$$\begin{aligned} a_1 &= 1 \\ a_{n+1}\cdot a_n &= 2^n\,(n\in\mathbb{N}^\star) \end{aligned}$$

$$\begin{aligned}
&\text{代入}n=1\text{，}\ a_2\cdot a_1=2\rightarrow a_2=2\\
&\text{代入}n=2\text{，}\ a_3\cdot a_2=2^2\rightarrow a_3=2\\
&\text{代入}n=3\text{，}\ a_4\cdot a_3=2^3\rightarrow a_4=2^2\\
&\text{代入}n=4\text{，}\ a_5\cdot a_4=2^4\rightarrow a_5=2^2\\
&\text{代入}n=5\text{，}\ a_6\cdot a_5=2^5\rightarrow a_6=2^3\\
&\text{代入}n=6\text{，}\ a_7\cdot a_6=2^6\rightarrow a_7=2^3
\end{aligned}$$

发现了规律

当n为奇数时，$a_n=2^{\frac{n-1}{2}}$

当n为偶数时，$a_n=2^{\frac{n}{2}}$

于是

$$\begin{aligned}
S_n &= a_1+a_2+a_3+a_4+a_5+\cdots+a_{n-1}+a_n\\
S_{2019} &= 1+2+2+2^2+2^2+\cdots+a_{2018}+a_{2019}\\
&= 1+\underbrace{2+2+2^2+2^2+\cdots+2^{1009}+2^{1009}}_{\text{总共2018项}}\\
&= 1+2\cdot\underbrace{\left(2+2^2+\cdots+2^{1009}\right)}_{\text{总共有1009项}}\\
&= 1+2\cdot 2\frac{1-2^{1009}}{1-2}\\
&= 4\cdot(2^{1009}-1)+1\\
&= 4\cdot 2^{1009}-3\\
&= 2^2\cdot 2^{1009}-3\\
&= 2^{2+1009}-3\\
&= 2^{1011}-3
\end{aligned}$$