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1.) Tentukan solusi masalah nilai awal untuk persamaan klein - Gordon $u_{tt}(x,t) - \gamma^2 u_{xx}(x,y) + c^2 u_t = 0$, dengan kondisi awal $u(x,0) = f(x)$ dan $u_t(x,0) = g(x)$

Jawab: Dengan menggunakan $u(x,0) = f(x)$ dan $u_t(x,0) = g(x)$, $t=0$ ambil $U(\lambda,t)$

$$U(\lambda,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\lambda x}\, u(x,t)\, dx$$

a.) Persamaan ~~tr~~ transformasi fourier

$$\frac{\partial u(\lambda,t)}{\partial t^2} + (y^2\lambda^2 + e^2)\, u(\lambda,t) = 0 \quad , t > 0$$

Sehingga dengan mudah dapat mendetermine bahwa

$$u(\lambda,t) = f+(\lambda)\exp\left(i\sqrt{y^2\lambda^2+e}\ t\right) + f-(\lambda)\exp\left(-i\sqrt{y^2\lambda^2+e^2}\ t\right)$$

sehingga $\frac{dx}{dt}$ dengan $\emptyset \pm (x,t,\lambda)$ dapat ditulis sebagai

$$\frac{dx}{dt} = \frac{\omega(\lambda)}{\lambda} = \pm\frac{\sqrt{y^2\lambda^2+e^2}}{\lambda}$$

b.) Statement $\emptyset + (x,t,\lambda)$ didapat dari

$$\frac{\partial \emptyset}{\partial \lambda} - \frac{\partial}{\partial\lambda}\left[\omega(\lambda)t - \lambda x\right] = \omega'(\lambda)t - x$$

$$= \frac{y^2\lambda t}{\sqrt{y^2\lambda^2 e^2}} - x$$

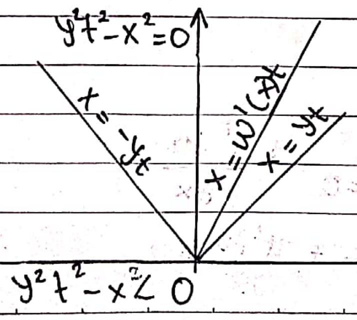
$$= 0$$

Hal itu menunjukkan bahwa, $\frac{x}{t} = \omega'(\lambda) = \frac{y^2\lambda}{\sqrt{y^2\lambda^2+e^2}}$

Nilai dari λ ditulis, $\lambda = \frac{c}{y}\frac{x}{t}\left[y^2 - \left(\frac{x}{t}\right)^2\right]^{-1/2}$

$\omega'(\lambda) = \frac{y^2\lambda}{\sqrt{y^2\lambda^2 e^2}}$ merupakan grup velocity

c.)



2) Diberikan persamaan difusi berikut $u_t(x,t) + u^2(x,t)\,u_x(x,t) = 0$ dengan kondisi awal $u(x,0) = x$

a.) Solusi nilai awal dan mengecek keabsahannya.

→ Melalui kurva & karakteristik, dari $u_t(x,t) + u^2(x,t)\,u_x(x,t) = 0$ dapat ditulis

$$u_t + u^2 u_x = 0 \text{ atau } \frac{\partial u}{\partial t} + u^2 \frac{\partial u}{\partial x} = 0 \qquad (1)$$

$$\frac{dt}{ds} = 1, \ \frac{dx}{ds} = u^2, \ \frac{du}{ds} = 0$$

diintegralkan, $\frac{dt}{ds} = 1 \Leftrightarrow \int dt = \int ds \Leftrightarrow t = s$

$$\frac{dx}{ds} = u^2 \Leftrightarrow \int dx = \int u^2\, ds \Leftrightarrow x = u^2 s$$

$$\frac{du}{ds} = 0 \Leftrightarrow \int du = \int 0.\,ds \Leftrightarrow u = 0$$

Jadi $x_1 = u^2 t$ dan $u_1 = 0$

→ Ingat bahwa $x_0 = w$, → $x = x_0 + x_1$

$$= w + u^2 t$$

$$w = x - u^2 t$$

$u_0(x,0) = f(x) = f(x(w)) = f(w) = w$, maka $u = u_0 + u_1$

$$= f(w) + 0$$

$$= x - u^2 t$$

Sehingga $u(x,t) = x - u^2 t$ dianggap sebagai solusi persamaan (1)

→ Perlu validasi solusi $u(x,t) = x - u^2 t$

(i) $u(x,t)$ harus memenuhi kondisi awal untuk $u(x,0) = x$

$$u(x,t) = x - u^2 t$$

$$u(x,0) = x - u^2(0)$$

$$u(x,0) = x$$

(ii) $u(x,t)$ harus kembali kebentuk asal persamaan (1)

$$u(x,t) = f(x - u^2 t)$$

$$\left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = -u^2 \cdot f'(x - u^2 t)$$

$$= -u^2 - f'(x)$$

$$= -u^2 \frac{\partial u}{\partial x}$$

Substitusi, $\frac{\partial u}{\partial t} + u^2 \frac{\partial u}{\partial x} = \left.\frac{\partial u}{\partial t}\right|_{t=0} + u' \frac{\partial u}{\partial x}$

$$= -u^2 \frac{\partial u}{\partial x} + u^2 \frac{\partial u}{\partial x} = 0 \qquad \therefore \text{terbukti}$$

b.) Buatlah analisis waktu patah serta gambar.

waktu patah $\frac{\partial u}{\partial x} = \frac{\frac{\partial u}{\partial w}}{\frac{\partial x}{\partial w}}$ dimana, $u = f(w) = w$

$\frac{\partial u}{\partial w} = 1$

$x = w + u^2 t$

$\frac{\partial x}{\partial w} = 1$

sehingga, $\frac{\partial u}{\partial x} = \frac{\frac{\partial u}{\partial w}}{\frac{\partial x}{\partial w}} = \frac{1}{1} = 1$

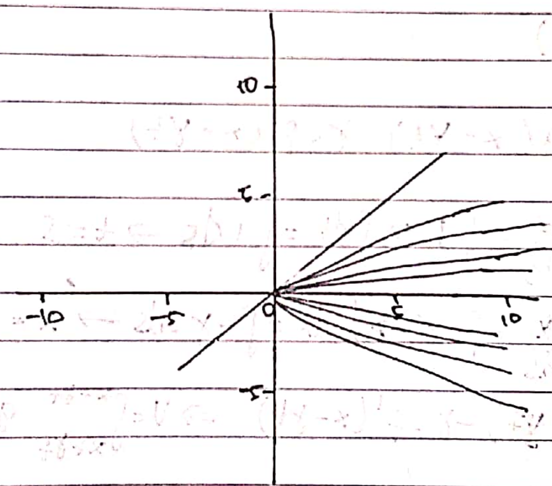
Gambar, untuk $t=0 \rightarrow u(x,0) = x$

$t=1 \rightarrow u(x,1) = x - u^2$

$t=2 \rightarrow u(x,2) = x - 2u^2$

$t=3 \rightarrow u(x,3) = x - 3u^2$

$t=4 \rightarrow u(x,4) = x - 4u^2$



3.) Diberikan $u_{tt}(x,t) - y^2 v_{xx}(x,t) = 0$ dimana kondisi awalnya berbentuk $v(x,0) = f(x) = \cos x$ dan $v_t(x,0) = \sin x$

Jawab:

a.) Solusi masalah nilai awal, Reduksi PDP orde 2 ke PDP orde 1

$$\left(\left(\frac{\partial}{\partial t} + \gamma \frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - \gamma \frac{\partial}{\partial x}\right)\right) v = 0 \iff \left(\frac{\partial^2}{\partial t^2} - \gamma^2 \frac{\partial^2}{\partial x^2}\right) v = 0$$

$$\begin{cases} \frac{\partial v}{\partial t} - \gamma \frac{\partial v}{\partial x} = v & \ldots (*) \\ \frac{\partial v}{\partial t} + \gamma \frac{\partial v}{\partial x} = 0 & \ldots (**) \end{cases}$$

- Initial Value

$v_0(x,0) = f(x) = \cos x$, $v_x(x,0) = -\sin x$, ketika $x = T$, $\frac{\partial v}{\partial x}(T,0) = -\sin T$

- $V_t(x,0) = g(x) = \sin x$, $x_0 = \tau$ dan $t = 0$

sehingga , $\frac{\partial v}{\partial t}(\tau,0) = g(\tau) = \sin \tau$

- kurva - kurva karakteristik

- Persamaan (**) $\frac{\partial v}{\partial t} + \gamma \frac{\partial v}{\partial x} = 0$

$$\frac{dt}{ds} = 1 \rightarrow \int \frac{dt}{ds} = \int 1 \rightarrow \int dt = \int 1 . ds \rightarrow t = s$$

$$\frac{dx}{ds} = \gamma \rightarrow \int \frac{dx}{ds} = \int \gamma \rightarrow \int dx = \int \gamma . ds \rightarrow x = \gamma s$$

$$\frac{dv}{ds} = 0 \rightarrow \int \frac{dv}{ds} = \int 0 \rightarrow \int dv = \int 0 . ds \rightarrow v = 0$$

$t = s, x = \gamma s \rightarrow x_1 = \gamma t$

$x = x_0 + x_1 = \tau + \gamma t$

$\tau = x - \gamma t$

$v_1 = 0$

$v = \cos(x - \gamma t)$

- Persamaan (*)

$$\frac{\partial v}{\partial t} - \gamma \frac{\partial v}{\partial x} = g(x - \gamma t) - \gamma . f'(x - \gamma t)$$

$$\frac{dt}{ds} = 1 \rightarrow \int \frac{dt}{ds} = \int 1 \rightarrow \int dt = \int 1 ds \rightarrow t = s$$

$$\frac{dx}{ds} = -\gamma \int \frac{dx}{ds} = \int -\gamma \Rightarrow \int dx = \int -\gamma . ds \rightarrow x = -\gamma s$$

$$\frac{dv}{ds} = g(x - \gamma t) - \gamma . f'(x - \gamma t) \Rightarrow v = \int_{x+\gamma t}^{x-\gamma t} g(x-\gamma t) - \gamma . f'(x - \gamma t) . d$$

$$+ f(x + \gamma t)$$

$\frac{d\lambda}{ds} = 2.\gamma \rightarrow ds = \frac{d\lambda}{2.\gamma}$, sehingga

$$v = \int_{x+\gamma t}^{x-\gamma t} (-g(\lambda) + \gamma . f'(\lambda)) . \left(\frac{d\lambda}{2.\gamma}\right) + f(x + \gamma t)$$

$$= -\frac{1}{2.\gamma}\left[G(x-\gamma t) - G(x+\gamma t)\right] + \frac{1}{2}\left[\left[f(x-\gamma t) - f(x+\gamma t)\right] + f(x+\gamma t)\right]$$

solusinya,

$$v(x,t) = \left[\frac{\gamma+1}{2.\gamma} \cos(x - \gamma t)\right] + \left[\frac{\gamma - 1}{2\gamma} \cos(x + \gamma t)\right]$$

b.) Cek keabsahan solusi

1.) Memenuhi initial value

$$v(x,0) = \left[\frac{\gamma+1}{2\gamma}\cos(x-\gamma 0)\right] + \left[\frac{\gamma-1}{2\gamma}\cos(x+\gamma 0)\right]$$

$$= \frac{2\cdot\gamma}{2\cdot\gamma}(\gamma)\cos(x) = \cos(x) = f(x) \qquad \text{(sesuai)}$$

$$\left.\frac{\partial v}{\partial t}\right|_{t=0} = \left[\frac{\gamma+1}{2\gamma}(-\gamma)(-\sin(x-\gamma 0))\right] + \left[\frac{\gamma-1}{2\gamma}(\gamma)(-\sin(x+\gamma 0))\right]$$

$$= \frac{2}{2\gamma}(\gamma)\sin(x) = \sin(x) = g(x) \qquad \text{(sesuai)}$$

2.) Kembali kebentuk dasar

$$\left.\frac{\partial v}{\partial t}\right|_{t=0} = \left[\frac{\gamma+1}{2\gamma}(-\gamma)(-\sin(x-\gamma 0))\right] + \left[\frac{\gamma-1}{2\gamma}(\gamma)(-\sin(x+\gamma 0))\right]$$

$$= \frac{2}{2\cdot\gamma}(\gamma)\sin(x) = g(x)$$

$$\left.\frac{\partial^2 v}{\partial t^2}\right|_{t=0} = \cos(x) = g'(x) \qquad \text{(sesuai)}$$

$$\left.\frac{\partial v}{dx}\right|_{t=0} = -\sin(x) = f'(x), \quad \left.\frac{\partial^2 v}{\partial x^2}\right|_{t=0} = -\cos(x) = f''(x) \qquad \text{(sesuai)}$$

c.) Grafik solusi

$$v(x,t) = \left[\frac{\gamma+1}{2\gamma}\cos(x-\gamma t)\right] + \left[\frac{\gamma-1}{2\gamma}\cos(x+\gamma t)\right]$$

untuk $\gamma = 2$