

考点1：等比数列通项公式

$$a_n = a_1 q^{n-1}$$

结合题目，看 $T_n$ 定义

$$\begin{aligned} T_n &= a_1 a_2 a_3 \cdots a_n \\ &= a_1 \underbrace{a_1 q}_{a_2} \underbrace{a_1 q^2}_{a_3} \cdots \underbrace{a_1 q^{n-1}}_{a_n} \\ &= \underbrace{a_1 a_1 a_1 \cdots a_1}_{n \text{ 个 } a_1 \text{ 相乘}} \underbrace{q q^2 q^3 \cdots q^{n-1}}_{q \text{ 一直乘到 } q^{n-1}} \\ &= a_1^n q^{1+2+3+\cdots+n-1} \quad (1+2+3+\cdots+n-1, \text{ 用等差数列求和的相关公式}) \\ &= a_1^n q^{\frac{n(n-1)}{2}} \end{aligned}$$

结合题目，用题目条件

$$\begin{aligned} T_2 &= a_1^2 q \\ T_9 &= a_1^9 q^{36} \\ T_2 &= T_9 \end{aligned}$$

代入计算

$$\begin{aligned} a_1^2 q &= a_1^9 q^{36} \\ 1 &= a_1^7 q^{35} = (a_1 q^5)^7 \\ \text{即 } 1 &= a_1 q^5 \\ \text{即 } a_1 &= \frac{1}{q^5} \quad \text{即 } \left(\frac{1}{q^5}\right)^2 q = 512 \\ \text{即 } q^{-9} &= 512 \quad \text{即 } q = \frac{1}{2} \end{aligned}$$

代入 $T_n$ 计算式子中

$$\begin{aligned} T_n &= a_1^n q^{\frac{n(n-1)}{2}} \\ &= \left(\frac{1}{q^5}\right)^n q^{\frac{n(n-1)}{2}} \\ &= q^{-5n} q^{\frac{n(n-1)}{2}} \\ &= q^{\frac{n(n-11)}{2}} \end{aligned}$$

$$\begin{aligned} \text{代入 } q &= \frac{1}{2} \\ \text{即 } T_n &= \left(\frac{1}{2}\right)^{\frac{n(n-11)}{2}} \\ &= 2^{\frac{n(11-n)}{2}} \end{aligned}$$

n取5或者n取6时， $T_n$ 最大值取 $2^{15}$ ，选择**A**

$a_1=32$ 且 $q = \frac{1}{2}$