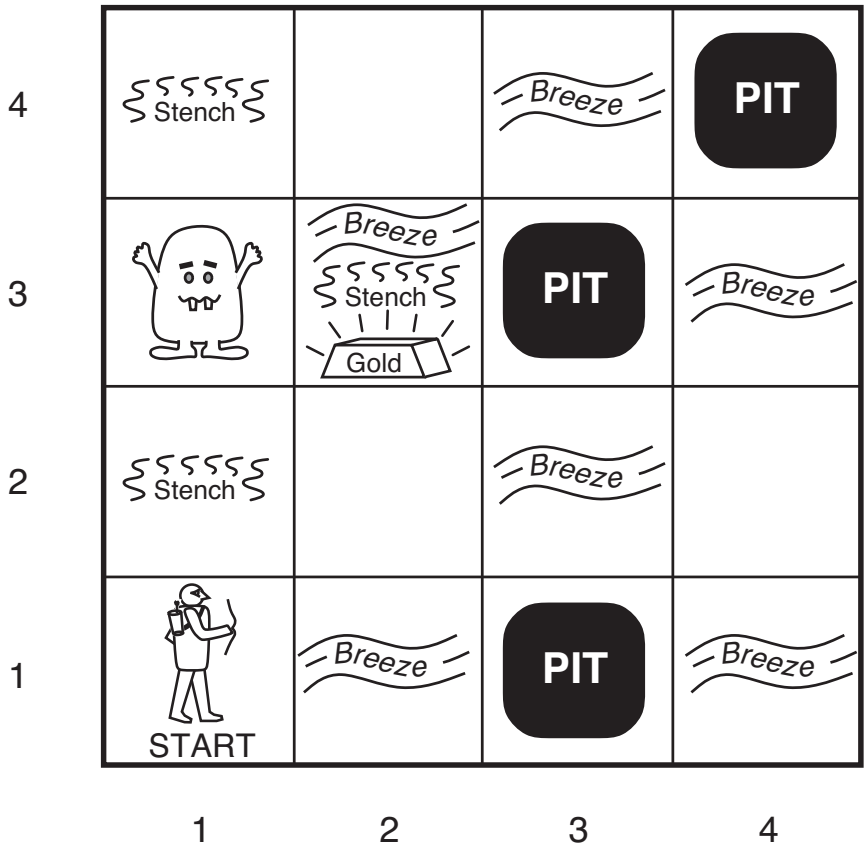
'logic' is one of the formal languages

- Knowledge base is a set of sentences in a formal language.
- Declarative approach to develop an agent: Tell it what it needs to know. 重點不是尋找解決方法，而是用 正規語言描述一個問題

KB-Agent(*percept*)

```
1  Tell(KB, Make-Percept-Sentence(percept, t))
2  action = Ask(KB, Make-Action-Query(t))
3  Tell(KB, Make-Action-Sentence(action, t))
4  t = t + 1
5  return action
```

Wumpus World



Wumpus World (PEAS)

Performance measure: gold +1000, death -1000, -1 per move, and -10 for using the arrow.

Environment: 4×4 grid. Agent starts at $[1, 1]$, facing right. One gold and one wumpus are uniformly randomly located. Any square can be a pit with probability of 0.2.

Actuators: FORWARD, TURNLEFT, TURNRIGHT, GRAB, SHOOT (only one arrow, going straight until it kills the wumpus or hits the wall), CLIMB (only works at $[1, 1]$).

Sensors: STENCH when adjacent to the wumpus. BREEZE when adjacent to a pit. GLITTER when reaching the gold. BUMP when walking into a wall. SCREAM when the wumpus dies.

Wumpus World

- The agent sometimes needs to decide to go home empty-handed or risk for the gold.
  - This environment does not guarantee the agent can always get the gold.
  - If at $[1, 1]$ the agent receives BREEZE, the agent does not know which direction to FORWARD is fatal and which is safe (can be both fatal).
  - With a probability of about 21%, the gold is in a pit, or surrounded by pits.
- The agent's initial *KB* contains the rules.
- It also knows it's in $[1, 1]$, and it's a safe square (marked OK).

Wumpus World

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2	3,2	4,2
1,1 A OK	2,1 OK	3,1	4,1

A = *Agent*
B = *Breeze*
G = *Glitter, Gold*
OK = *Safe square*
P = *Pit*
S = *Stench*
V = *Visited*
W = *Wumpus*

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

1^{st} step

- Percept: NONE.
- $\rightarrow$ $[1,2]$ and $[2,1]$ are OK.
- Action: FORWARD.

2^{nd} step

- Percept: BREEZE.
- $\rightarrow$ $[2,2]$ or $[3,1]$ are pits.
- $\rightarrow$ go back to $[1,1]$ then $[1,2]$.
- Action: TURNLEFT, TURNLEFT, FORWARD, TURNRIGHT, FORWARD.

Wumpus World

1,4	2,4	3,4	4,4
1,3 W!	2,3	3,3	4,3
1,2 A S OK	2,2 OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

A = *Agent*
B = *Breeze*
G = *Glitter, Gold*
OK = *Safe square*
P = *Pit*
S = *Stench*
V = *Visited*
W = *Wumpus*

1,4	2,4 P?	3,4	4,4
1,3 W!	2,3 A S G B	3,3 P?	4,3
1,2 S V OK	2,2 V OK	3,2	4,2
1,1 V OK	2,1 B V OK	3,1 P!	4,1

3^{rd} step

- Percept: Stench.
- $\rightarrow$ $[2,2]$: OK; $[1,3]$: wumpus.
- $\rightarrow$ Kill wumpus; go to $[2,2]$.
- Action: Shoot, TurnRight, Forward.

5^{th} step

- Percept: Stench, Glitter, Breeze.
- $\rightarrow$ $[2,4]$ or $[3,3]$: pits; $[2,3]$: gold.
- $\rightarrow$ Get gold and go back.
- Action: Grab, ...

Logics

- Logics are formal languages.
- Syntax defines the sentence structures in the language.
- Semantics defines the meanings of sentences.
  - Semantics defines the truth of each sentence w.r.t. each possible world.
  - $x + y = 4$ is true in a world where $x = 1$ and $y = 3$.
- We use the term model in place of possible world.

Models

If a sentence α is true in model m,

- m satisfies α. m 使 a 為真
- m is a model of α. m 是所有使a為真的集合的之一
- $m \in M(\alpha)$, where $M(\alpha)$ is the set of all models of α.

Entailment and Models

Entailment

Knowledge base KB entails sentence α **if and only if** α is true in all worlds where KB is true, denoted as:

前面是正後面是正

似implies，但又有點不一樣

$$KB \models \alpha$$

KB裡 所有a都要為真

- $(x + y = 4) \models (4 = x + y)$.
- $x = 0 \models xy = 0$.

是非

最好用truth table看
前面對後面也要對
否則false

Q: (a^B) |= r then al=r or B |= r ? F
implies: T, Entail: F

Theorem: $\alpha \models \beta$ iff $M(\alpha) \subseteq M(\beta)$.

only if: $\forall m \in M(\alpha)$, β is true in $m \Leftrightarrow \forall m \in M(\alpha), m \in M(\beta) \Leftrightarrow M(\alpha) \subseteq M(\beta)$
if: $\forall m \in M(\alpha), m \in M(\beta) \Leftrightarrow \forall m$ where α is true, β is true $\Leftrightarrow \alpha \models \beta$. □

entailment的定義

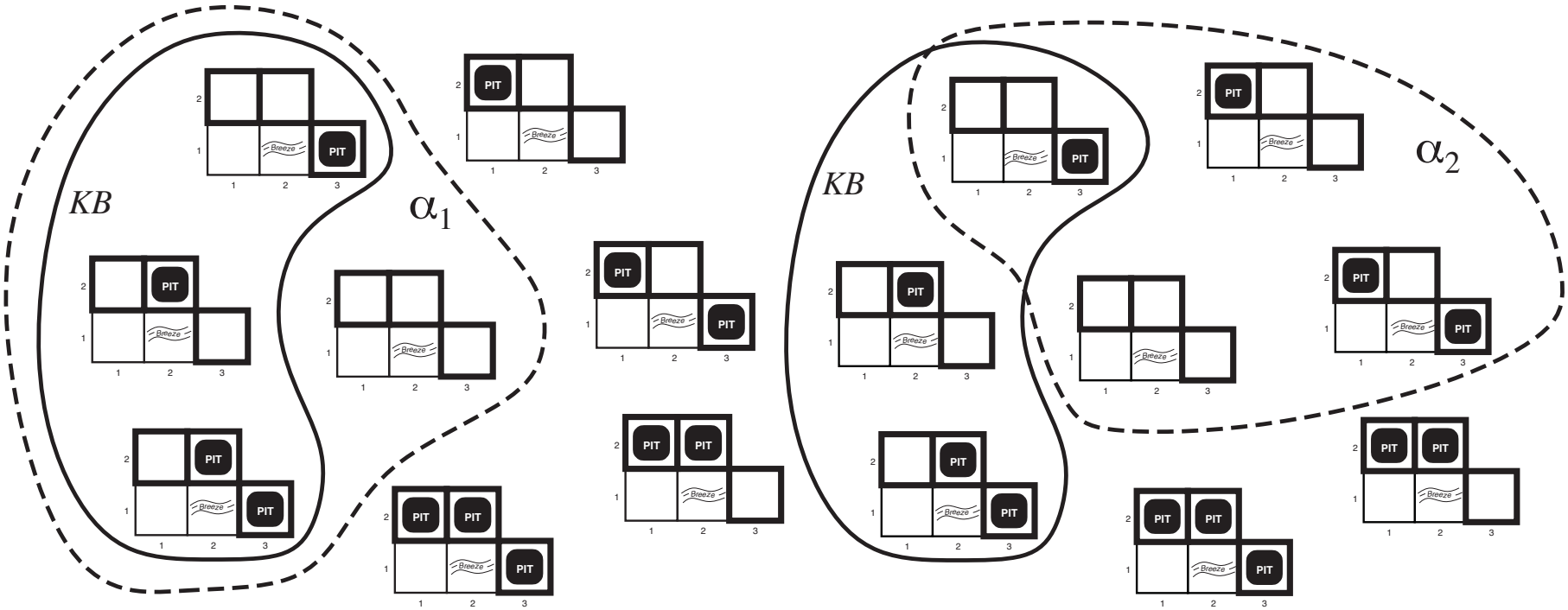
M(B)≥M(a)

Back to Wumpus World

1,4	2,4	3,4	4,4
1,3	2,3	3,3	4,3
1,2 OK	2,2 P?	3,2	4,2
1,1 V OK	2,1 A B OK	3,1 P?	4,1

- $[1, 1]$ percepts NONE.
- $[2, 1]$ percepts BREEZE.
- The agent wants to know unexplored adjacent squares $[1, 2]$, $[2, 2]$, $[3, 1]$ contains pits or not.

- The agent wants to know unexplored adjacent squares $[1,2]$, $[2,2]$, $[3,1]$ contains pits or not.
- $2^3 = 8$ possible models.
- Consider two sentences: α_1: no pit in $[1,2]$; α_2: no pit in $[2,2]$.



- $KB \models \alpha_1$; $KB \not\models \alpha_2$.

alpha NOT entail beta 和 alpha entail ~beta 兩件事很不一樣
這邊是 KB NOT entail alpha2，代表不知道alpha2是真的還假的
given entail 的定義alpha1應該是對的

Propositional Logic

- Proposition: a declarative sentence that is either true or false.
  - $\mathcal{P} = \mathcal{NP}$ is a proposition.
  - "How are you?" is not.
- Propositional logic usually does not consider time.
- If the truth of a proposition varies over time, we call it fluent.
  - "Today is Monday" is a fluent.
- Atomic propositions are minimum propositions.
- Literals are atomic propositions or their negations (p or $\neg p$).

Propositional Logic

- The following grammar in Backus-Naur form (BNF) for syntax.
- Truth tables for semantics.

$$
\begin{array}{rcl}
\mathit{Sentence} & \rightarrow & \mathit{AtomicSentence} \mid \mathit{ComplexSentence} \\
\mathit{AtomicSentence} & \rightarrow & \mathit{True} \mid \mathit{False} \mid P \mid Q \mid R \mid \ldots \\
\mathit{ComplexSentence} & \rightarrow & (\mathit{Sentence}) \mid [\mathit{Sentence}] \\
& \mid & \neg \mathit{Sentence} \\
& \mid & \mathit{Sentence} \wedge \mathit{Sentence} \\
& \mid & \mathit{Sentence} \vee \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Rightarrow \mathit{Sentence} \\
& \mid & \mathit{Sentence} \Leftrightarrow \mathit{Sentence} \\
\text{Operator Precedence} & : & \neg, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

Inference by Enumeration

TT-Entails(KB, α)

```
1  symbols = a list of the proposition symbols in KB and α
2  return TT-Check-All(KB, α, symbols, {})
```

TT-Check-All($KB, \alpha, symbols, model$)

```
1  if Empty(symbols)
2        if PL-True(KB, model)
3              return PL-True(α, model)
4        else return true
5  else
6        P = First(symbols)
7        rest = Rest(symbols)
8        return TT-Check-All(KB, α, rest, model ∪ {P = true})
               and TT-Check-All(KB, α, rest, model ∪ {P = false})
```

Standard Logical Equivalences

- $\alpha \equiv \beta$ **iff** $\alpha \models \beta$ and $\beta \models \alpha$.

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

Validity and Satisfiability

是非

- A sentence is valid if it is true in all models.

  $\textsc{True}$, $A \vee \neg A$
- Validity is connected to inference via the Deduction Theorem:

  $KB \models \alpha$ **iff** $(KB \Rightarrow \alpha)$ is valid.
- A sentence is satisfiable if it is true in some model.

  $A \wedge B$, A
- A sentence is unsatisfiable if it is true in no model.

  $A \wedge \neg A$
- Satisfiability is connected to inference via Reductio ad Absurdum (proof by contradiction):

  $KB \models \alpha$ **iff** $(KB \wedge \neg\alpha)$ is unsatisfiable.

Inference

- Inference i can derive α from KB, denoted as

  $KB \vdash_i \alpha$

- Soundness: i is sound if

  $(KB \vdash_i \alpha) \Rightarrow (KB \models \alpha)$

- Completeness: i is complete if

  $(KB \models \alpha) \Rightarrow (KB \vdash_i \alpha)$

  在有限的範圍裡面，有可能同時 sound and complete

- For KB consisting of only propositional logic or first-order logic (FOL), there exists a sound and complete inference procedure.
- FOL is expressive enough to express many things in the real world.

Simple Knowledge Base Using Propositional Logic

- $P_{x,y}$ is true there is a pit in $[x, y]$.
- $W_{x,y}$ is true there is a wumpus in $[x, y]$.
- $B_{x,y}$ is true if the agent perceives BREEZE in $[x, y]$.
- $S_{x,y}$ is true if the agent perceives STENCH in $[x, y]$.

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1,2 OK	2,2 P?	3,2	4,2
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KB

- $R_1 : \neg P_{1,1}$.
- $R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$.
- $R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$.
- $R_4 : \neg B_{1,1}$.
- $R_5 : B_{2,1}$.

Inference of the Wumpus World

Biconditional elimination from R_2.

- $R_6 : (B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$.

And-elimination from R_6.

- $R_7 : (P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1}$.

Contrapositive from R_7.

- $R_8 : \neg B_{1,1} \Rightarrow \neg(P_{1,2} \vee P_{2,1})$.

Modus Ponens from R_8.

- $R_9 : \neg(P_{1,2} \vee P_{2,1})$.

De Morgan's rule from R_9.

- $R_{10} : \neg P_{1,2} \wedge \neg P_{2,1}$.

Proof by Resolution

如何讓電腦做到上一頁那件事，方法：resolution

- Convert a sentence into conjunctive normal form (CNF).
  - Conjunction of $\underbrace{\text{disjunctions of literals}}_{\text{clauses}}$.
  - $(A \vee \neg B) \wedge (B \vee \neg C \vee D)$

Resolution Inference Rule

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \quad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n},$$

where ℓ_i and m_j are complementary literals.

- $\dfrac{P_{1,1} \vee P_{3,1}, \quad \neg P_{1,1} \vee \neg P_{2,2}}{P_{3,1} \vee \neg P_{2,2}}$

Conjunctive Normal Form (CNF) Conversion

會考，是送分題要會！！First Order 那裡也有

$B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$

1. Eliminate $\Leftrightarrow$ by bidirectional elimination:

   $(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$ by $(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$:

   $(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$

3. "Move $\neg$ inwards" by double-negation elimination and De Morgan:

   $(\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$

4. Distribute $\vee$ over $\wedge$ and "flatten":

   $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$

Resolution Algorithm

- Proof by contradiction — showing $KB \wedge \neg\alpha$ is unsatisfiable.

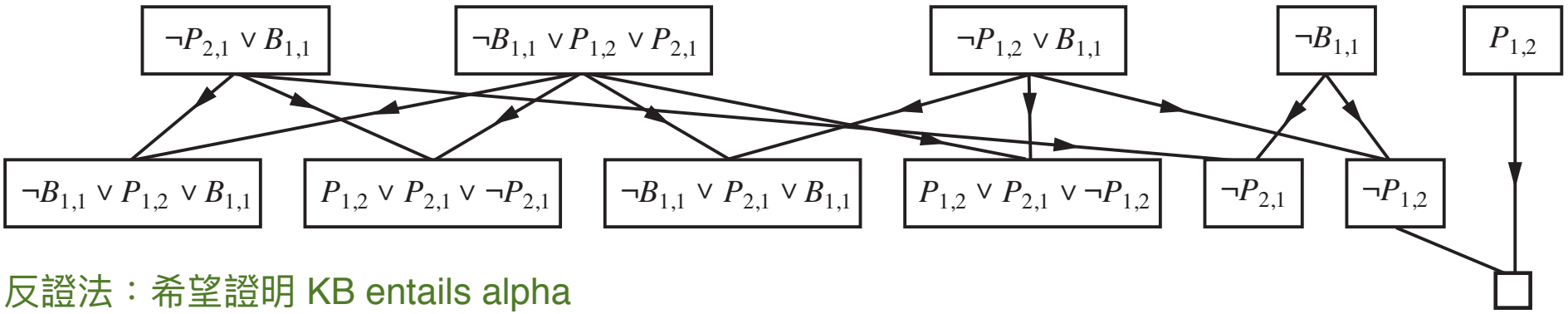
PL-Resolution(KB, α)

```
1  clauses = the set of clauses in the CNF representation of KB ∧ ¬α
2  new = φ
3  repeat
4      for each pair of clauses C_i, C_j in clauses do
5          resolvents = PL-Resolve(C_i, C_j)
6          if resolvents contains the empty clause
7              return TRUE                          矛盾
8          new = new ∪ resolvents
9      if new ⊆ clauses
10         return FALSE
11     clauses = clauses ∪ new
```

Resolution Example

- KB : $B_{1,1} \Leftrightarrow P_{1,2} \vee P_{2,1}$.
- $KB(CNF)$: $(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$.
- After we know (add into KB) $\neg B_{1,1}$, we'd like to assert $\alpha = \neg P_{1,2}$.
- PL-Resolution resolves $KB \wedge \neg\alpha$ to the empty clause.

= at odds
e.g. A ^ A'



反證法：希望證明 KB entails alpha
只要 resolution 過程看到 empty 就證明 KB entails alpha
如果到最後都沒看到，就無法證明

Resolution is Sound and Complete

- Soundness is not surprising since inference rules are sound (check the truth table). 先證 base case，假設 N 個成立，用數歸證 N+1 個也成立
- Resolution is also complete.
  - Resolution closure $RC(S)$ of a set of clauses S: the set of all clauses derivable by resolution.
  - Final value of *clauses* in PL-Resolution is $RC(S)$.
  - $RC(S)$ is finite, and hence PL-Resolution always terminates.

Ground Resolution Theorem

S is unsatisfiable $\Rightarrow$ $RC(S)$ contains the empty clause.

Ground Resolution Theorem

不會考證明（？

Proof by Contrapositive.

$RC(S)$ does not contains the empty set $\Rightarrow$ S is satisfiable.

If $\phi \notin RC(S)$, construct a model for S with suitable values for literals $P_1, \ldots, P_k$:
For i from 1 to k,

- If a clause in $RC(S)$ contains $\neg P_i$ and all its other literals are FALSE, assign FALSE to P_i.
- Otherwise, assign TRUE to P_i.

For a clause in S to be close to FALSE, it must be either $(\text{FALSE} \vee \cdots \text{FALSE} \vee P_i)$ or $(\text{FALSE} \vee \cdots \text{FALSE} \vee \neg P_i)$.
However, our assignment will make the clause to be true. Therefore, such assignment is a model of S. □

Horn and Definite Clauses

- The completeness of resolution is good.
- For many real-world applications, if we add some restrictions, more efficient inference can be achieved.
- Definite clause: a disjunction of literals where exactly one is positive.
- Horn clause: a disjunction of literals where at most one is positive.
- Horn clauses are closed under resolution:

  Resolving two Horn clauses yields a Horn clause.
- Another way to view Horn clauses:
  - TRUE $\Rightarrow$ symbol.
  - (Conjunction of symbols) $\Rightarrow$ symbol.

A ^ B => C
~(A ^ B) v C
(~A v ~B v C) only one is positive

- Deciding entailment with Horn clauses can be done in linear time!

  Forward and backward chaining.

Forward Chaining

(Resolution 的特例)

- Resolution for Horn clauses (Modus Ponens):

$$\frac{\alpha_1, \cdots, \alpha_n, \quad (\alpha_1 \wedge \cdots \wedge \alpha_n) \Rightarrow \beta}{\beta}$$

- Main idea:
  - Counts the unknown premises in all clauses.
  - Decreases the count if a premise is known.
  - When a count becomes zero, the conclusion is added as a known fact.
  - Record the inferred symbols to avoid redundant work ($P \Rightarrow Q, Q \Rightarrow P$).

Forward Chaining

sound and complete

PL-FC-Entails(KB, q)

count: number of symbols in c's premise.
agenda: a queue of symbols, initially known facts in KB.

```
1  while agenda is not empty do
2      p = Pop(agenda)
3      if p == q
4          return TRUE
5      if inferred[p] == FALSE
6          inferred[p] = TRUE
7          for each clause c in KB where p is in c.premise
8              decrement count[c]
9              if count[c] == 0
10                 add c.conclusion to agenda
11 return FALSE
```

Forward Chaining Example

- Fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found.

KB

$P \Rightarrow Q$
$L \wedge M \Rightarrow P$
$B \wedge L \Rightarrow M$
$A \wedge P \Rightarrow L$
$A \wedge B \Rightarrow L$
A
B

Step 1

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	2
$A \wedge P \Rightarrow L$	2
$A \wedge B \Rightarrow L$	2

$agenda : [A, B]$

事實丟到 queue 裡面

Step 2

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	2
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	1

$agenda : [B]$

所有前面有 A 的 counter - 1

Step 3

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	2
$B \wedge L \Rightarrow M$	1
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

$agenda : [L]$

所有前面有 B 的 counter - 1
如果 counter = 0 代表得到這個結論 => 丟到 queue

Forward Chaining Example

- Fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found.

Step 4

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	1
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

$agenda : [M]$

Step 5

$P \Rightarrow Q$	1
$L \wedge M \Rightarrow P$	0
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	1
$A \wedge B \Rightarrow L$	0

$agenda : [P]$

Step 6

$P \Rightarrow Q$	0
$L \wedge M \Rightarrow P$	0
$B \wedge L \Rightarrow M$	0
$A \wedge P \Rightarrow L$	0
$A \wedge B \Rightarrow L$	0

$agenda : [Q, L]$

所有進過 queue 的都是得到的結論
要注意進過 queue 的就不要再進去了
避免 counter 減到負的

Completeness of Forward Chaining

- FC reaches a fixed point where no new inferences are possible.
- Consider a model m which assigns TRUE to every symbol inferred and FALSE to others.
- Every clause in KB is TRUE in m:

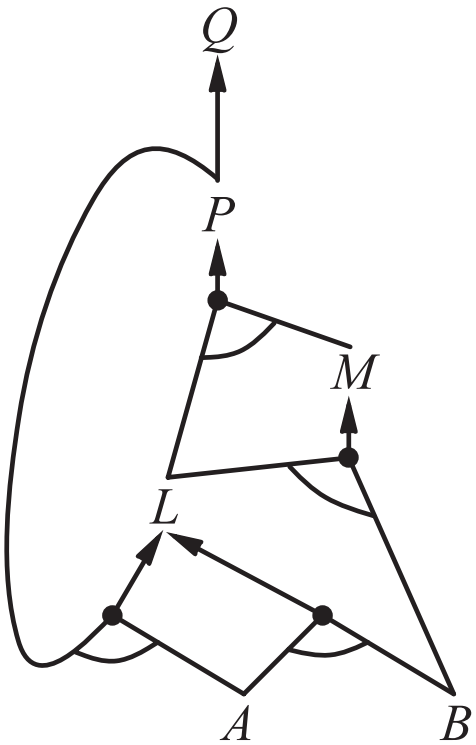
Proof.

- Suppose $\alpha_1 \wedge \cdots \wedge \alpha_k \Rightarrow \beta$ is FALSE in m.
- $\alpha_1 \wedge \cdots \wedge \alpha_k$ is TRUE and β is FALSE.
- FC has not reached a fixed point. □

- $m \in M(KB)$ (m is a model of KB)
- If $KB \models q$, q is TRUE in m.
- Therefore, FC derives every atomic sentence that is entailed by KB.

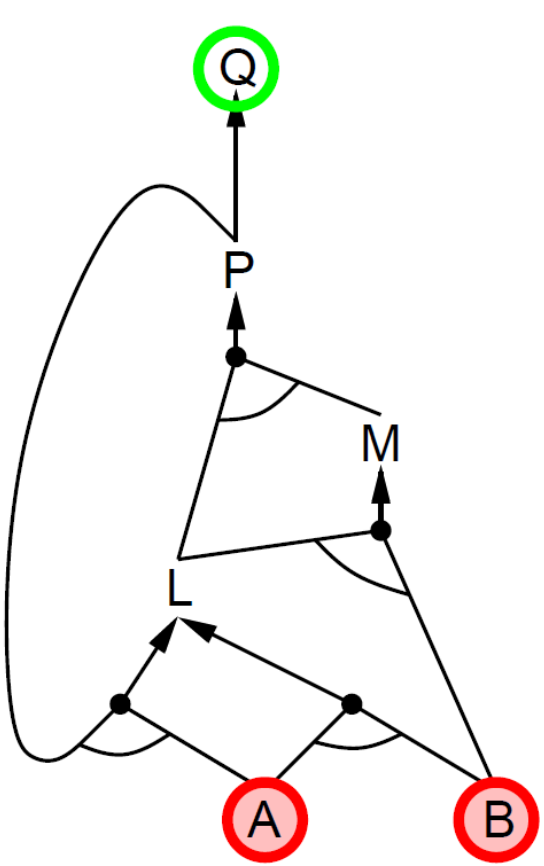
Backward Chaining

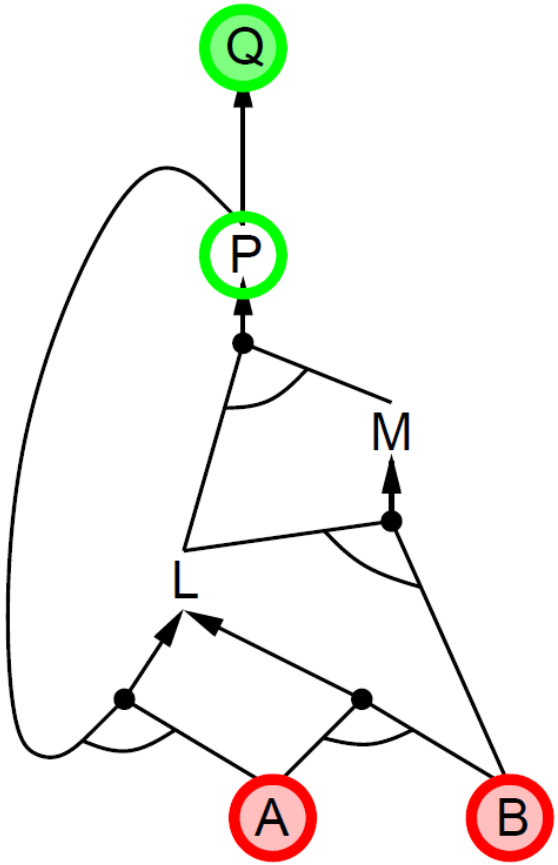
- Work backward from Q.
- If Q is known, done.
- Otherwise, check it's premise P.
- Next step checks L and M.
- Identical to the AND-OR-GRAPH-SEARCH in the textbook.



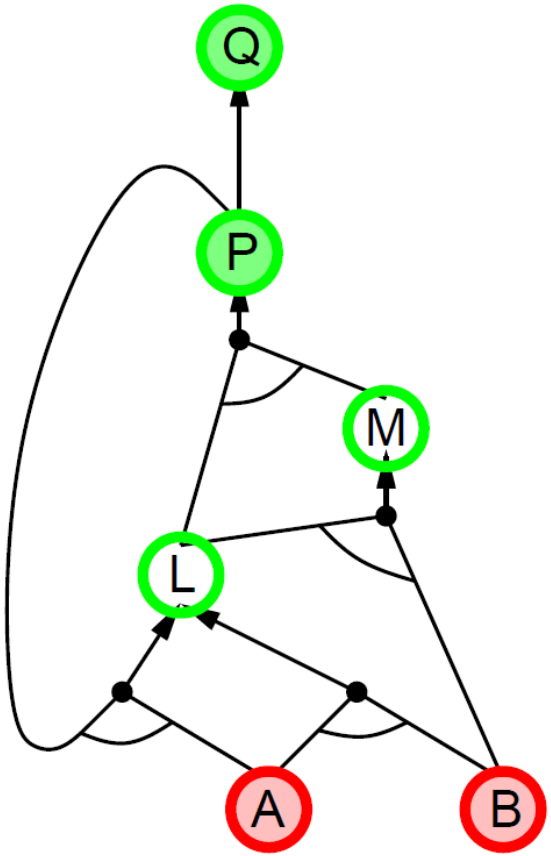
Backward Chaining Example

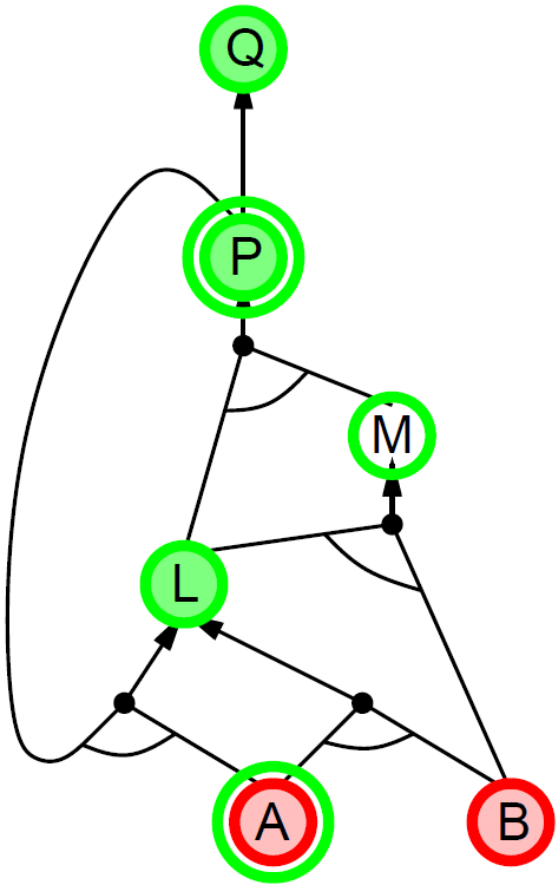
Green ring: Checking.
Green circle: Checked.
Red circle: Facts.

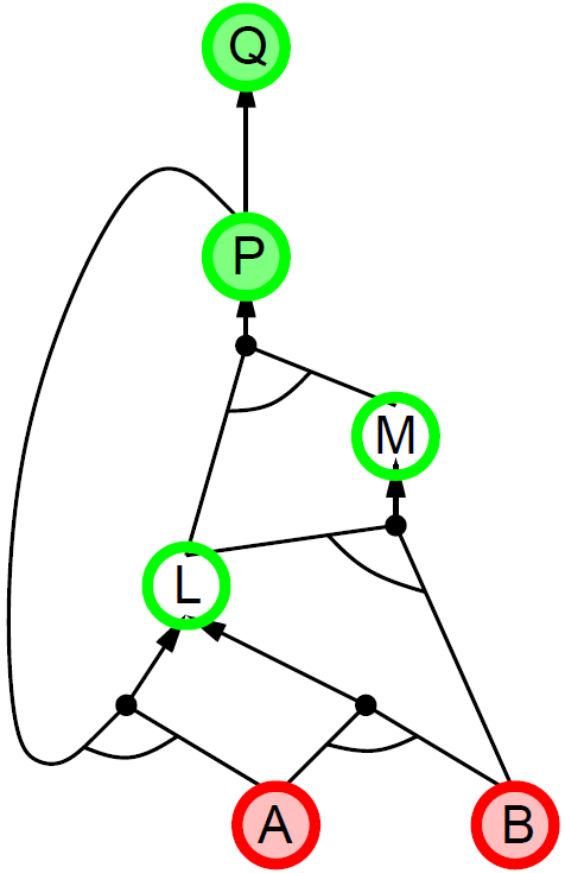




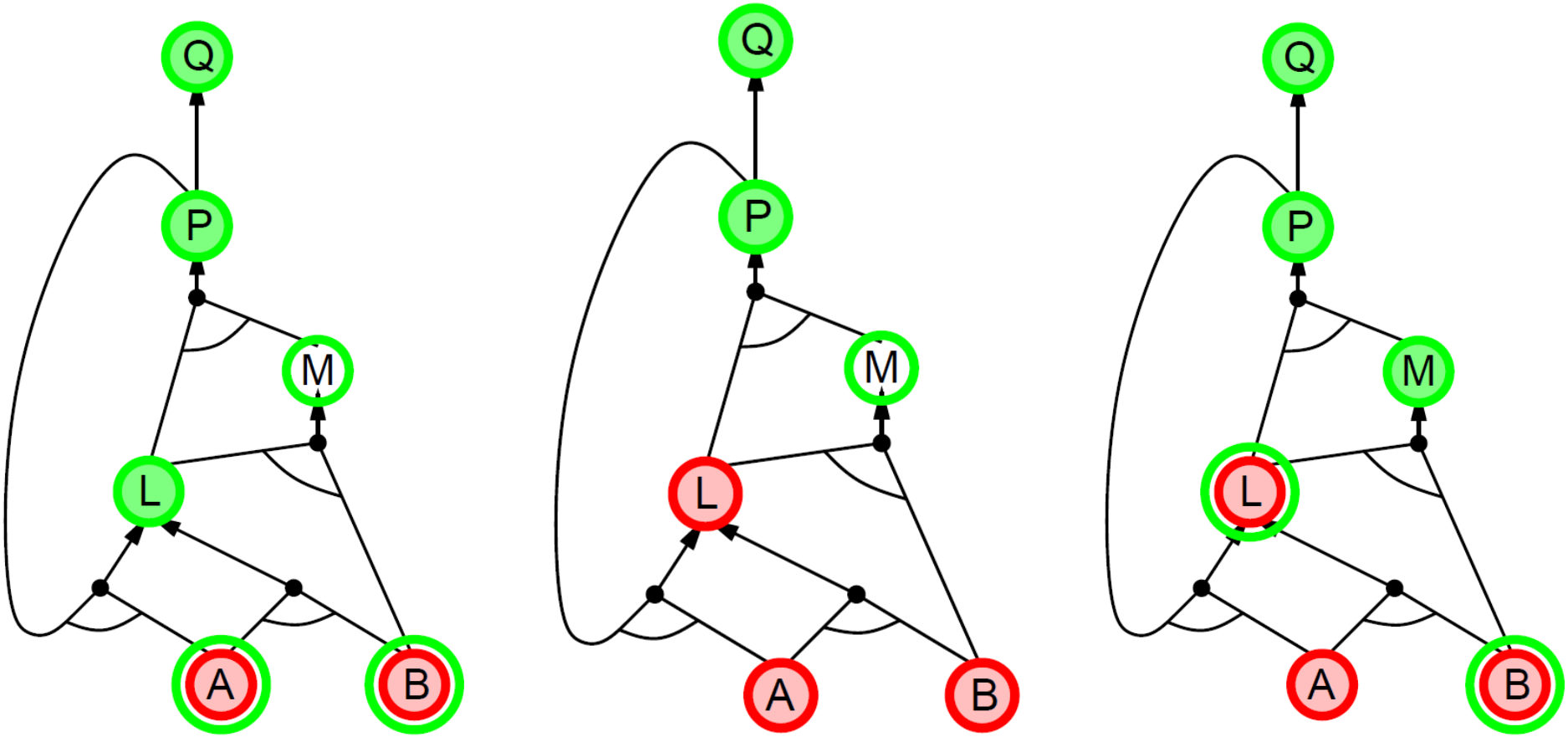
Backward Chaining Example



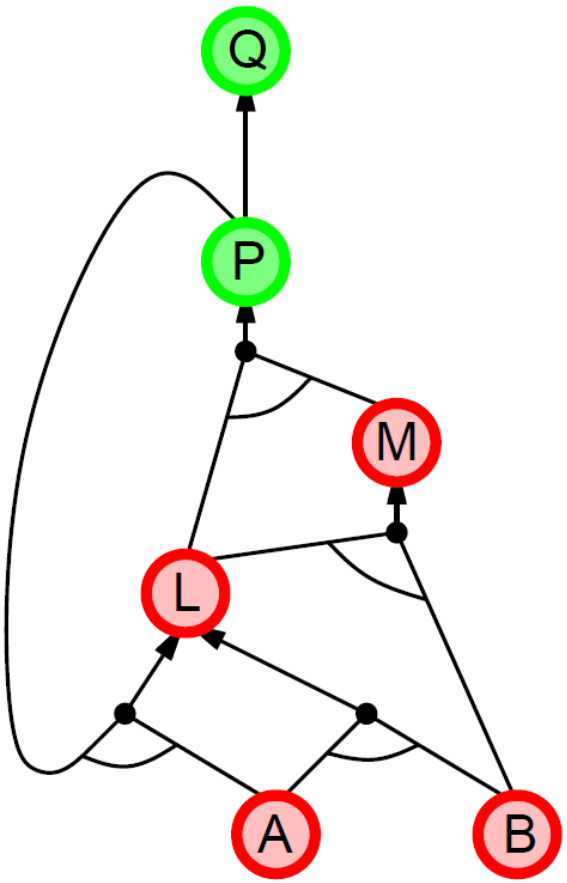


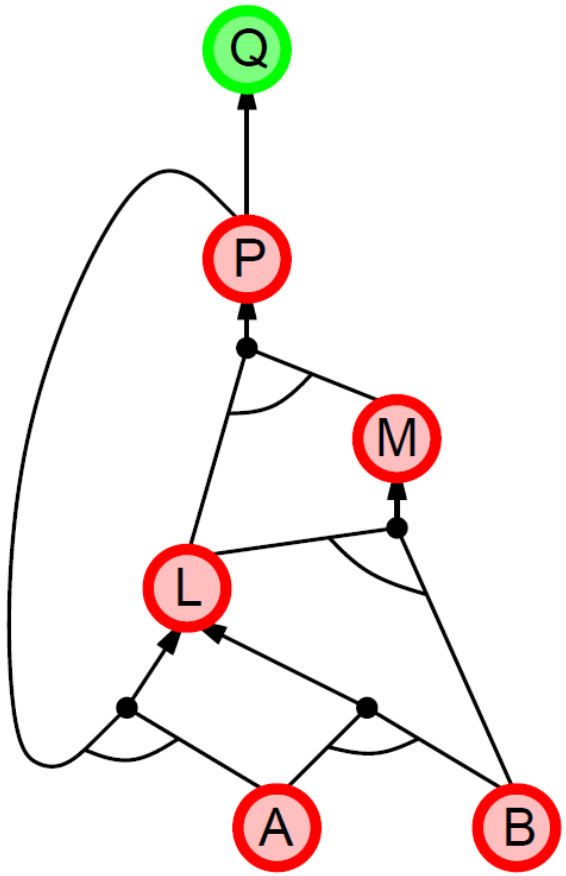


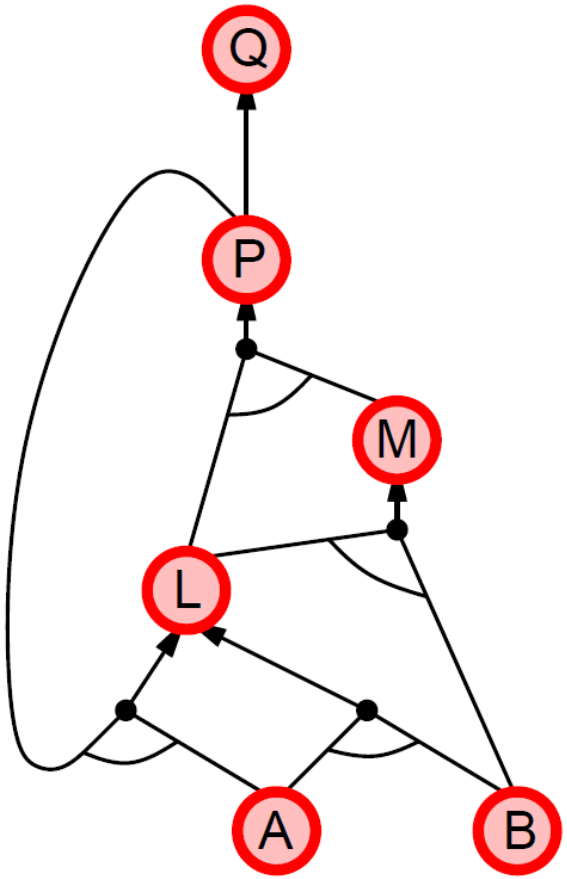
Backward Chaining Example



Backward Chaining Example







Forward vs. Backward Chaining

- Both time complexities are linear to the size of KB.
- Forward chaining is data-driven.
- Backward chaining is goal-driven.
- In general, backward chaining is more appropriate for problem solving
  - Where's my key?
  - How to pass this course?
- Forward chaining may generate many conclusions irrelevant to the goal.
- In general, time complexity of backward chaining is much less than linear of the size of KB.

Pros and Cons of Propositional Logic

- Pro
  - Propositional logic is declarative: pieces of syntax correspond to facts.
  - Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases).
  - Propositional logic is compositional:
    Meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$.
  - Meaning in propositional logic is context-independent (unlike natural language, where meaning depends on context)
- Con
  - Propositional logic has very limited expressive power.
    Cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square.

context-independent: 轉成邏輯之後就不用管在證明什麼

propositional logic 能力有限

Summary

propositional logic 能力有限

- Knowledge base contains sentences in a knowledge representation language.
- A representation language is defined by its syntax and semantics, which defines the truth of each sentence in each model.
- α entails β if β is true in all models where α is true. Equivalent definitions: validity of $\alpha \Rightarrow \beta$; unsatisfiability of $\alpha \wedge \neg\beta$.
- Sound inferences derive only sentences that are entailed; complete inferences derive all sentences that are entailed.
- Resolution is sound and complete inference for propositional logics, where KB can be expressed by CNF.
- Forward and backward chaining are sound and complete for KB in Horn form (more restrict than propositional logics).