第四章 导热问题的数值解

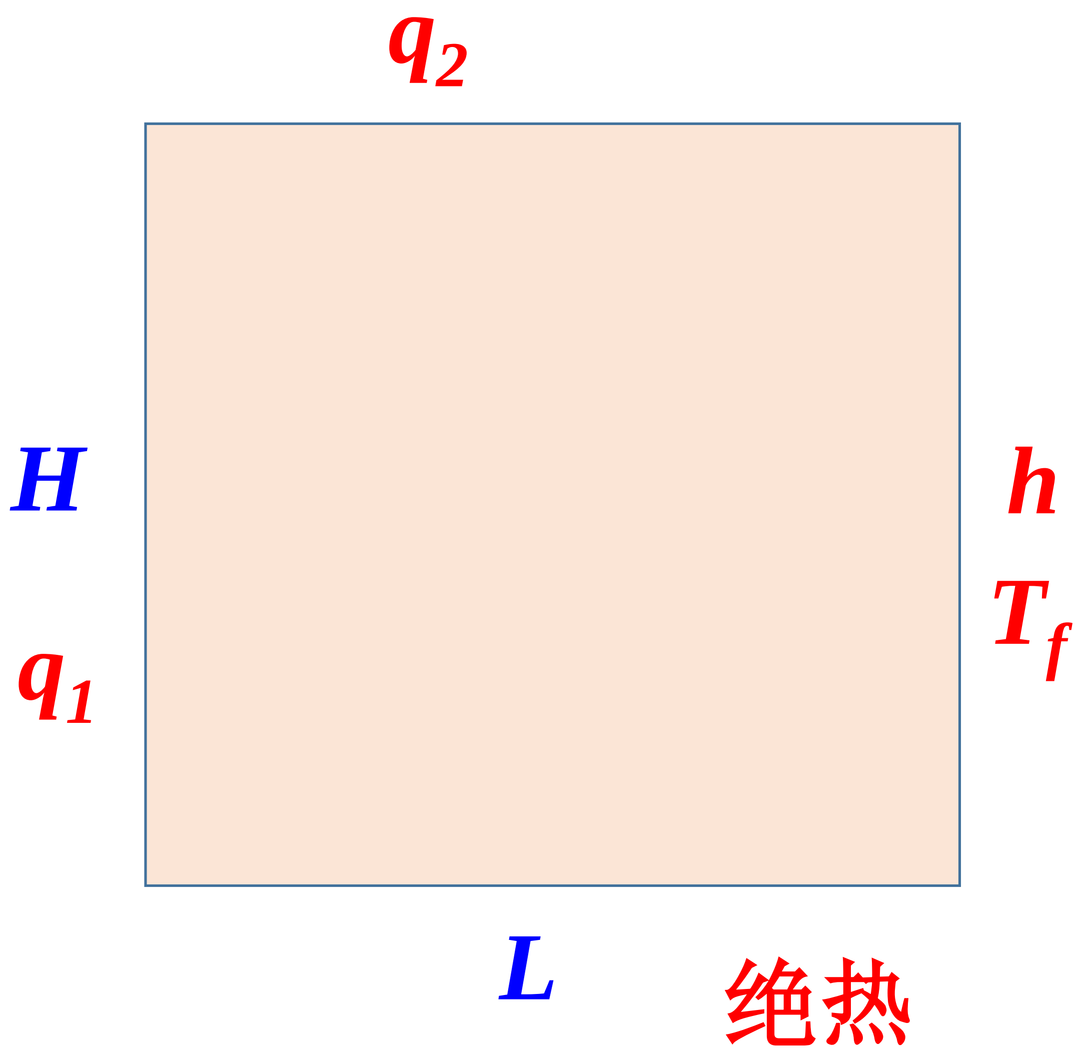
——复杂导热问题的近似解法

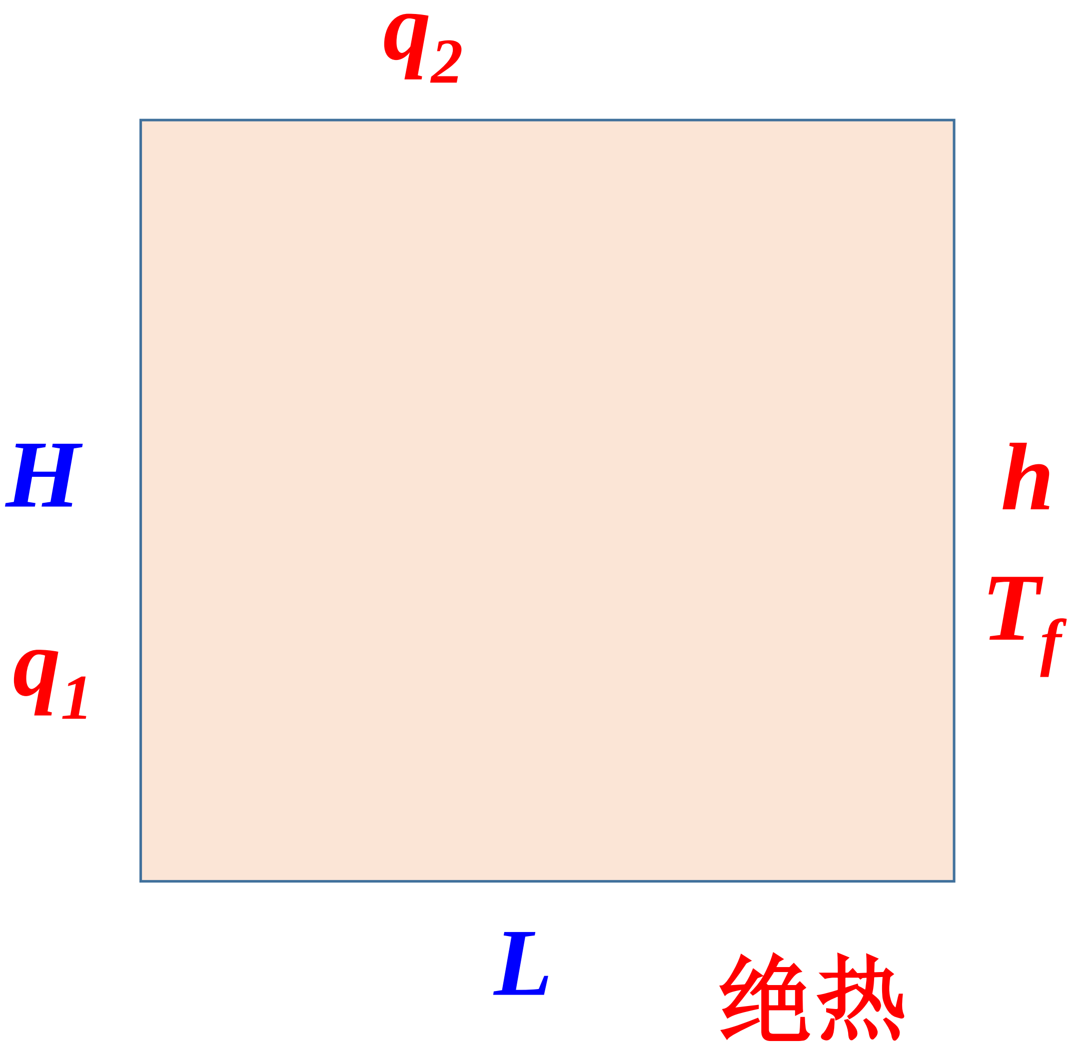
非稳态热传导

- 无限大平板的非稳态导热
- 集总参数法
- 半无限大物体的非稳态导热
- 第三类边界条件下的二维和三维非稳态导热

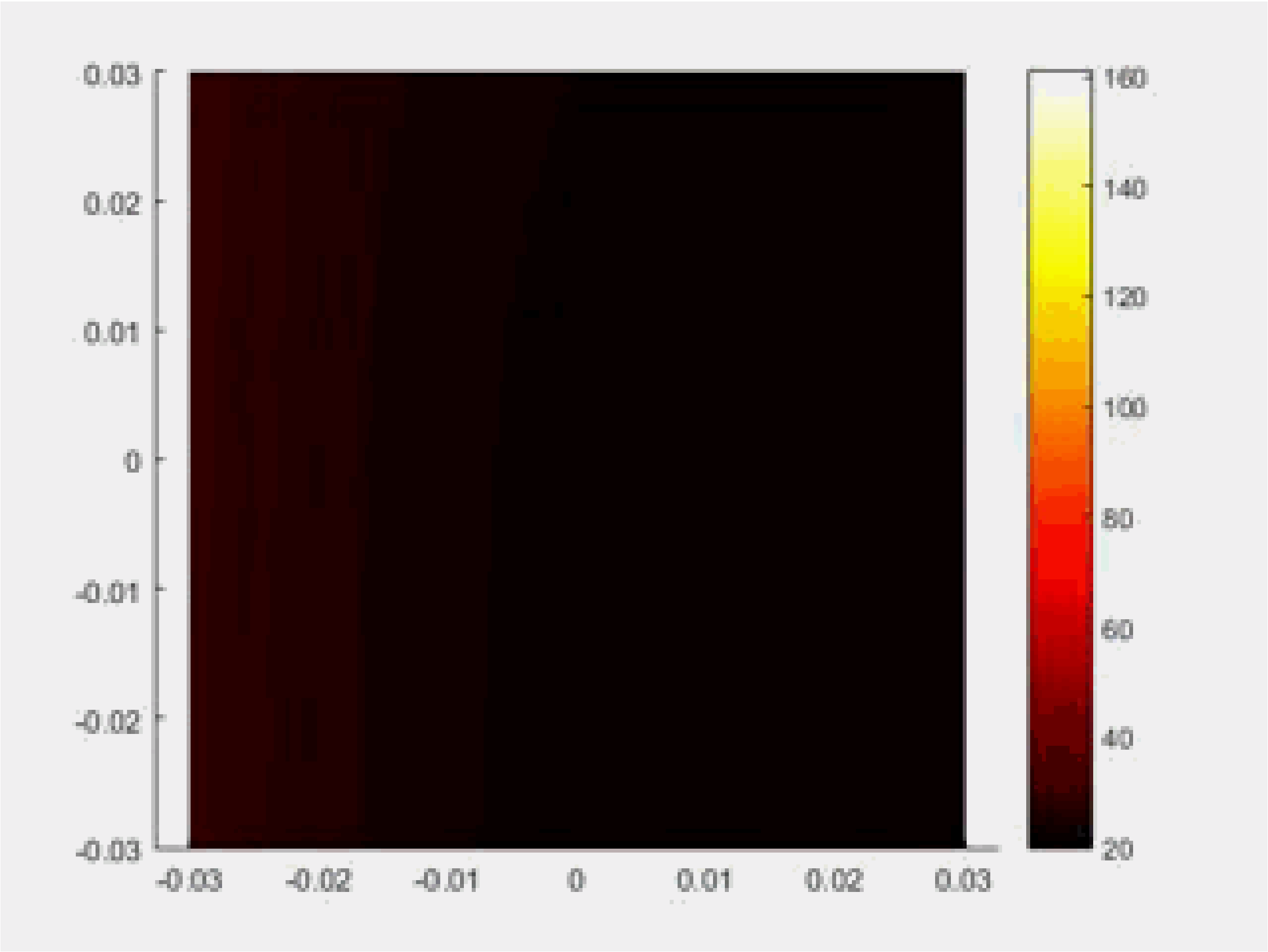
实际计算案例

一个金属固体，长6cm，宽6cm，垂直纸面方向无限长，左端面边界条件：q_1=10 W/cm^2，下端面绝热，上端面边界条件：q_2=1 W/cm^2，右端面边界条件：h=1000 W/m^2·K， T_f= 20°C，导热系数λ=200 W/m·K，





q2
H
q1
h
Tf
L
绝热

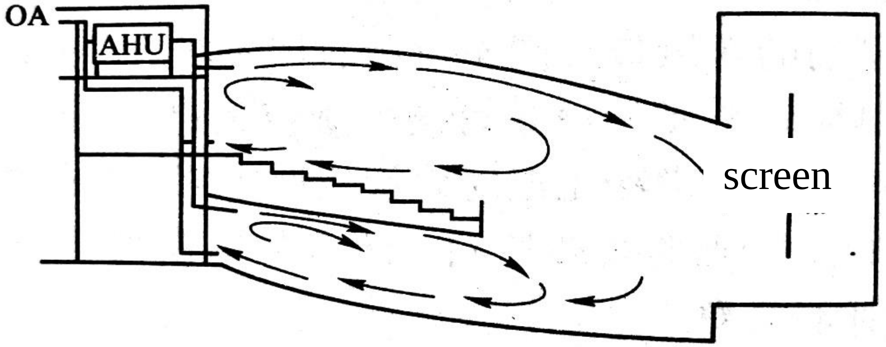


0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
160
140
120
100
80
60
40
20

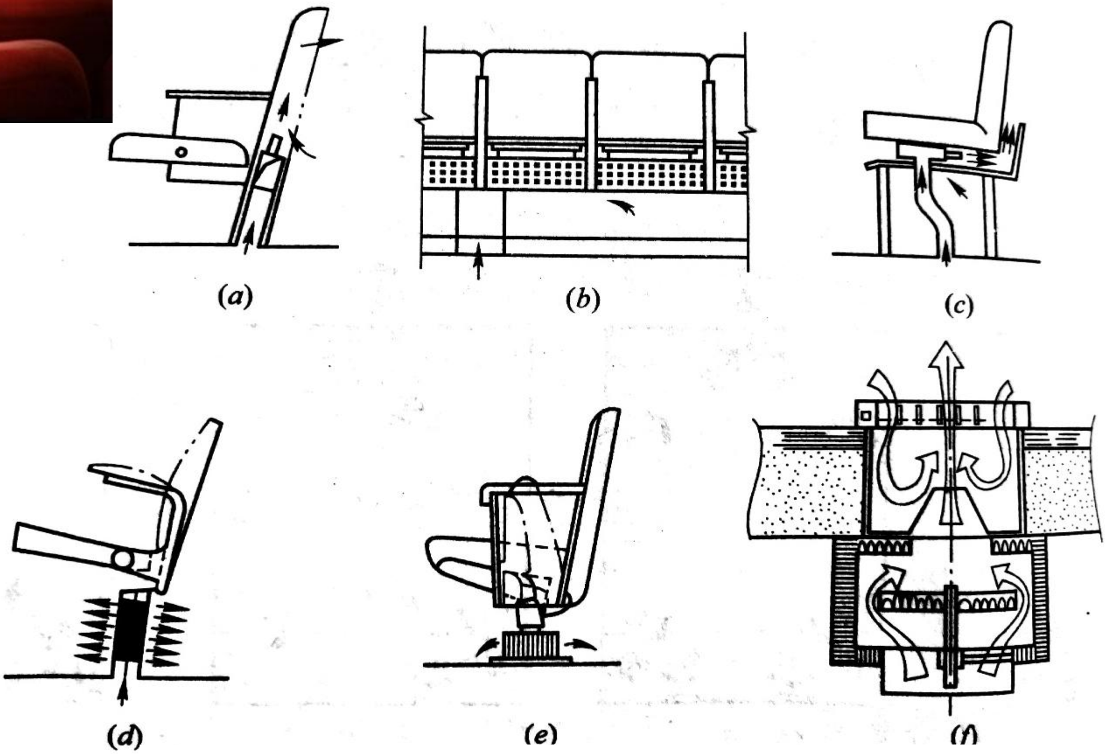
热传导
求解方法

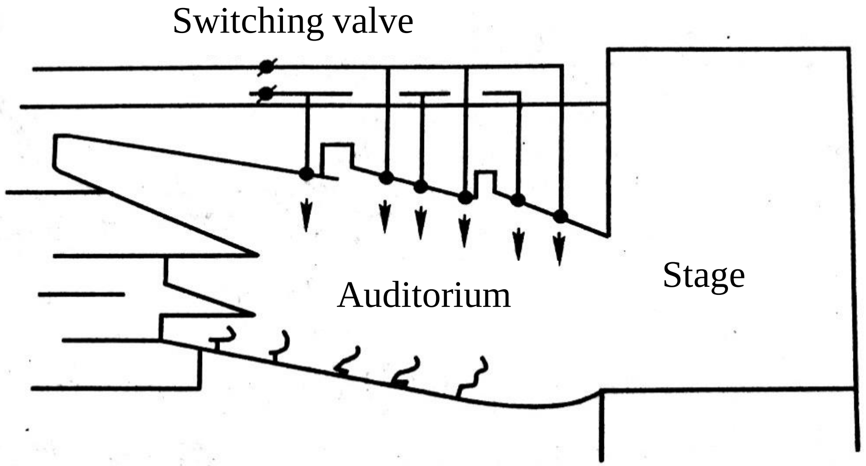
- 理论分析
- 实验方法
- 数值计算
  - 几何条件复杂
  - 边界条件复杂
  -

Radiation cooling technology in Theater （剧场辐射制冷技术）



Nozzle deliver model

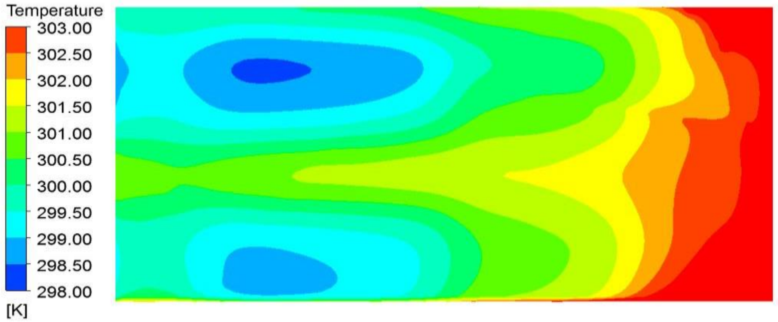




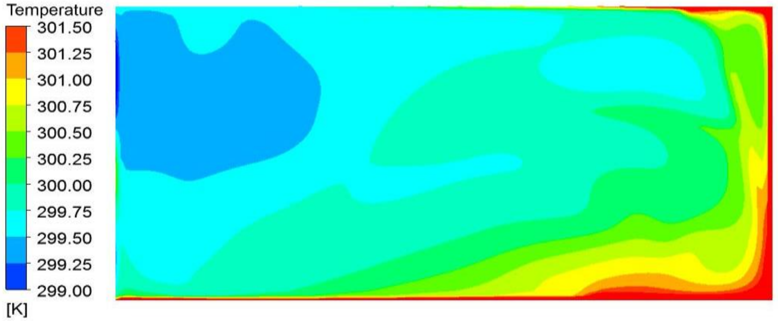
Top-supply and down-return air supply model

■ Distribution of indoor temperature

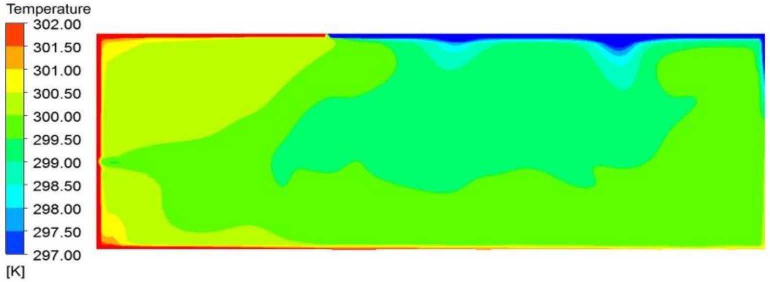
➢ Roof radiation system（顶部）



Y=0.1m

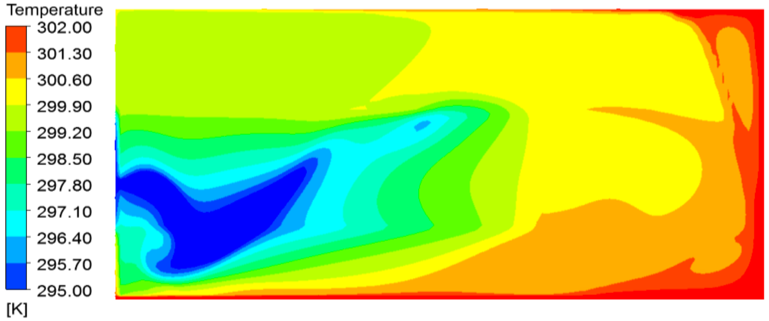


Y=1.1m

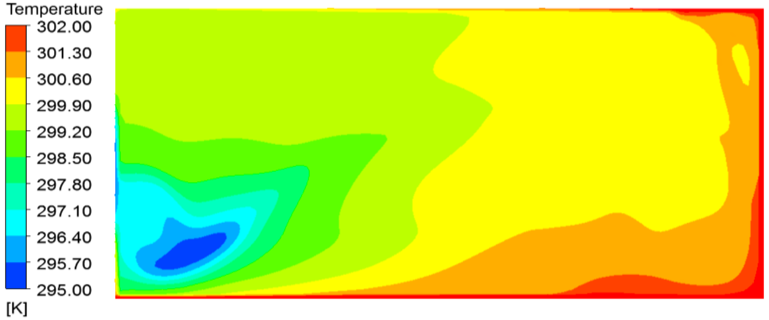


Z=10m

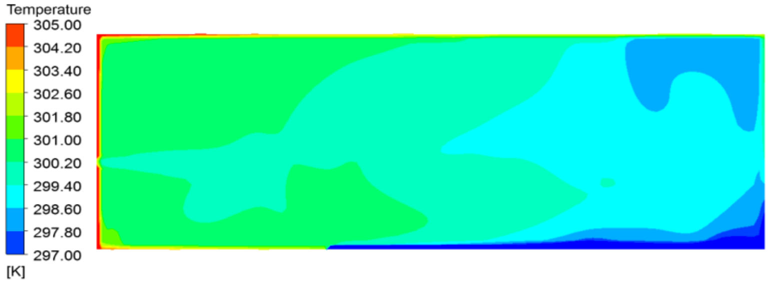
➢ Floor radiation system（地面）



Y=0.1m

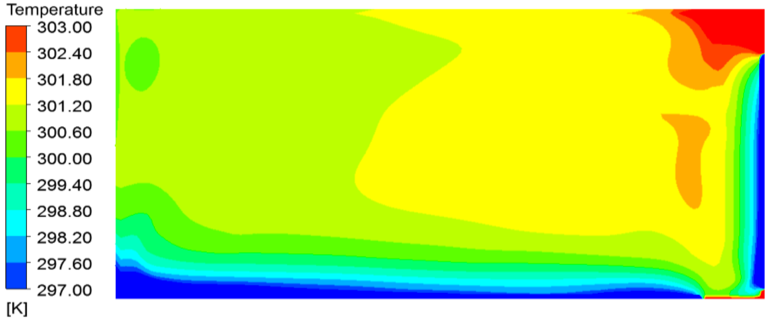


Y=1.1m

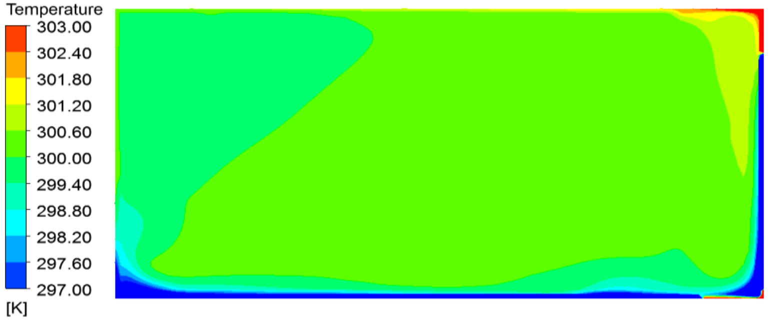


Z=10m

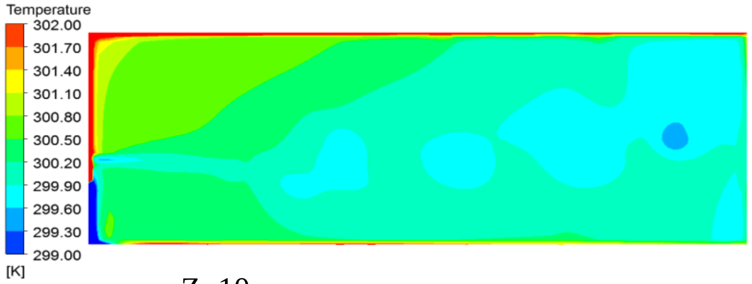
➢ Wall radiation system（壁面）



Y=0.1m



Y=1.1m

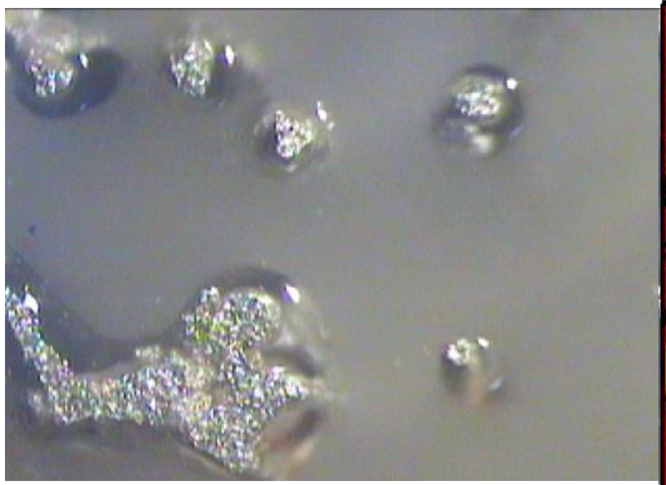


Z=10m

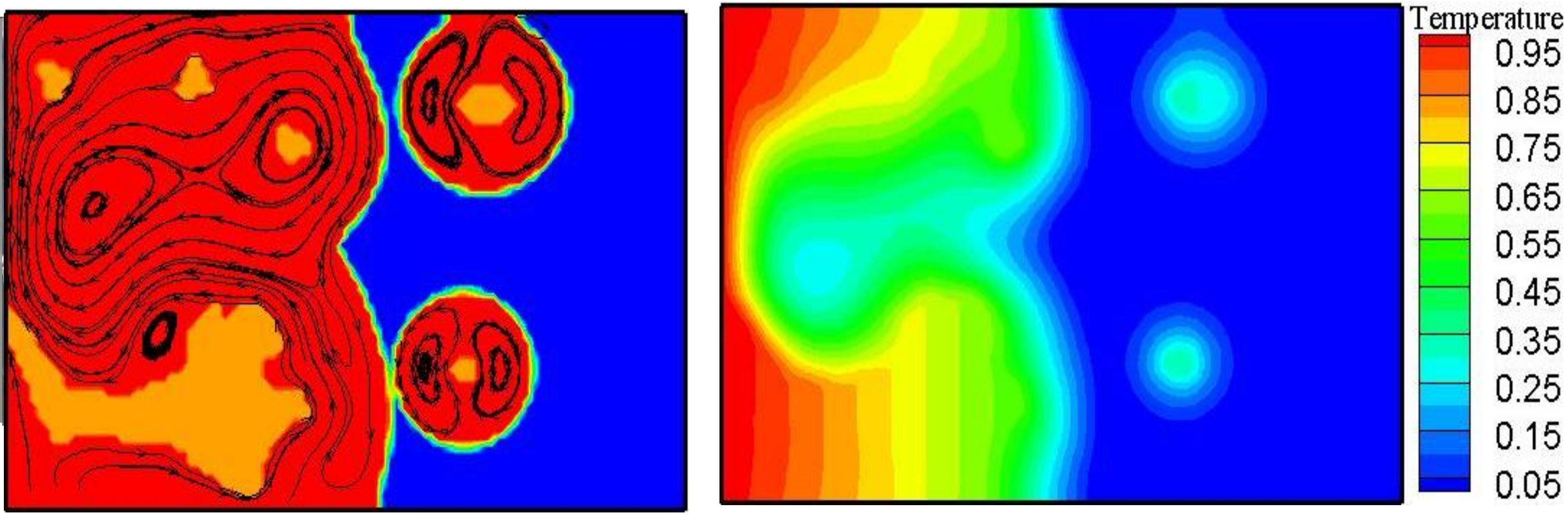
热传导求解方法

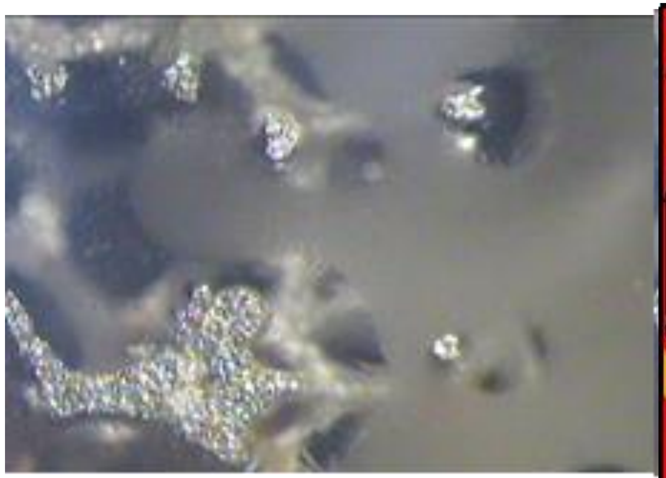
- **理论分析**
- **实验方法**
- **数值计算**
  - 几何条件复杂
  - 边界条件复杂
  - 热物性随位置变化
  - 热物性随温度变化
  - 移动边界

泡沫金属内石蜡融化过程

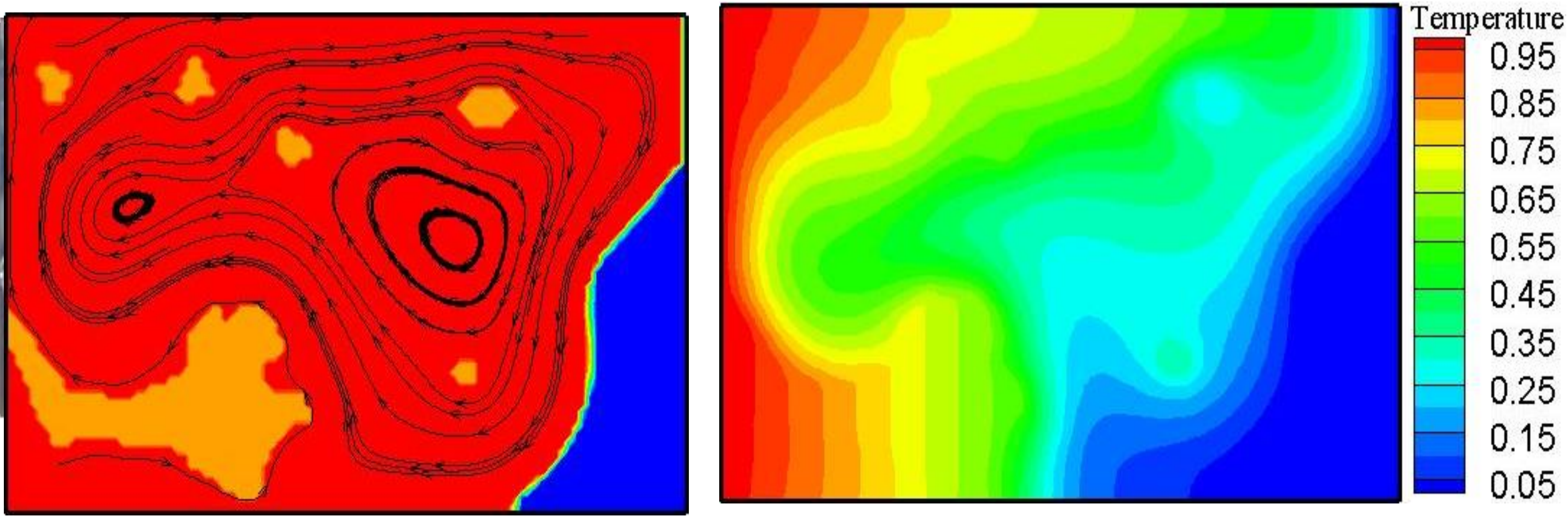


0 min

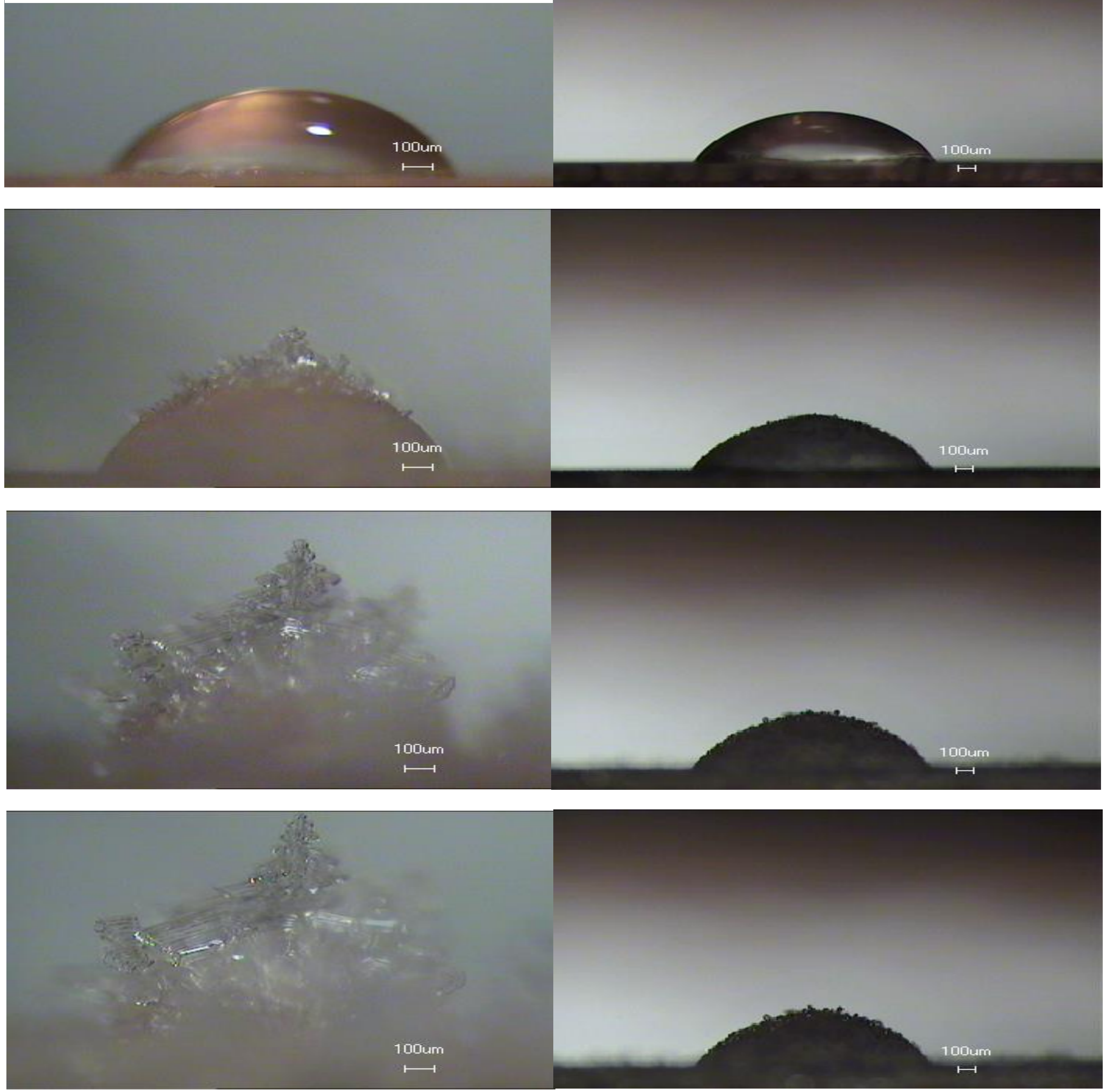




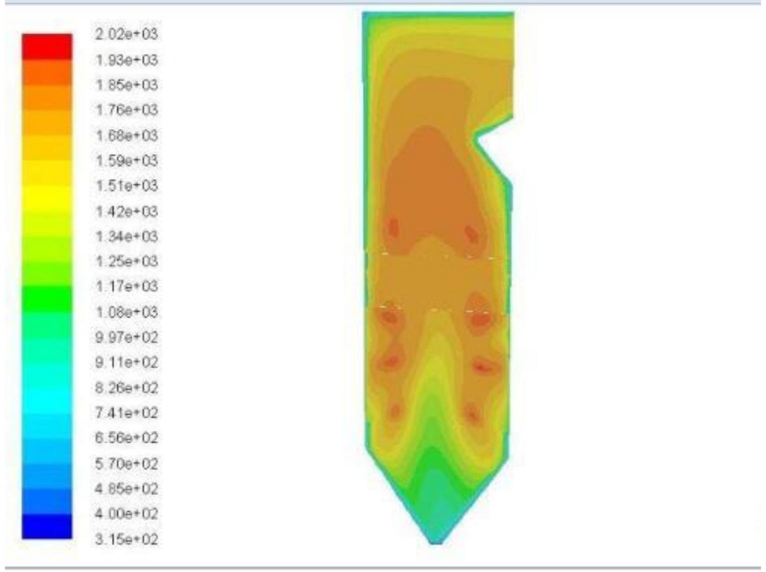
27 min



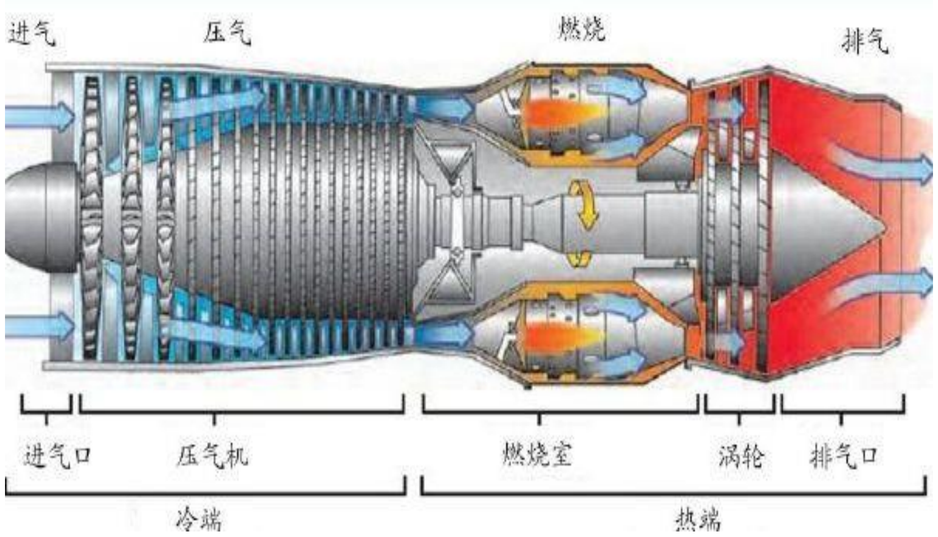
冰晶生成过程观测

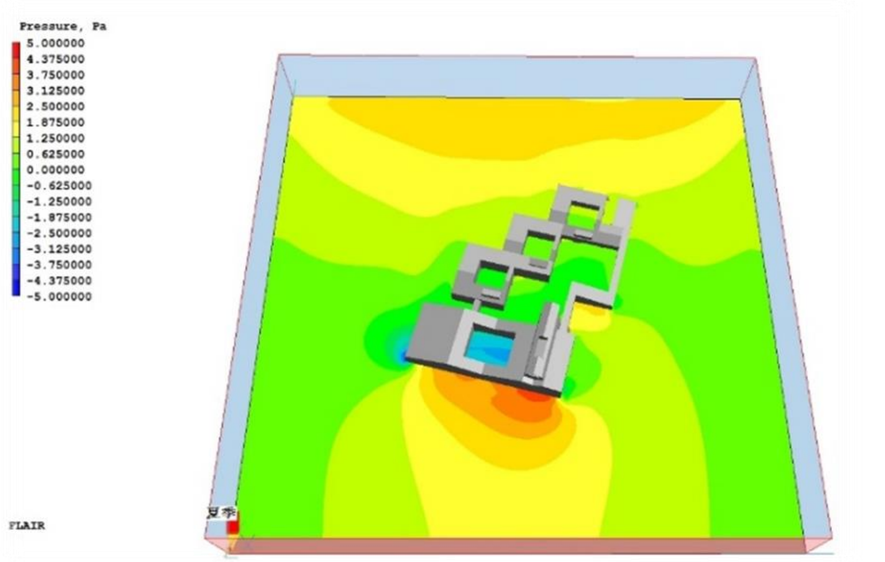


计算传热学的应用

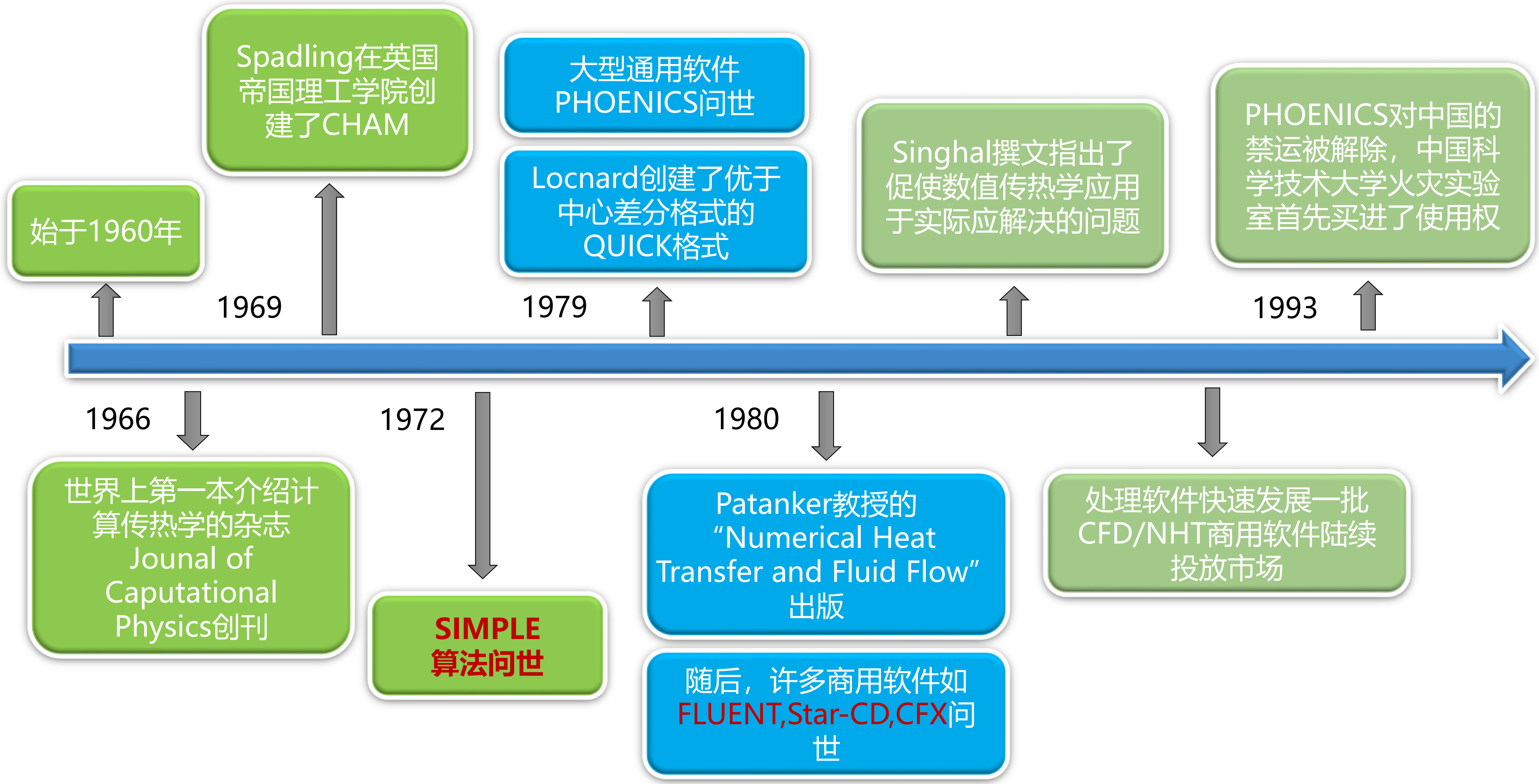


- 冶金工业中的铸轧
- 模拟航空发动机
- 模拟大型锅炉的燃烧
- 商用软件Fluent等





计算传热学发展简史


始于1960年
1966
世界上第一本介绍计算传热学的杂志Jounal of Caputational Physics创刊
1969
Spadling在英国帝国理工学院创建了CHAM
1972
SIMPLE算法问世
1979
大型通用软件PHOENICS问世
Locnard创建了优于中心差分格式的QUICK格式
1980
Patanker教授的“Numerical Heat Transfer and Fluid Flow”出版
随后，许多商用软件如FLUENT,Star-CD,CFX问世
Singhal撰文指出了促使数值传热学应用于实际应解决的问题
处理软件快速发展一批CFD/NHT商用软件陆续投放市场
1993
PHOENICS对中国的禁运被解除，中国科学技术大学火灾实验室首先买进了使用权

4.1 数值求解的基本思想

4.2 离散方程的建立方法

4.3 二维稳态导热问题的计算

4.4 代数方程组的解法

4.5 一维非稳态导热问题的计算

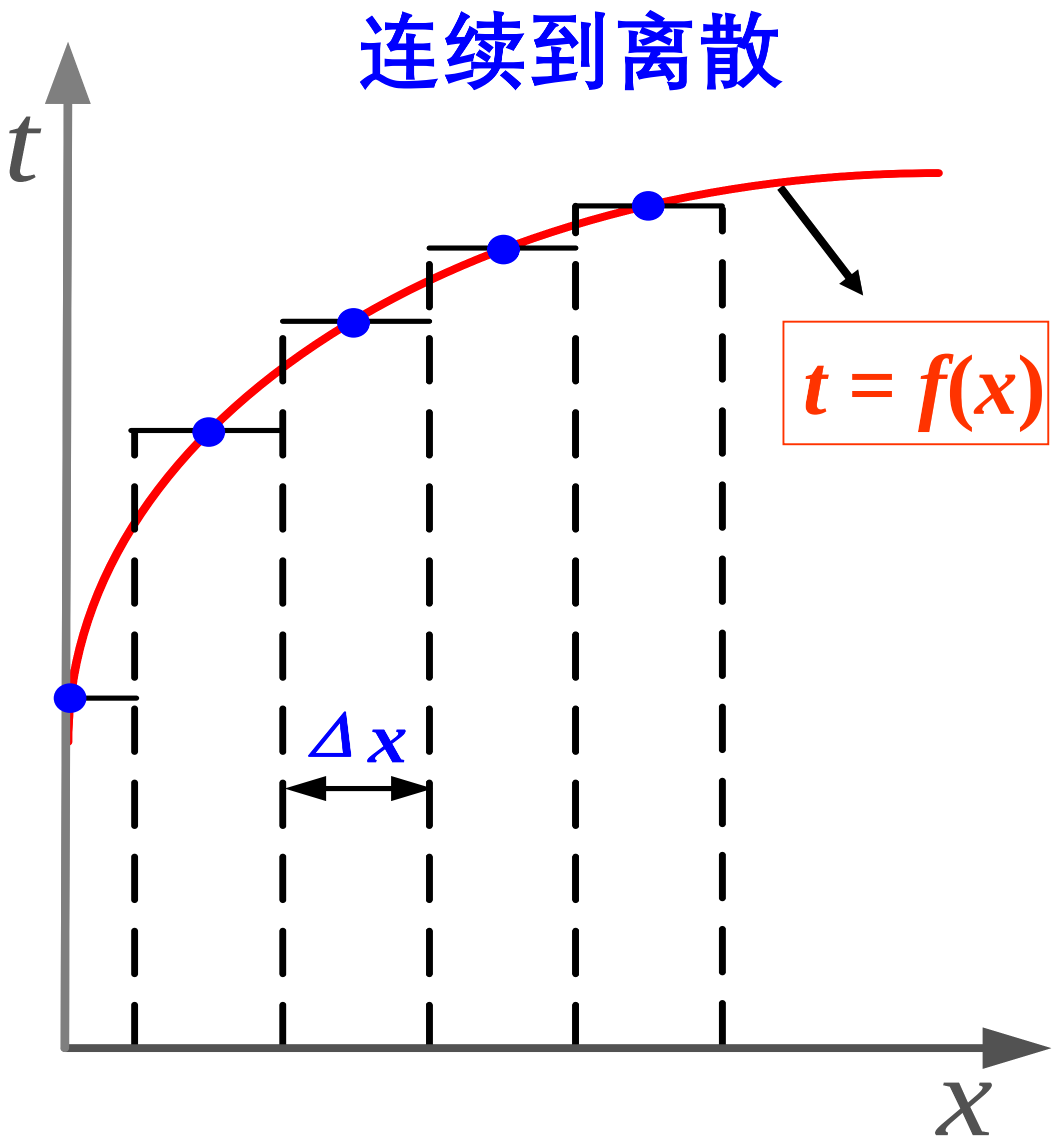
4.1 数值求解的基本思想

➢分析解——导热微分方程在定解条件下的积分求解

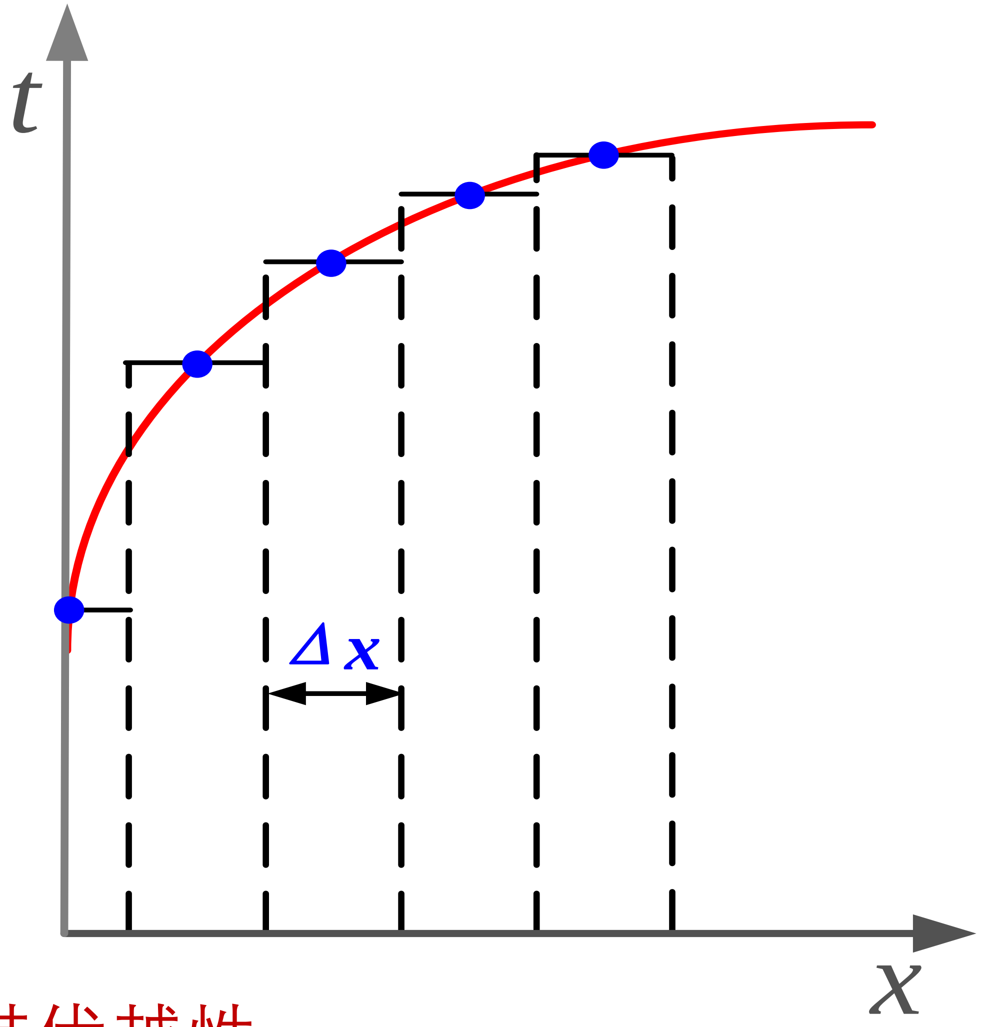
➢数值解——离散点上被求物理量的值的集合

连续 ⟹ 离散

微分方程 ⟹ 代数方程



分析解和数值解的比较



- **分析解**

(1) 精确解

(2) 对复杂问题无法求解

(3) 具有普遍性

- **数值解**

(1) 近似解

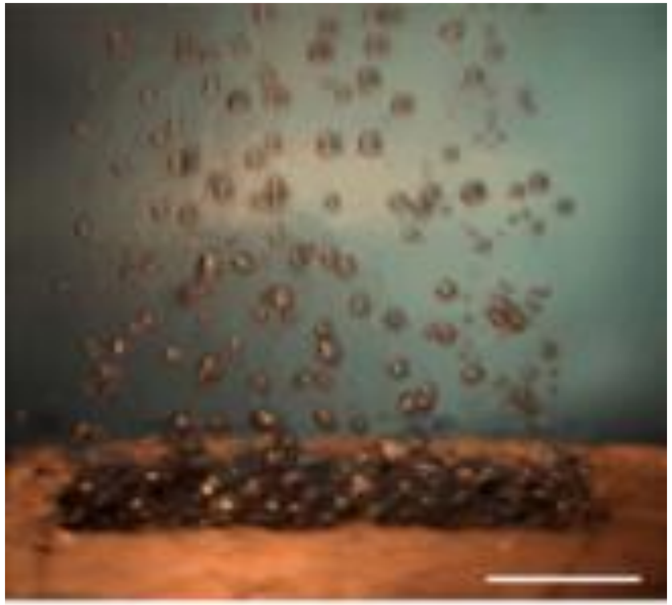
(2) 适应性强，特别对于复杂问题更显其优越性

(3) 与实验法相比成本低

(4) 具有一定精度

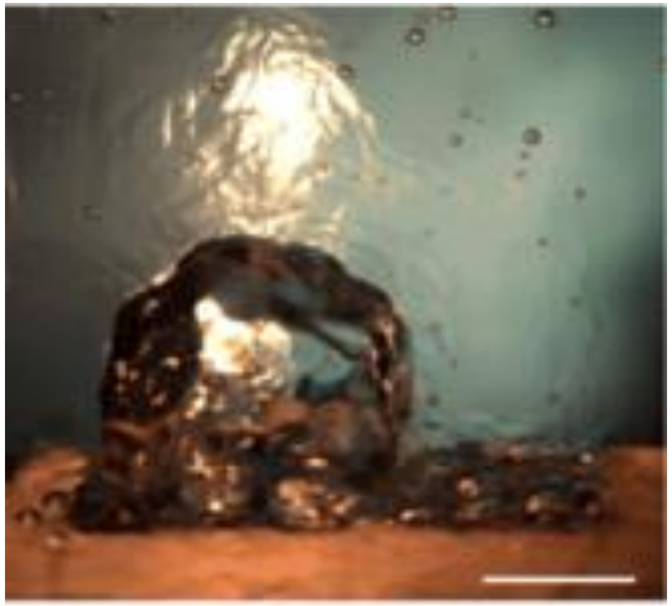
实际计算案例

航空航天工程中，航天器大多处于**微重力环境**，高集成度的电子元器件带来较大的散热需求，与常重力条件相比气泡行为变化明显。



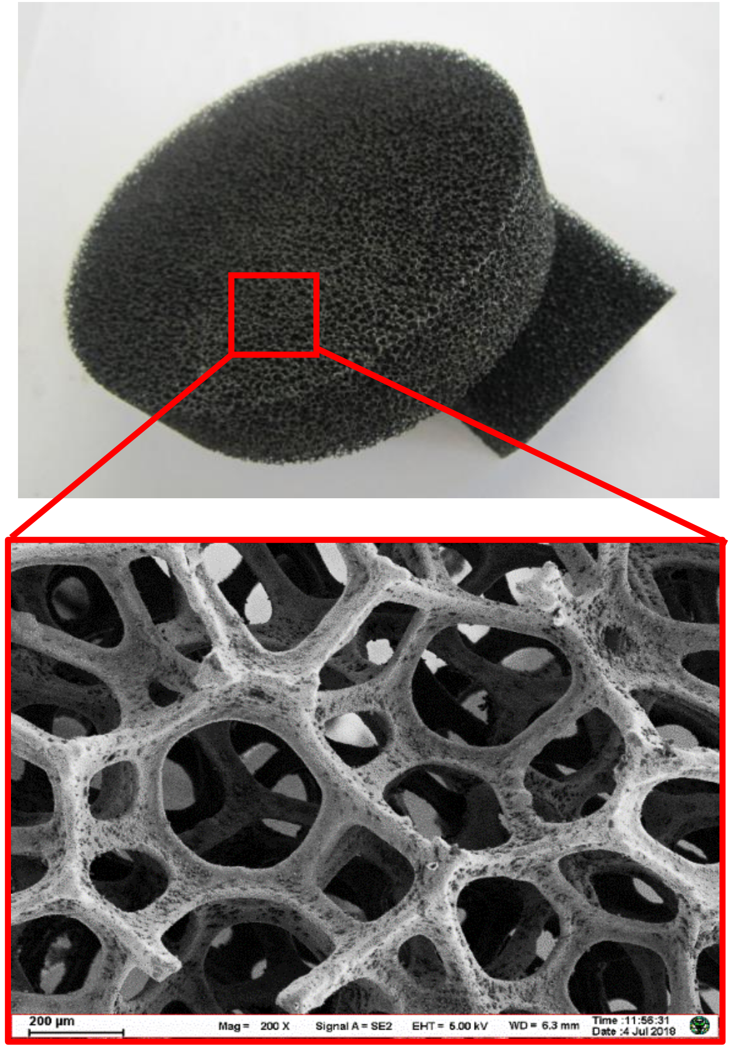
-0.714 s

常重力



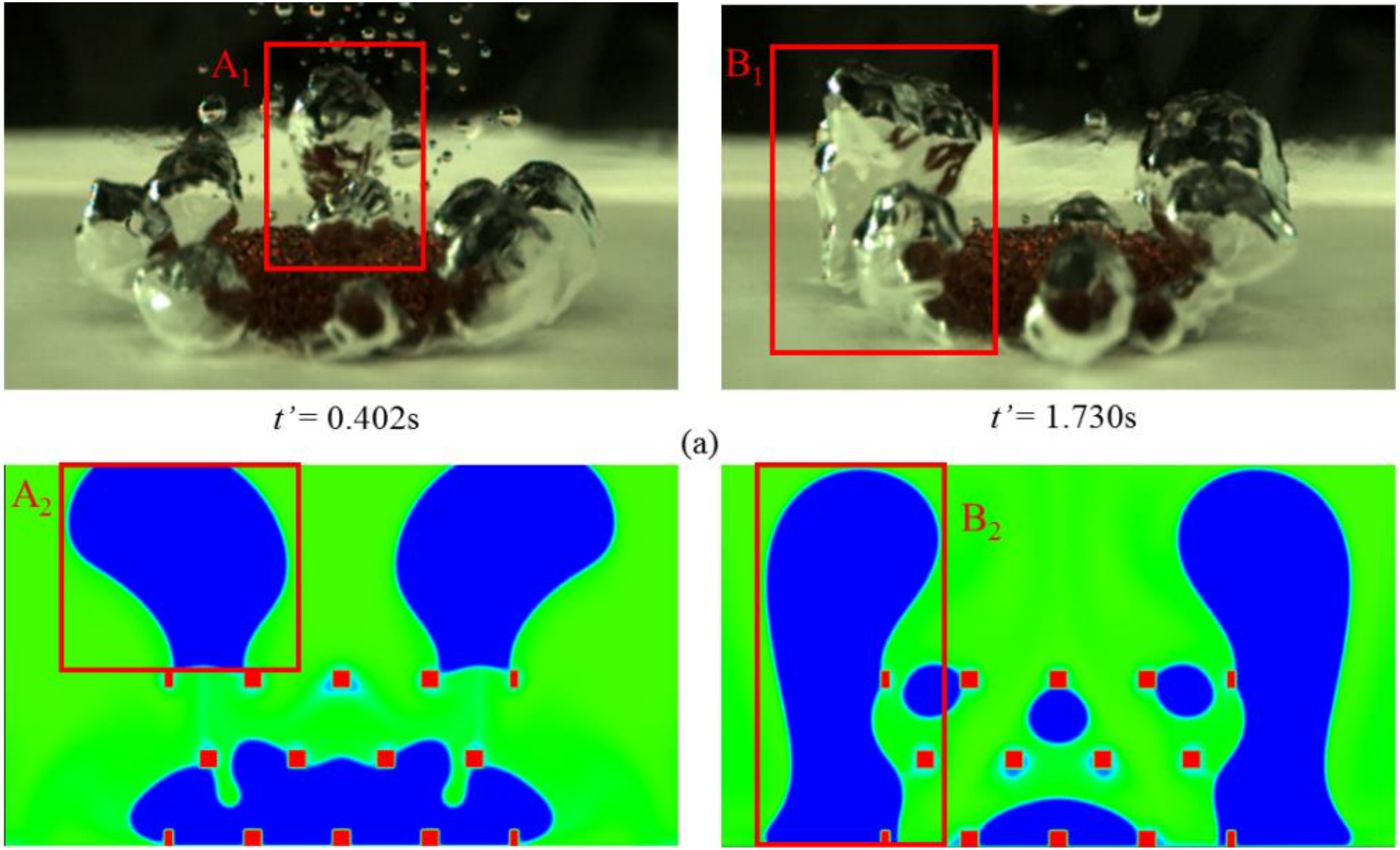
1.074 s

微重力



泡沫金属的宏观与微观图像

微重力池沸腾模拟



t'= 0.402s　　t'= 1.730s

(a)

t = 42000　　t = 67000

(b)

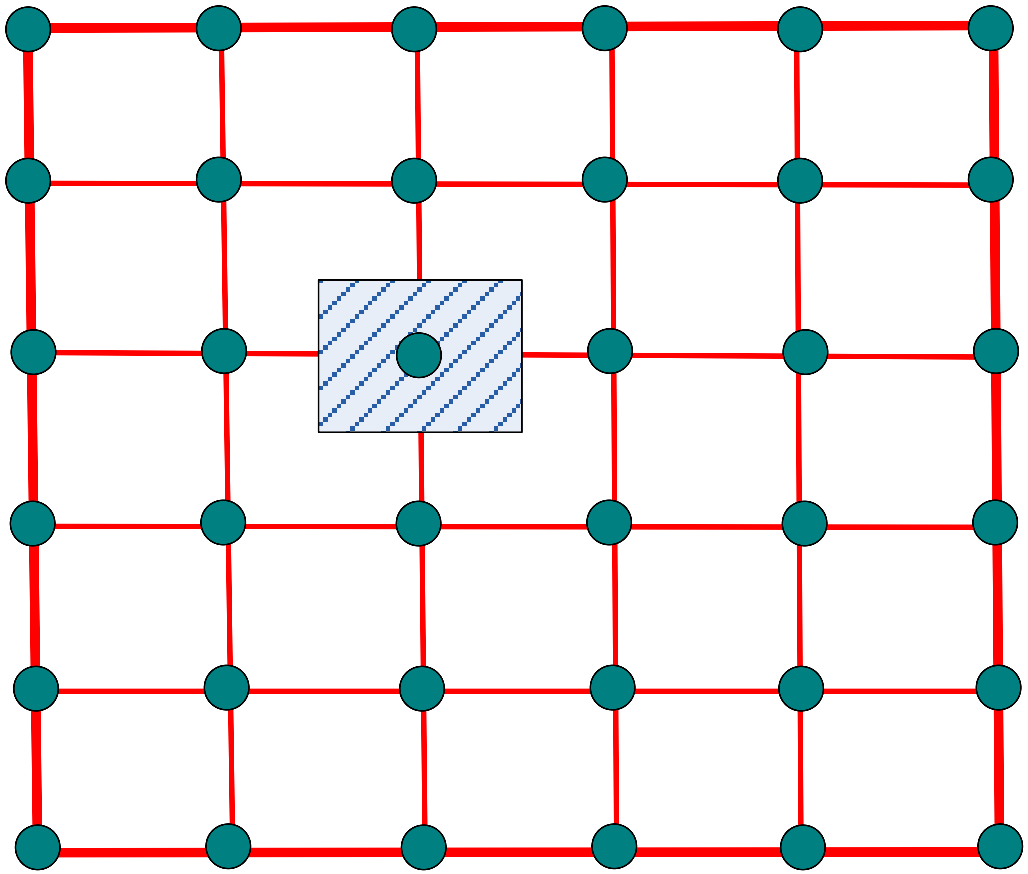
常用的数值计算方法

- 有限差分法
- 有限元法
- 有限体积法
- 边界元法
- 等等

有限差分法的基本思想

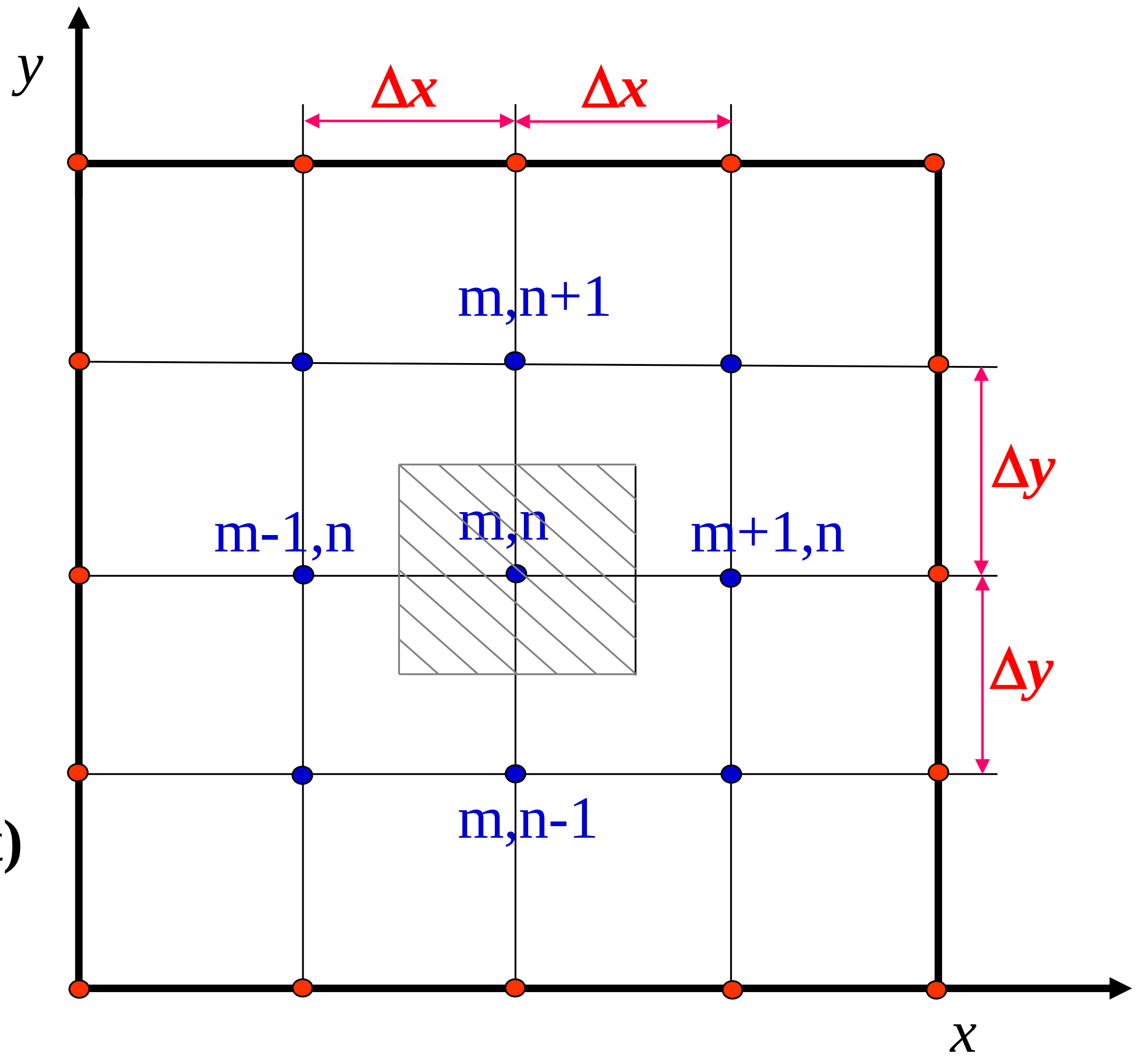
$$\frac{\partial t}{\partial \tau}=\frac{\lambda}{\rho c}\left(\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}+\frac{\partial^2 t}{\partial z^2}\right)+\frac{\dot{\Phi}_v}{\rho c}$$

- 用有限个网格节点的集合代替连续解；
- 用有限小的差商近似代替无限小的微商（导数）
- 用节点的离散化代数方程（差分方程）近似代替微分方程；
- 求解差分方程求取有限节点上的物理量。

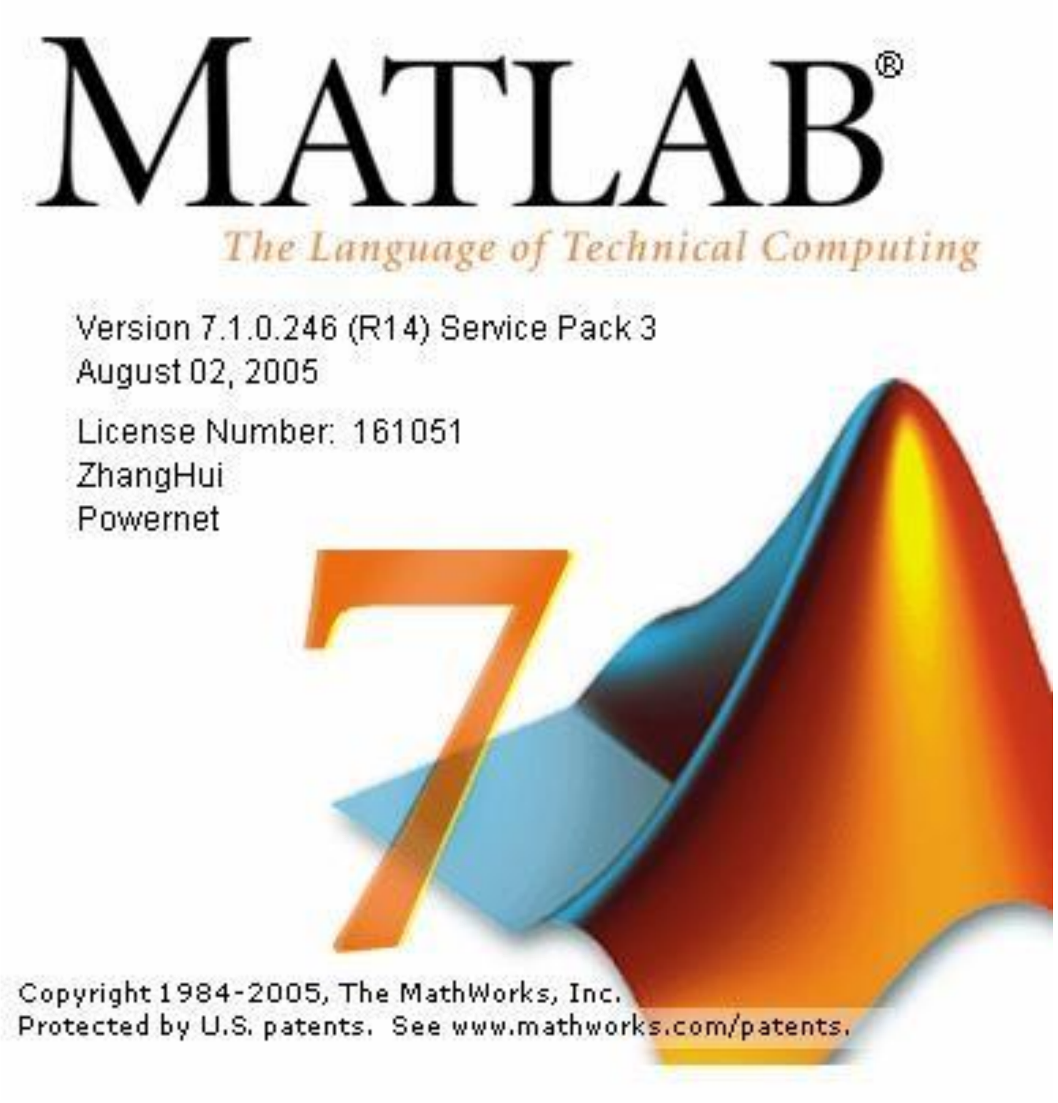


离散化方法中的常用术语

- 离散化(Discretization)
- 步长(step length)——即： Δx， Δy
- 节点(Nodes)——边界节点，内节点
- 网格(Grids)
- 控制容积(control volume)或单元体(element)



网格划分演示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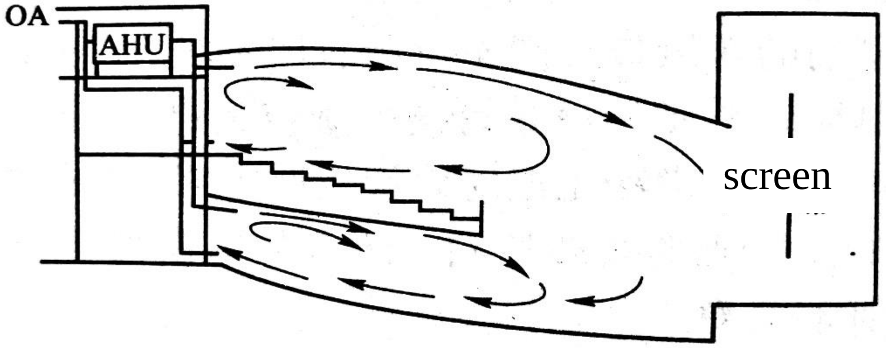
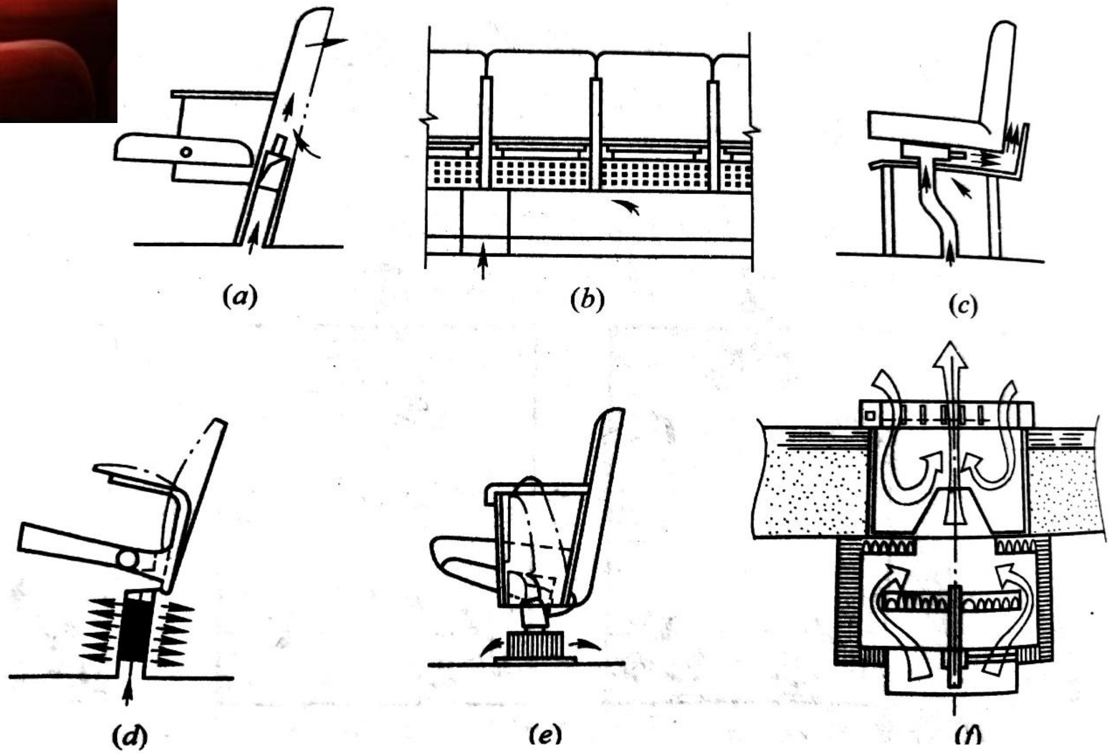
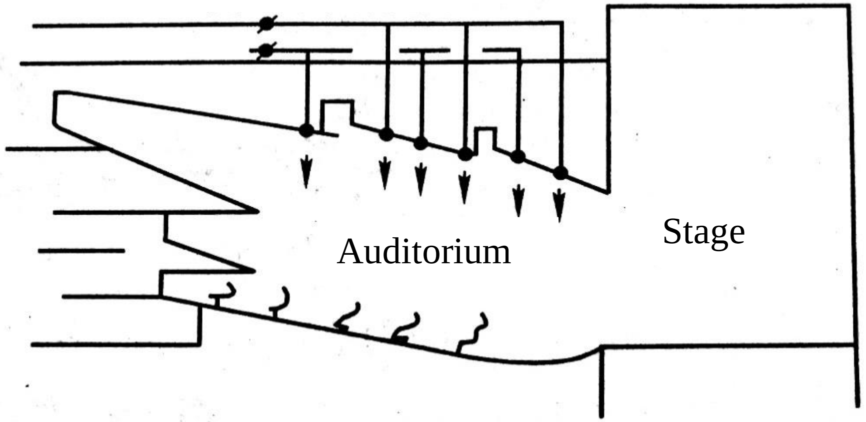
Radiation cooling technology in Theater （剧场辐射制冷技术）



Nozzle deliver model

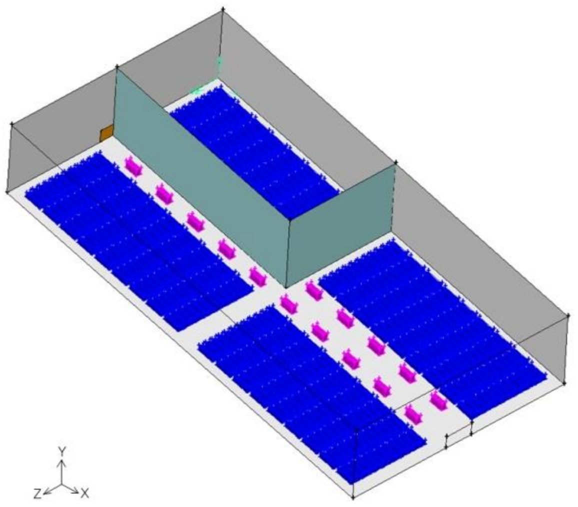


Switching valve



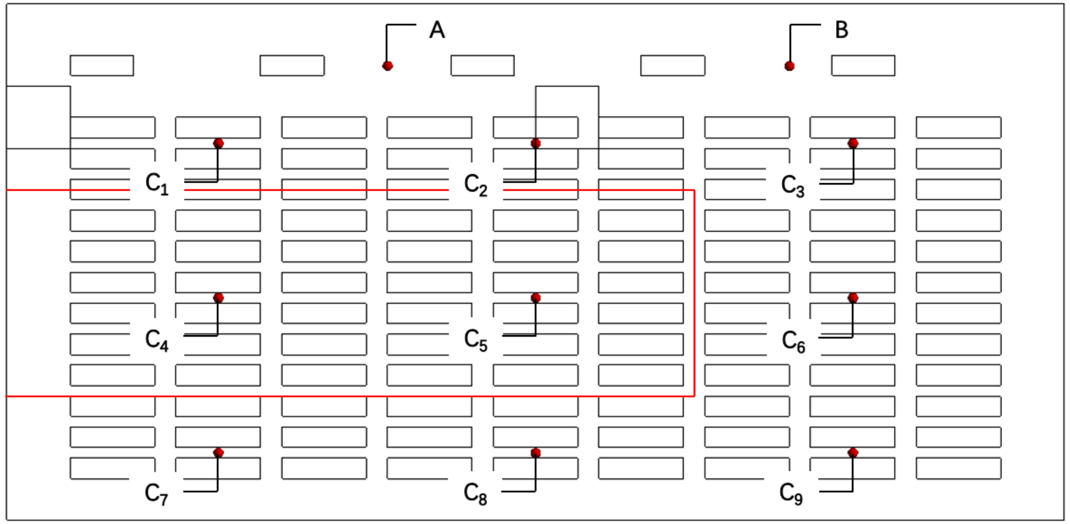
Top-supply and down-return air supply model

■ Establishment of heat transfer model

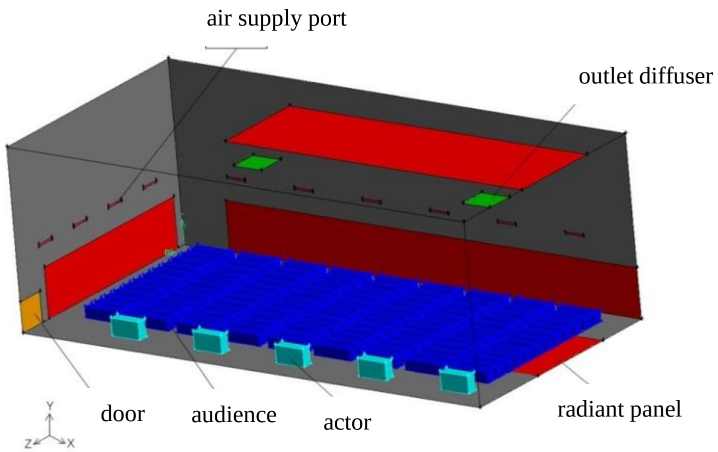


Stereogram of the theater building

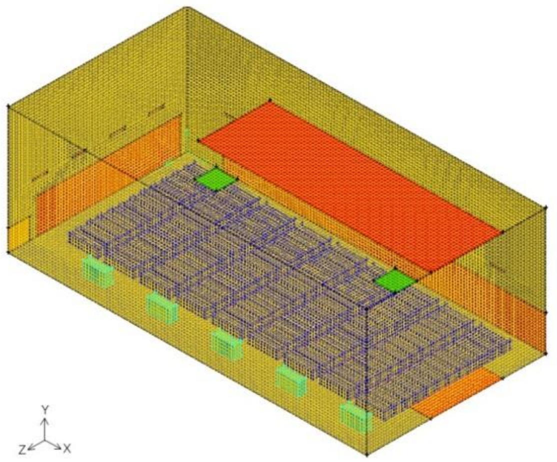
■ Setting of indoor monitoring point



- Audience area： C1~C9 nine monitoring points
- Performance area： A、B two monitoring points
- The inner area of the red line is where the radiant panel is placed on the ground.



Physical model diagram of the theater

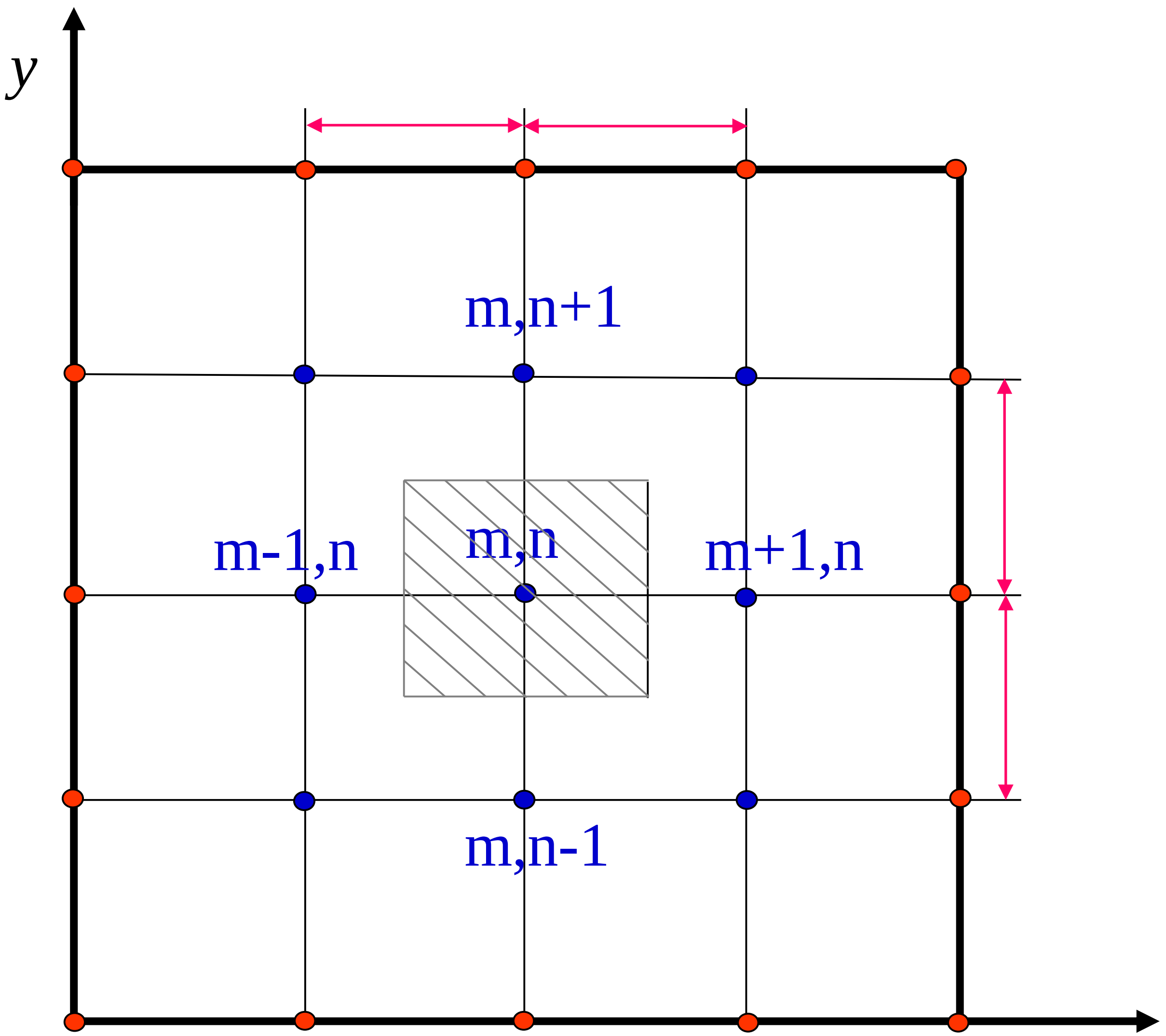


Mesh generation

Meshing under the premise of meeting the accuracy of the subject

Number of grids is 2034283

- 控制容积大小取决于节点数；
- 网格越细密，节点越多，计算时间越长，结果越接近分析解



有限差分法的基本思想

$$\frac{\partial t}{\partial \tau} = \frac{\lambda}{\rho c}\left(\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}\right) + \frac{\dot{\Phi}_v}{\rho c}$$

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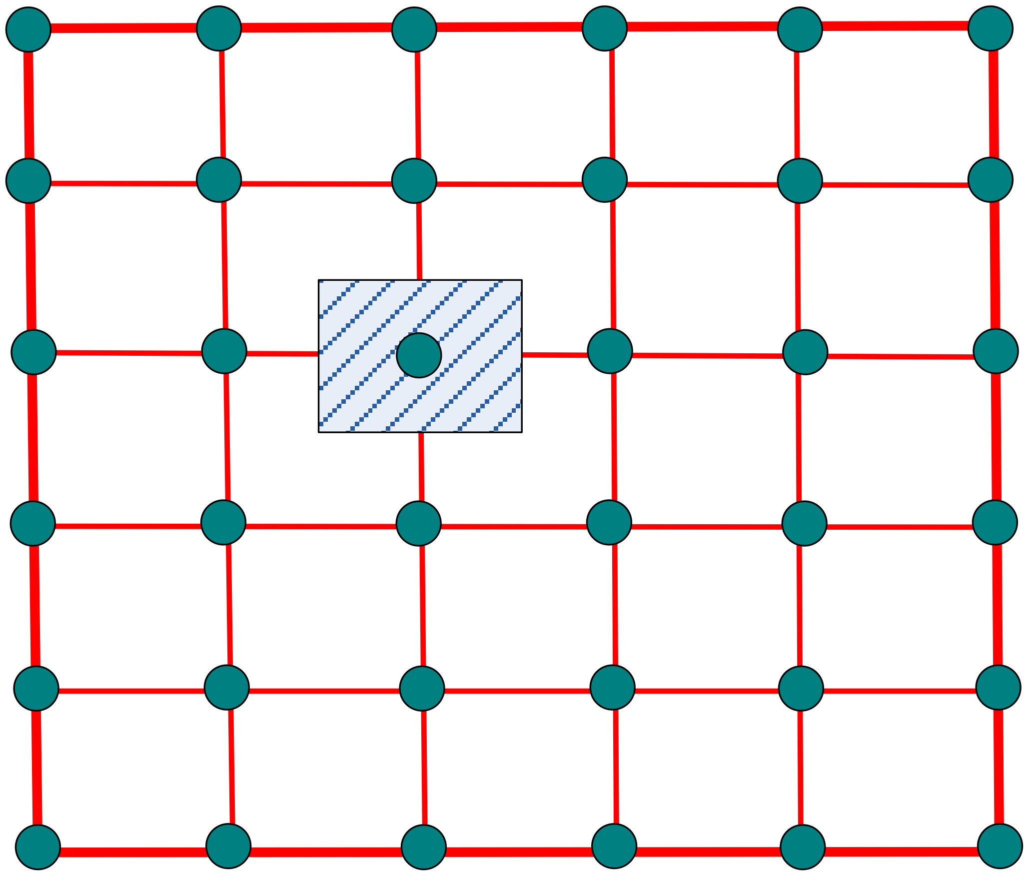


四种节点差分格式

微商形式（a）

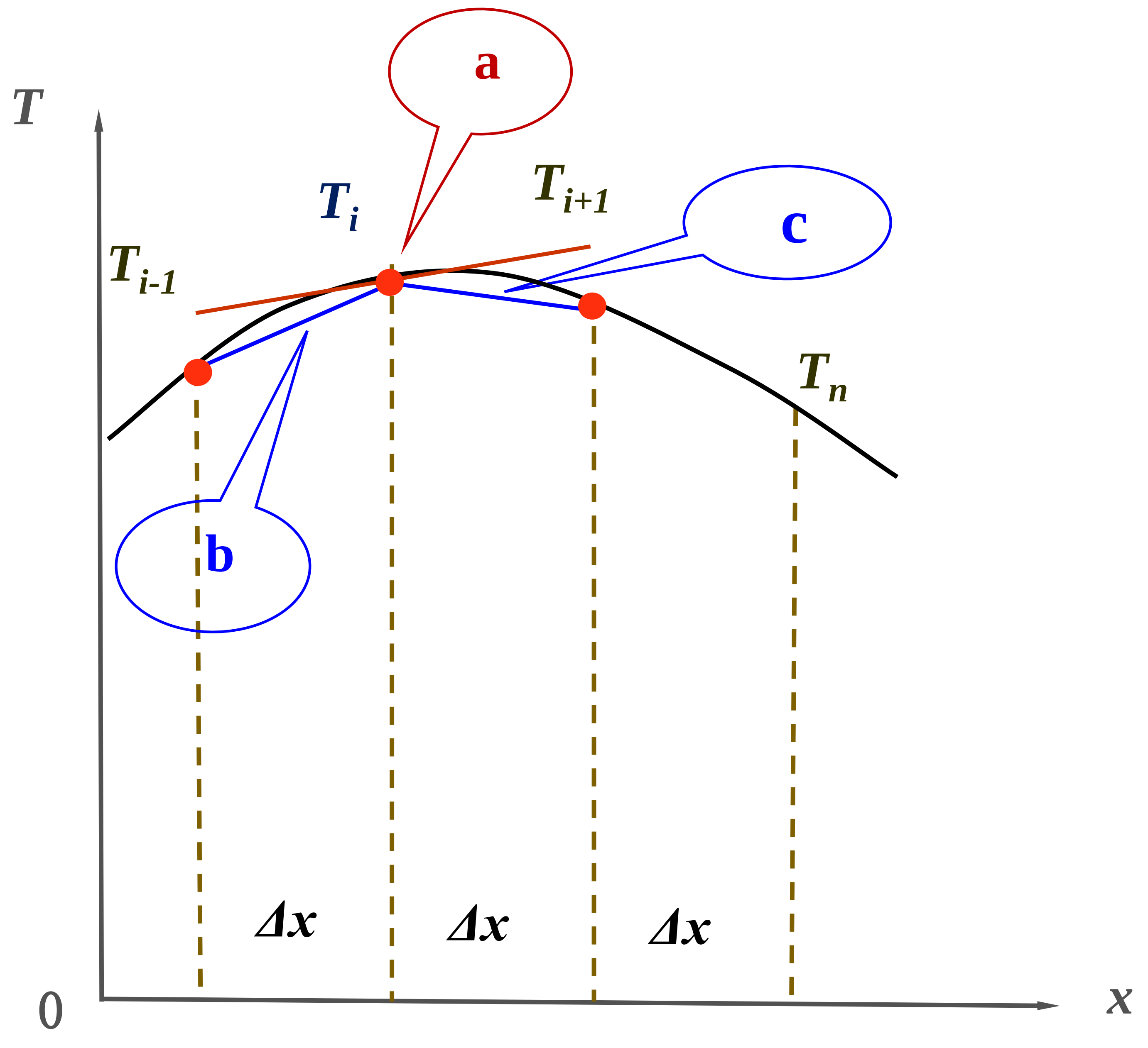
$$\frac{dT_i}{dx} = \lim_{\Delta x \to 0} \frac{\Delta T_i}{\Delta x}$$

1）向后差分格式(b：$T_i \to T_{i-1}$)

$$\frac{dT_i}{dx} \approx \frac{T(x_i) - T(x_{i-1})}{\Delta x}$$

2）向前差分格式(c：$T_i \to T_{i+1}$)

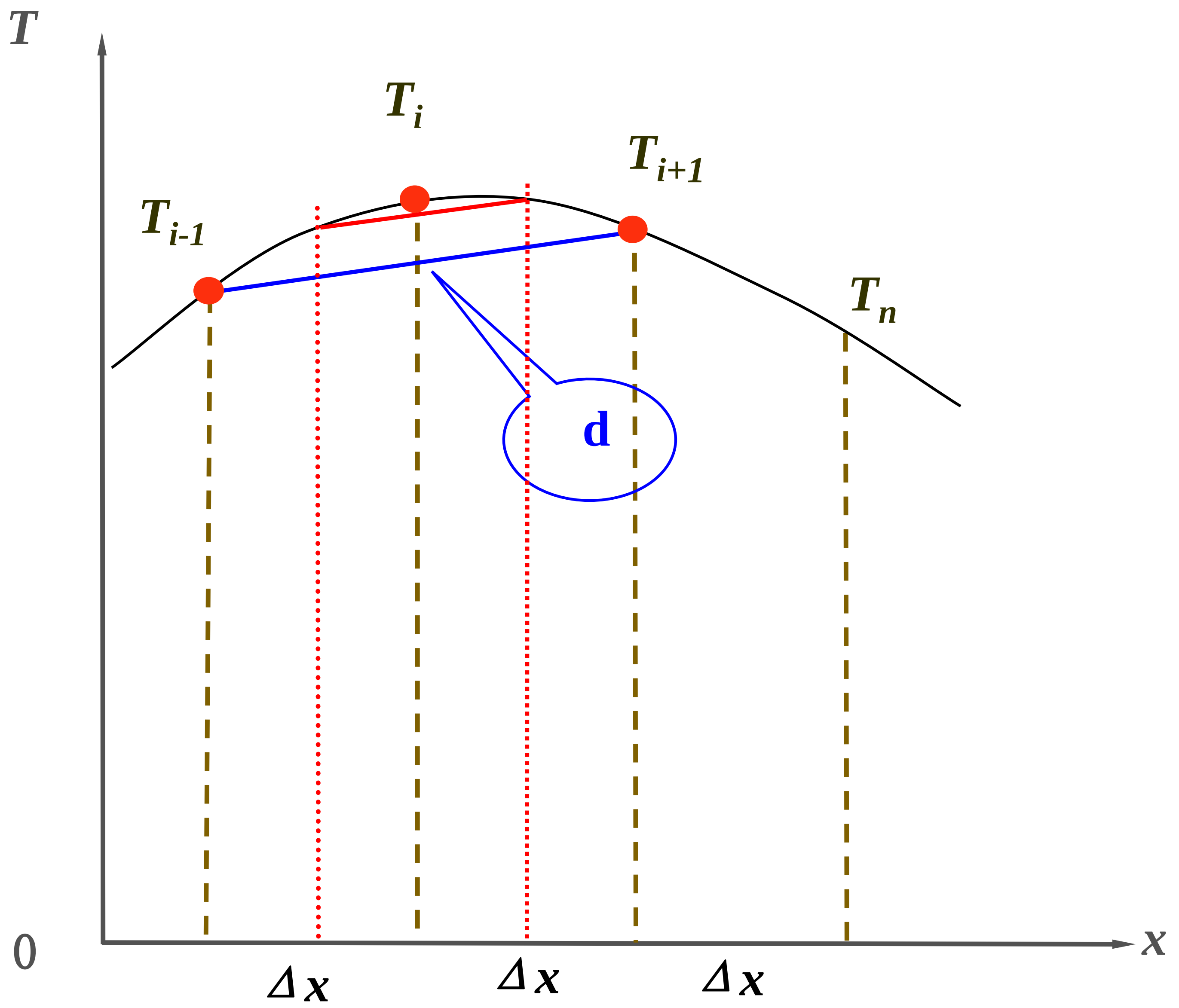
$$\frac{dT_i}{dx} \approx \frac{T(x_{i+1}) - T(x_i)}{\Delta x}$$



3）中心差分格式(d)

$$\frac{dT_i}{dx} \approx \frac{T(x_{i+1}) - T(x_{i-1})}{2\Delta x}$$

$$\frac{dT_i}{dx} \approx \frac{T(x_{i+1/2}) - T(x_{i-1/2})}{\Delta x}$$



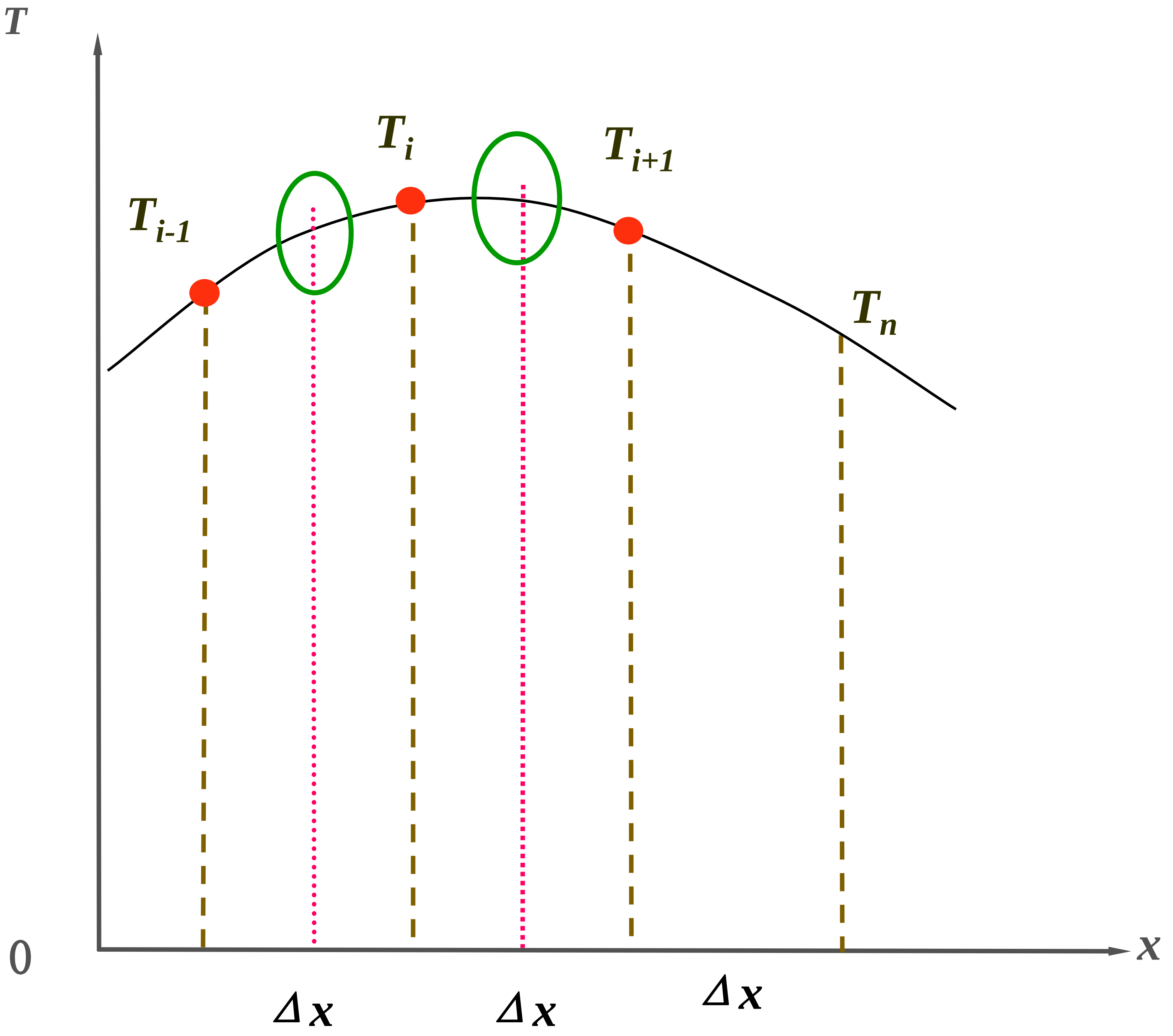
4）二阶微商（导数）的差分格式

$$\frac{d^2T_i}{dx^2}=\frac{d}{dx}\left(\frac{dT_i}{dx}\right)$$

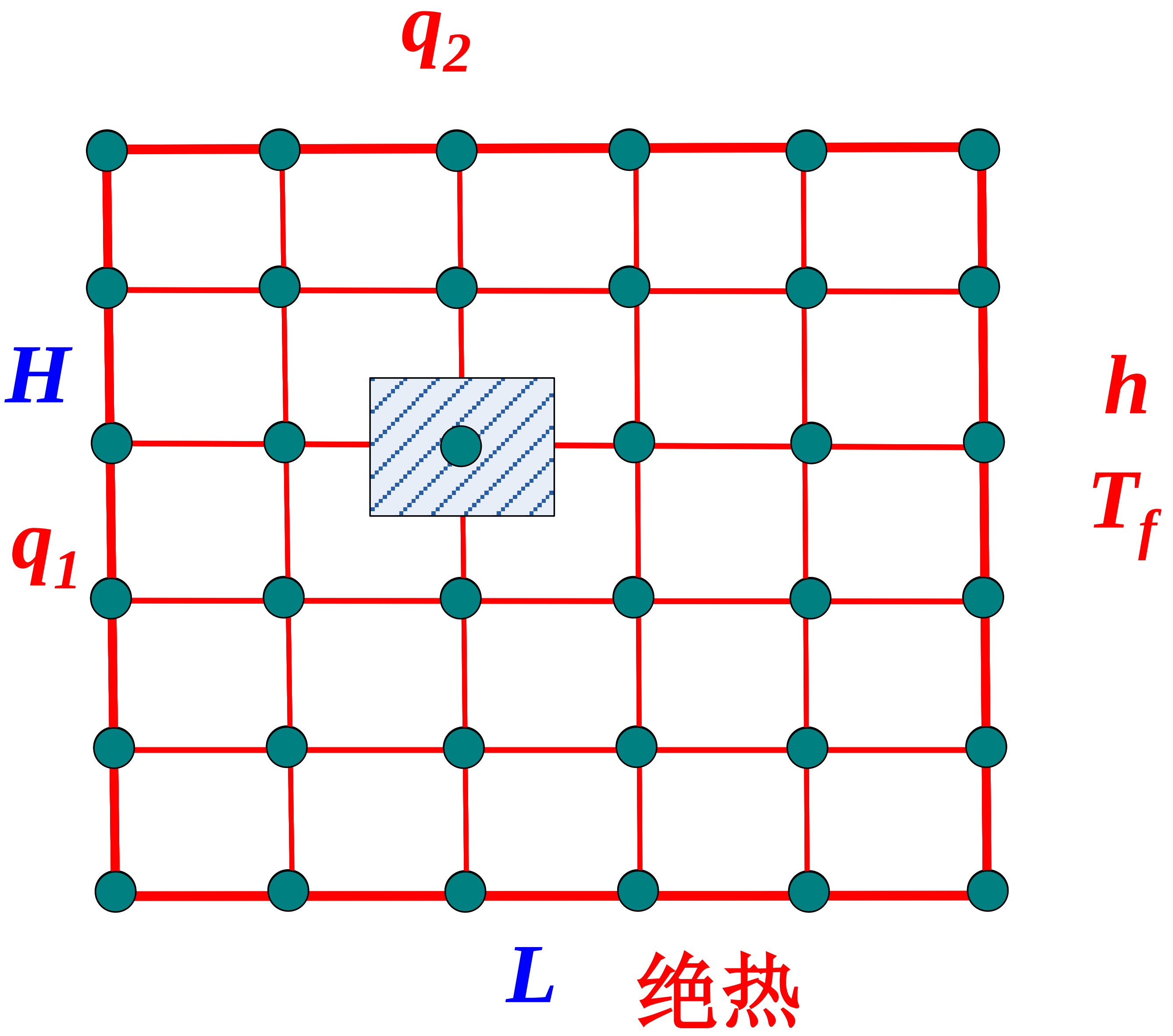
$$\approx\frac{\left(\frac{dT}{dx}\right)_{x=x_i+1/2}-\left(\frac{dT}{dx}\right)_{x=x_i-1/2}}{\Delta x}$$

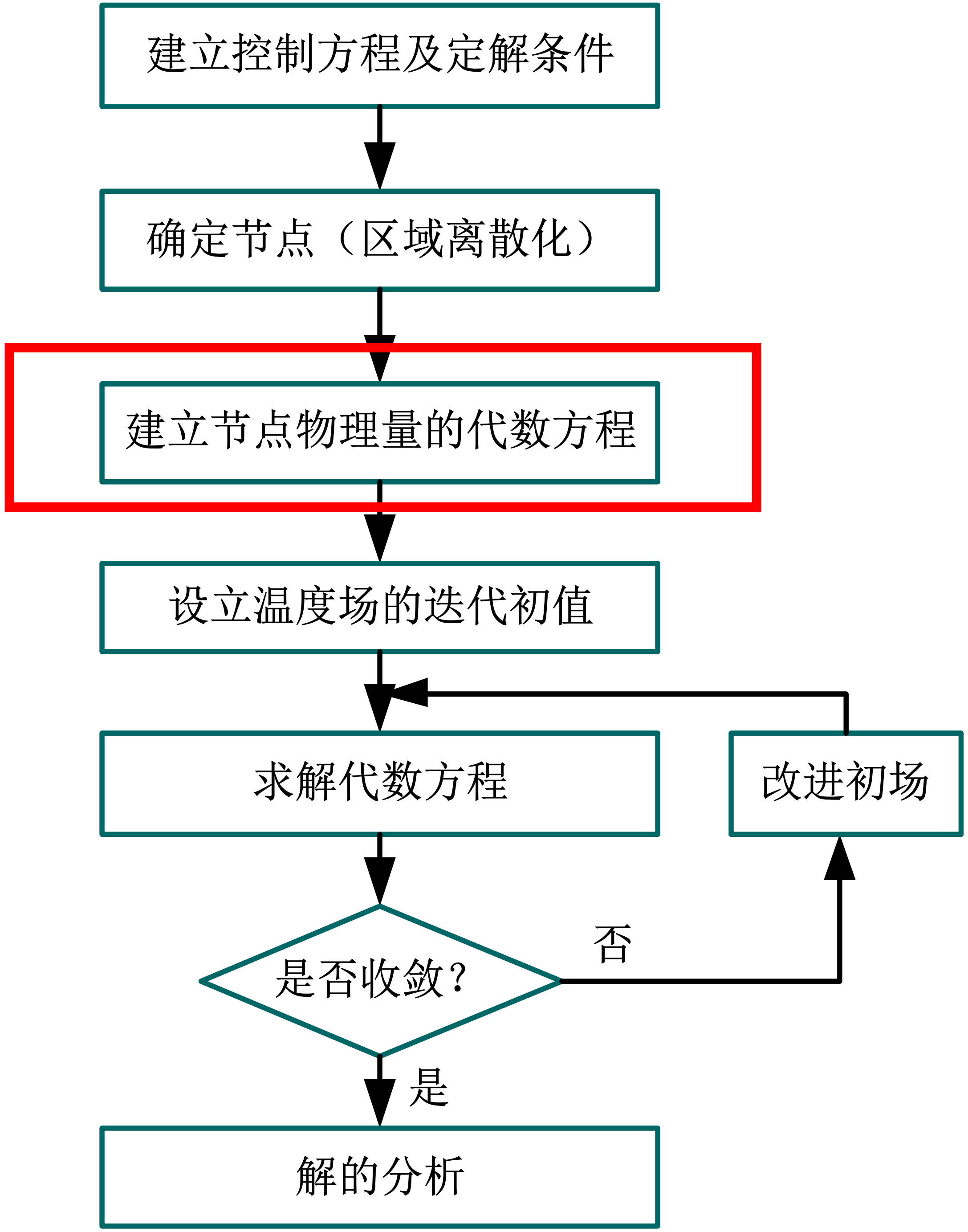
$$\approx\frac{\frac{T(x_{i+1})-T(x_i)}{\Delta x}-\frac{T(x_i)-T(x_{i\text{-}1})}{\Delta x}}{\Delta x}$$

$$\approx\frac{T(x_{i+1})-2T(x_i)+T(x_{i\text{-}1})}{(\Delta x)^2}$$



导热问题数值求解基本步骤





4.1 数值求解的基本思想

4.2 离散方程的建立方法

4.3 二维稳态导热问题的计算

4.4 代数方程组的解法

4.5 一维非稳态导热问题的计算

4.2 离散方程的建立方法

（1）控制容积热平衡法：着重于从物理的观点来分析

（2）Taylor（泰勒）级数展开法：偏重于从数学的角度进行推导

（3）多项式拟合法

（4）控制容积积分法

（1）热平衡法

基本思想

从基本物理现象和基本定律出发

依据能量守恒和傅立叶导热定律建立方程

能量守恒

从所有方向流入控制体的总热流量 ＋

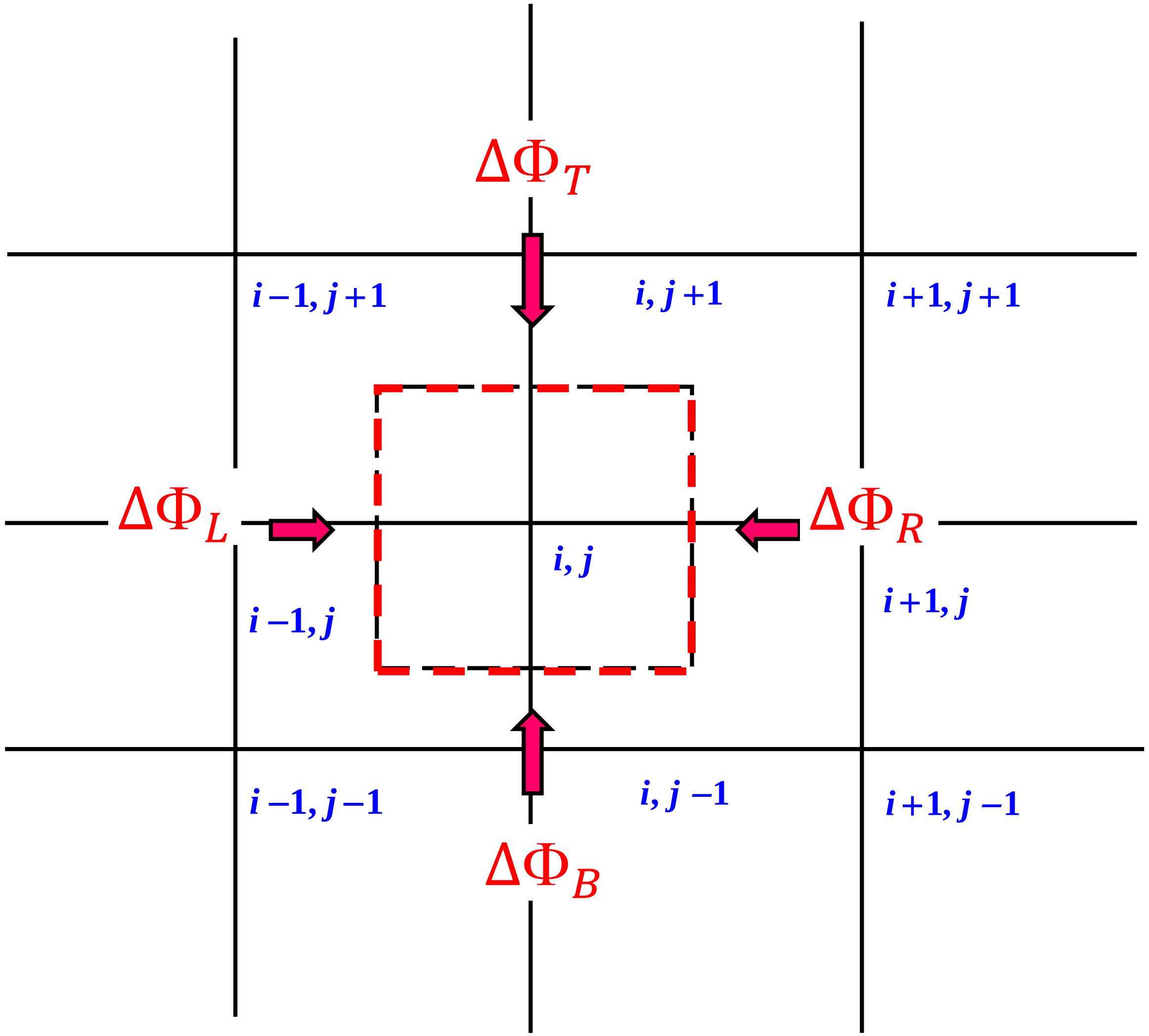
控制体内热源生成热 ＝ 控制体内能的增量

单位：[W]

$$\boxed{\Phi_i + (-\Phi_o) + \Phi_v = \Phi_\tau}$$

热平衡法

$$\Phi_i + (-\Phi_o) + \cancel{\Phi_v} = \cancel{\Phi_\tau}$$



稳态无内热源条件

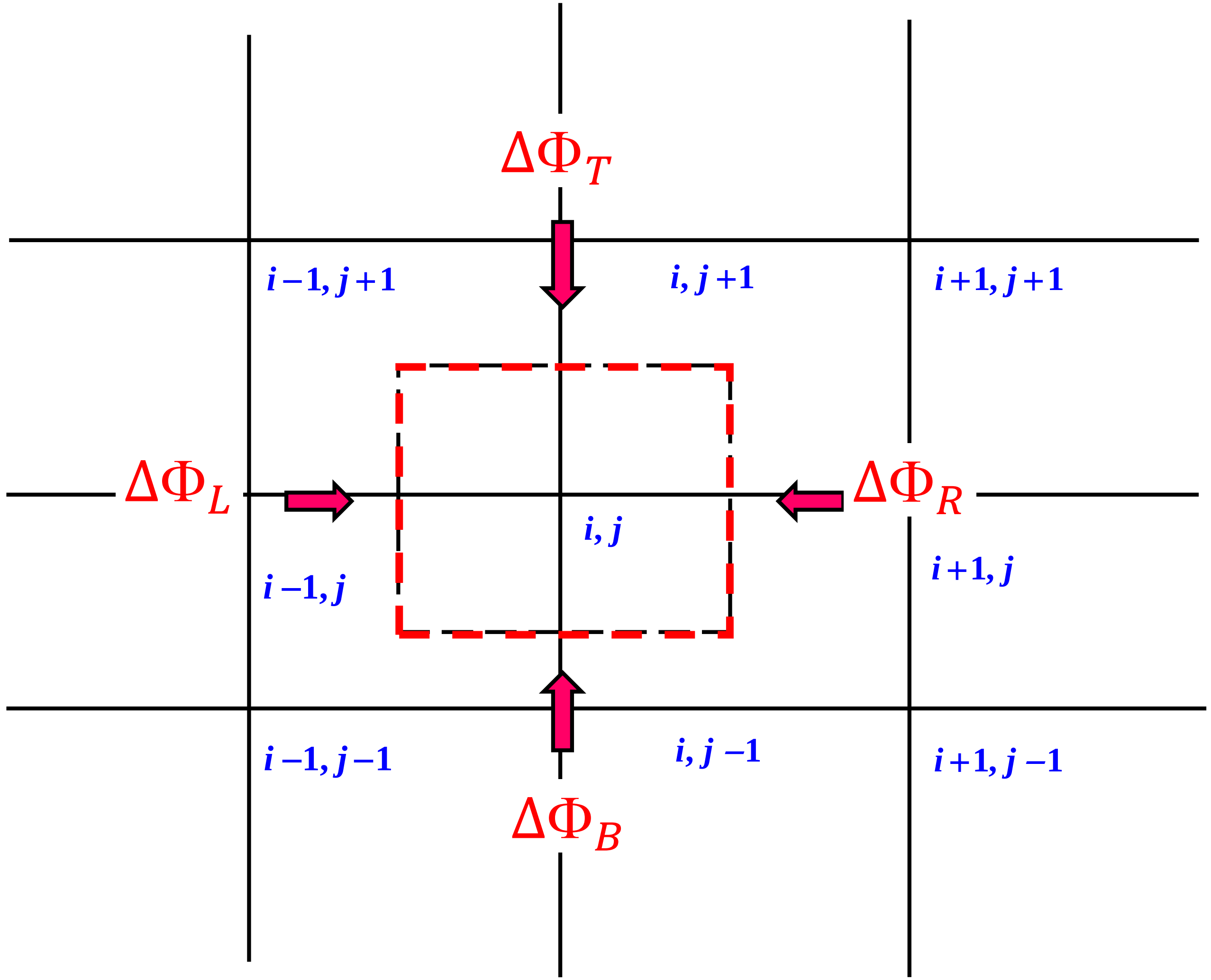
从所有方向导入控制体的总热流量=0

能量平衡方程

$$\Delta\Phi_L + \Delta\Phi_R + \Delta\Phi_B + \Delta\Phi_T = 0$$

能量平衡方程

$$\Delta\Phi_L + \Delta\Phi_R + \Delta\Phi_B + \Delta\Phi_T = 0$$



$$\Delta\Phi_L = q_L \Delta y = -\lambda \frac{\partial t}{\partial x}\Big|_L \Delta y$$

$$\Delta\Phi_R = q_R \Delta y = \lambda \frac{\partial t}{\partial x}\Big|_R \Delta y$$

$$\Delta\Phi_B = q_B \Delta x = -\lambda \frac{\partial t}{\partial y}\Big|_B \Delta x$$

$$\Delta\Phi_T = q_T \Delta x = \lambda \frac{\partial t}{\partial y}\Big|_T \Delta x$$

一阶导数的近似——差商

用差商代替微商（导数）

$$\Delta\Phi_L = q_L \Delta y = -\lambda \left.\frac{\partial t}{\partial x}\right|_L \Delta y$$

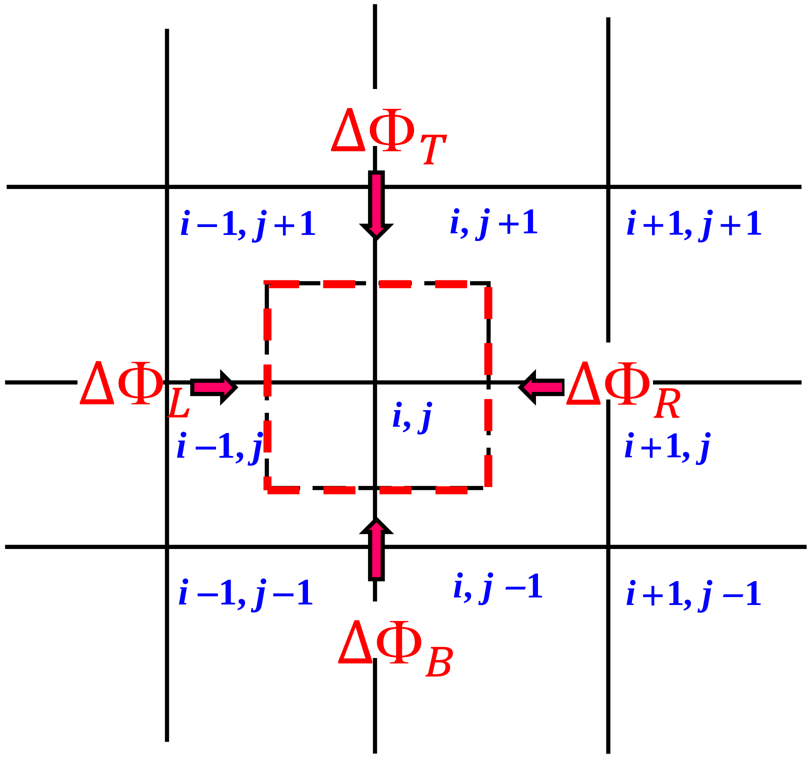
$$\Delta\Phi_R = q_R \Delta y = \lambda \left.\frac{\partial t}{\partial x}\right|_R \Delta y$$

$$\Delta\Phi_B = q_B \Delta y = -\lambda \left.\frac{\partial t}{\partial y}\right|_B \Delta x$$

$$\Delta\Phi_T = q_T \Delta y = \lambda \left.\frac{\partial t}{\partial y}\right|_T \Delta x$$

$$\left.\frac{\partial t}{\partial x}\right|_L \approx \frac{t_{i,j} - t_{i-1,j}}{\Delta x} \qquad \left.\frac{\partial t}{\partial x}\right|_R \approx \frac{t_{i+1,j} - t_{i,j}}{\Delta x}$$

$$\left.\frac{\partial t}{\partial y}\right|_B \approx \frac{t_{i,j} - t_{i,j-1}}{\Delta y} \qquad \left.\frac{\partial t}{\partial y}\right|_T \approx \frac{t_{i,j+1} - t_{i,j}}{\Delta y}$$



$$\Delta\Phi_L = q_L \Delta y = -\lambda \left.\frac{\partial t}{\partial x}\right|_L \Delta y$$

$$\Delta\Phi_R = q_R \Delta y = \lambda \left.\frac{\partial t}{\partial x}\right|_R \Delta y$$

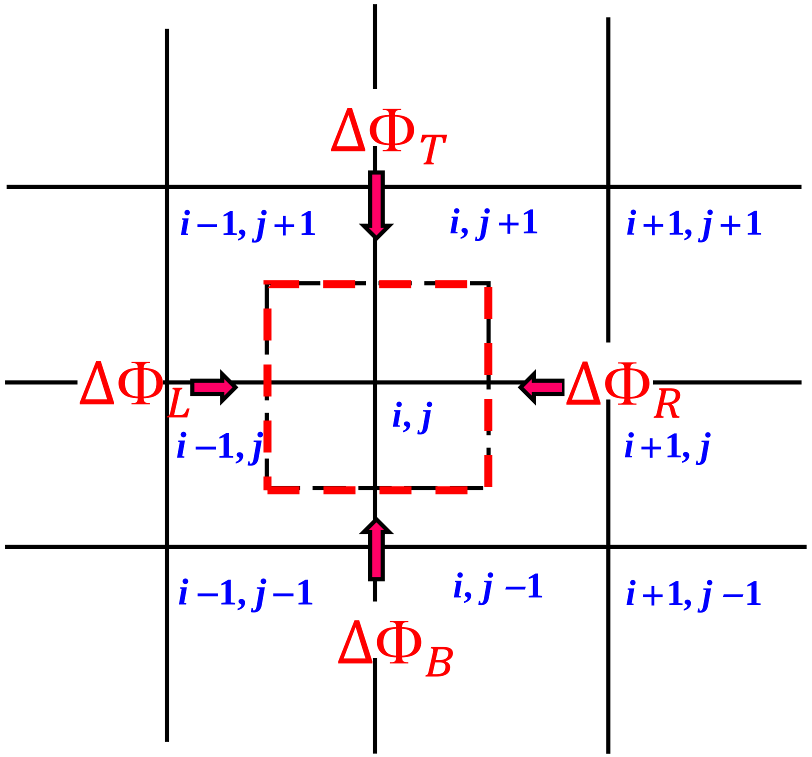
$$\Delta\Phi_B = q_B \Delta y = -\lambda \left.\frac{\partial t}{\partial y}\right|_B \Delta x$$

$$\Delta\Phi_T = q_T \Delta y = \lambda \left.\frac{\partial t}{\partial y}\right|_T \Delta x$$

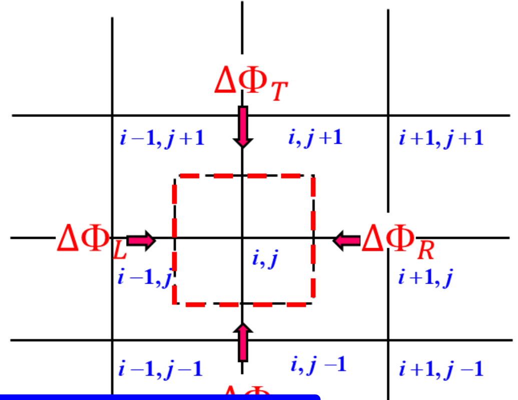
$$\left.\frac{\partial t}{\partial x}\right|_L \approx \frac{t_{i,j} - t_{i-1,j}}{\Delta x} \qquad \left.\frac{\partial t}{\partial x}\right|_R \approx \frac{t_{i+1,j} - t_{i,j}}{\Delta x}$$

$$\left.\frac{\partial t}{\partial y}\right|_B \approx \frac{t_{i,j} - t_{i,j-1}}{\Delta y} \qquad \left.\frac{\partial t}{\partial y}\right|_T \approx \frac{t_{i,j+1} - t_{i,j}}{\Delta y}$$

$$\begin{cases} \Delta\Phi_L = -\lambda\Delta y\left(\dfrac{t_{i,j} - t_{i-1,j}}{\Delta x}\right), \quad \Delta\Phi_R = \lambda\Delta y\left(\dfrac{t_{i+1,j} - t_{i,j}}{\Delta x}\right) \\ \Delta\Phi_B = -\lambda\Delta x\left(\dfrac{t_{i,j} - t_{i,j-1}}{\Delta y}\right), \quad \Delta\Phi_T = \lambda\Delta x\left(\dfrac{t_{i,j+1} - t_{i,j}}{\Delta y}\right) \end{cases}$$



微元体能量守恒的近似表达



$$
\begin{cases}
\Delta\Phi_L = -\lambda\Delta y\left(\dfrac{t_{i,j}-t_{i-1,j}}{\Delta x}\right), & \Delta\Phi_R = \lambda\Delta y\left(\dfrac{t_{i+1,j}-t_{i,j}}{\Delta x}\right) \\
\Delta\Phi_B = -\lambda\Delta x\left(\dfrac{t_{i,j}-t_{i,j-1}}{\Delta y}\right), & \Delta\Phi_T = \lambda\Delta x\left(\dfrac{t_{i,j+1}-t_{i,j}}{\Delta y}\right)
\end{cases}
$$

$$\boxed{\Delta\Phi_L + \Delta\Phi_R + \Delta\Phi_B + \Delta\Phi_T = 0}$$

$$-\cancel{\lambda}\cancel{\Delta y}\left(\frac{t_{i,j}-t_{i-1,j}}{\cancel{\Delta x}}\right)+\cancel{\lambda}\cancel{\Delta y}\left(\frac{t_{i+1,j}-t_{i,j}}{\cancel{\Delta x}}\right)-\cancel{\lambda}\cancel{\Delta x}\left(\frac{t_{i,j}-t_{i,j-1}}{\cancel{\Delta y}}\right)+\cancel{\lambda}\cancel{\Delta x}\left(\frac{t_{i,j+1}-t_{i,j}}{\cancel{\Delta y}}\right)=0$$

$\Delta x = \Delta y$

$$-\left(t_{i,j}-t_{i-1,j}\right)+\left(t_{i+1,j}-t_{i,j}\right)-\left(t_{i,j}-t_{i,j-1}\right)+\left(t_{i,j+1}-t_{i,j}\right)=0$$

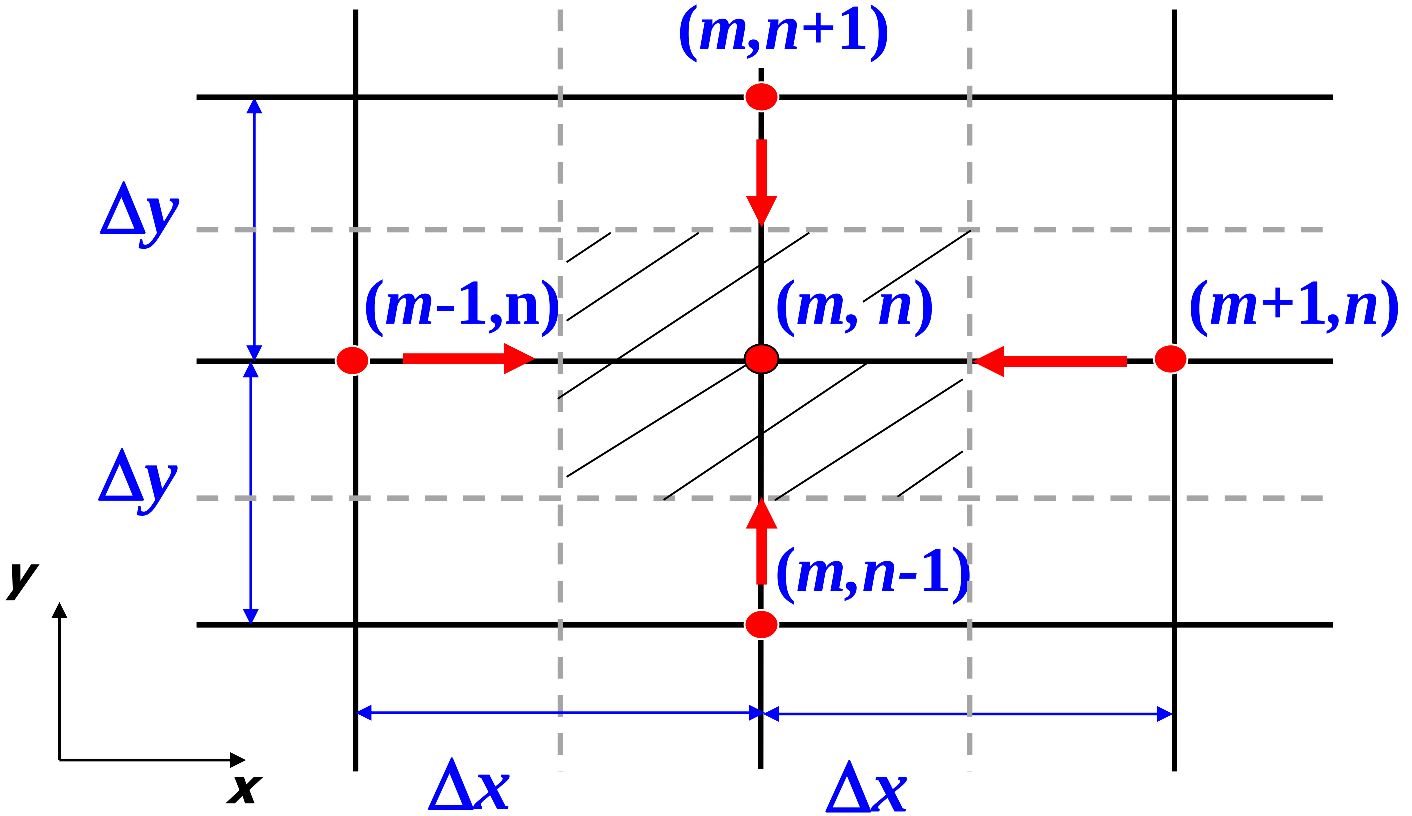
$$\boxed{t_{i,j}=\frac{1}{4}\left(t_{i-1,j}+t_{i+1,j}+t_{i,j-1}+t_{i,j+1}\right)}$$

(2) Taylor（泰勒）级数展开法

函数的泰勒级数展开式为

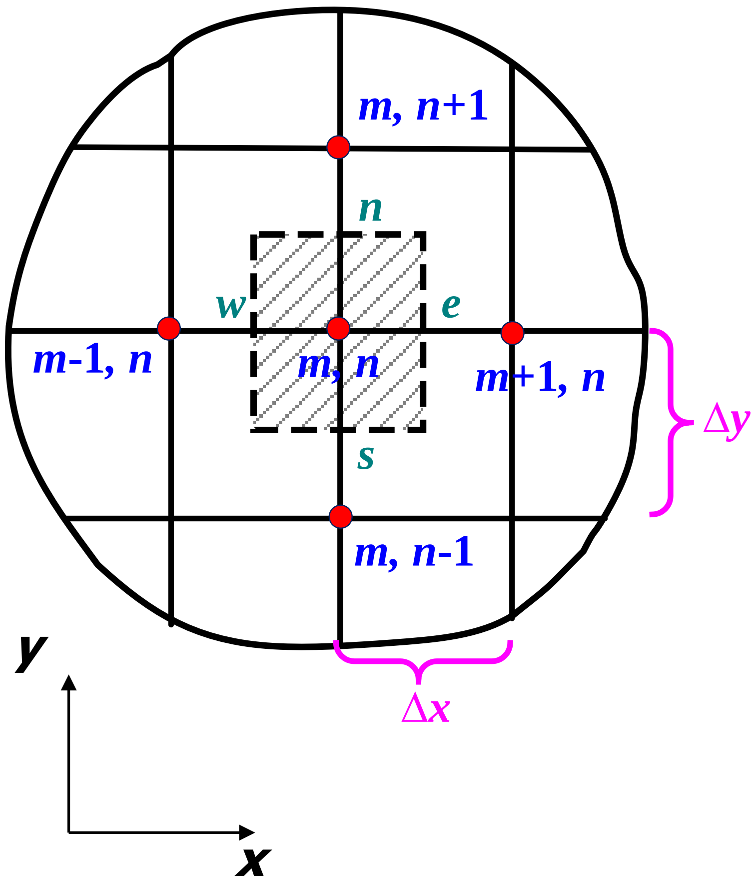
$$f\left(x+dx\right)=f\left(x\right)+\frac{df\left(x\right)}{dx}dx+\frac{1}{2!}\frac{d^2f\left(x\right)}{dx^2}dx^2+\frac{1}{3!}\frac{d^3f\left(x\right)}{dx^3}dx^3+\cdots$$

对相邻节点(m+1,n)及(m-1,n)分别写出温度对节点(m,n) 的泰勒级数展开式



$$f(x+dx)=f(x)+\frac{df(x)}{dx}dx+\frac{1}{2!}\frac{d^2f(x)}{dx^2}dx^2+\frac{1}{3!}\frac{d^3f(x)}{dx^3}dx^3+\cdots$$

（1）对相邻节点写出温度*t* 对内节点(*m*, *n*) 的泰勒级数展开式



➢用 $t_{m,n}$ 来表示 $t_{m+1,n}$

$$t_{m+1,n}=t_{m,n}+\left.\frac{\partial t}{\partial x}\right|_{m,n}\Delta x+\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n}\frac{\Delta x^2}{2!}+\left.\frac{\partial^3 t}{\partial x^3}\right|_{m,n}\frac{\Delta x^3}{3!}+\cdots$$

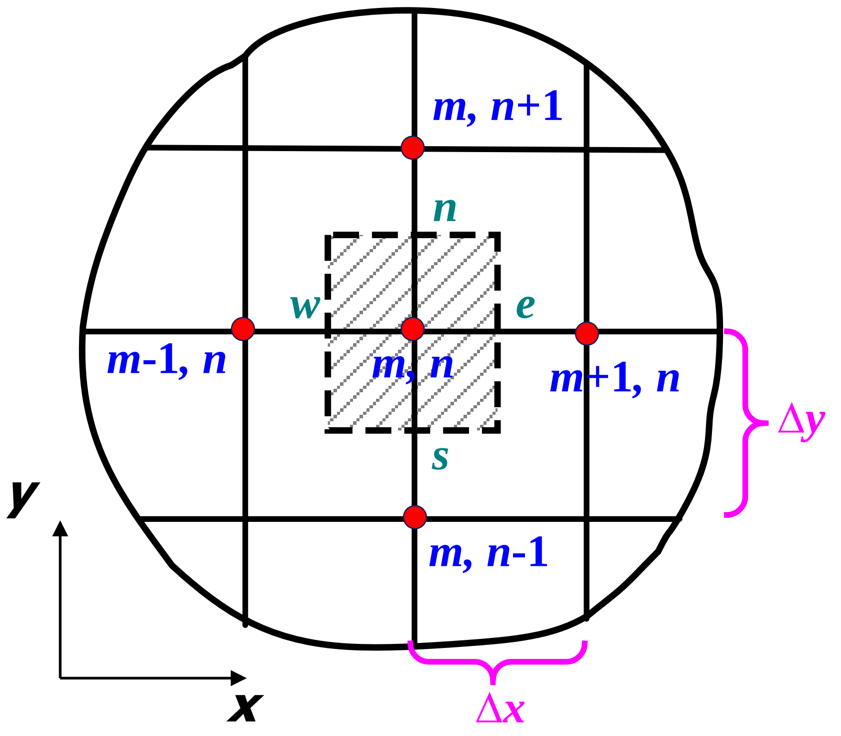
➢ 用 $t_{m,n}$ 来表示 $t_{m-1,n}$

$$t_{m-1,n}=t_{m,n}-\left.\frac{\partial t}{\partial x}\right|_{m,n}\Delta x+\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n}\frac{\Delta x^2}{2!}-\left.\frac{\partial^3 t}{\partial x^3}\right|_{m,n}\frac{\Delta x^3}{3!}+\cdots$$

内节点离散方程

$$t_{m+1,n} = t_{m,n} + \left.\frac{\partial t}{\partial x}\right|_{m,n} \Delta x + \left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n} \frac{\Delta x^2}{2!} + \left.\frac{\partial^3 t}{\partial x^3}\right|_{m,n} \frac{\Delta x^3}{3!} + \cdots$$

$$t_{m-1,n} = t_{m,n} - \left.\frac{\partial t}{\partial x}\right|_{m,n} \Delta x + \left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n} \frac{\Delta x^2}{2!} - \left.\frac{\partial^3 t}{\partial x^3}\right|_{m,n} \frac{\Delta x^3}{3!} + \cdots$$



$$t_{m+1,n} + t_{m-1,n} = 2t_{m,n} + \Delta x^2 \left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n} + \frac{\Delta x^4}{12} \left.\frac{\partial^4 t}{\partial x^4}\right|_{m,n} + \cdots$$

$$\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n} = \frac{t_{m+1,n} - 2t_{m,n} + t_{m-1,n}}{\Delta x^2} + \cancel{O\left(\Delta x^2\right)}$$

（2）由控制方程得到内节点(*m,n*)的离散代数方程

$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}=0$$

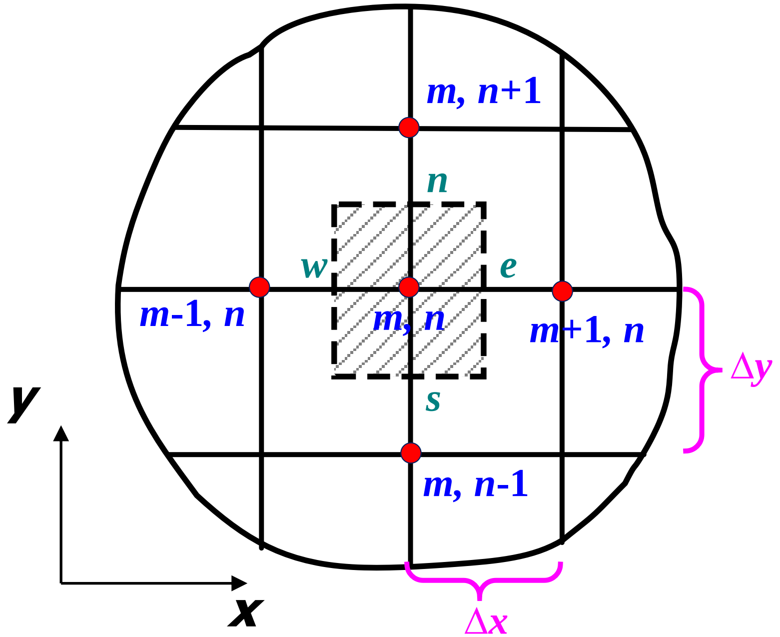
二阶导数的差分表达式

*x*方向

$$\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n}=\frac{t_{m+1,n}-2t_{m,n}+t_{m-1,n}}{\Delta x^2}$$

*y*方向

$$\left.\frac{\partial^2 t}{\partial y^2}\right|_{m,n}=\frac{t_{m,n+1}-2t_{m,n}+t_{m,n-1}}{\Delta y^2}$$



相加

$$\left.\frac{\partial^2 t}{\partial x^2}\right|_{m,n}+\left.\frac{\partial^2 t}{\partial y^2}\right|_{m,n}=\frac{t_{m+1,n}-2t_{m,n}+t_{m-1,n}}{\Delta x^2}+\frac{t_{m,n+1}-2t_{m,n}+t_{m,n-1}}{\Delta y^2}=0$$

$$\Delta x=\Delta y \quad \Longrightarrow \quad t_{m,n}=\frac{1}{4}\left(t_{m+1,n}+t_{m-1,n}+t_{m,n+1}+t_{m,n-1}\right)$$

4.2 离散方程的建立方法

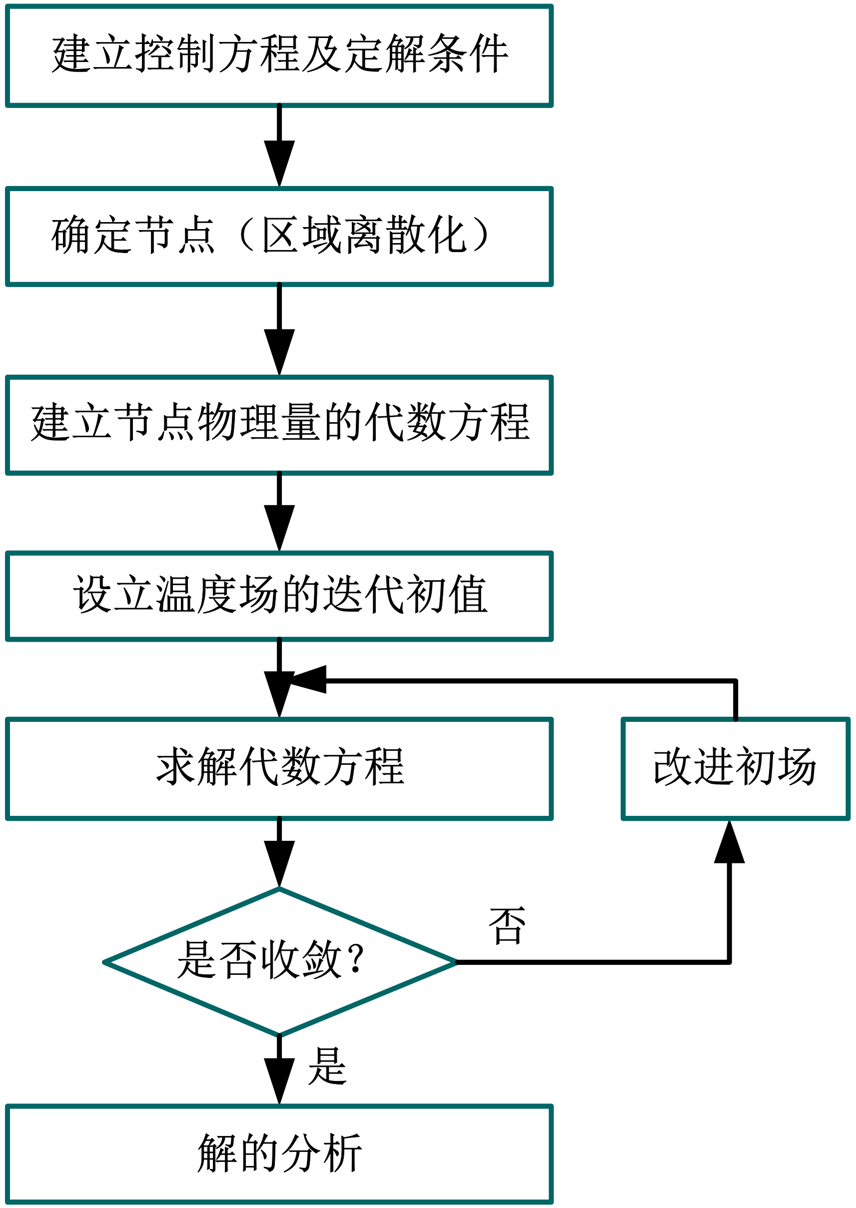
（1）控制容积热平衡法：着重于从物理的观点来分析

（2）Taylor（泰勒）级数展开法：偏重于从数学的角度进行推导

（3）多项式拟合法

（4）控制容积积分法

导热问题数值求解基本步骤



4.1 数值求解的基本思想

4.2 离散方程的建立方法

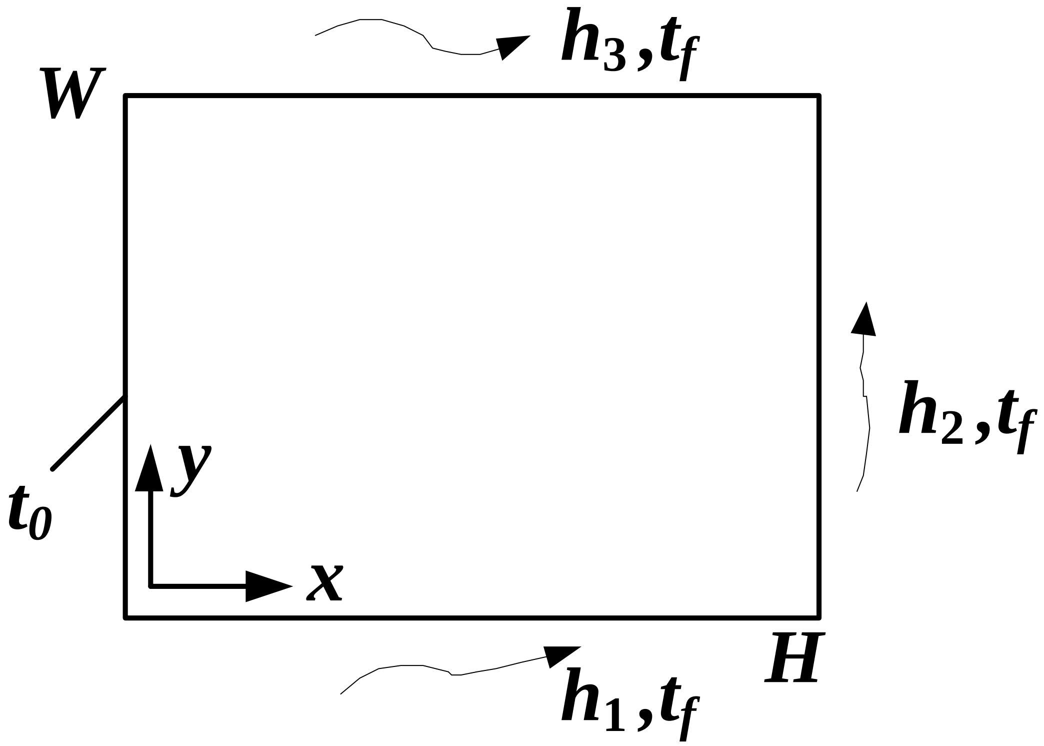
4.3 二维稳态导热问题的计算

4.4 代数方程组的解法

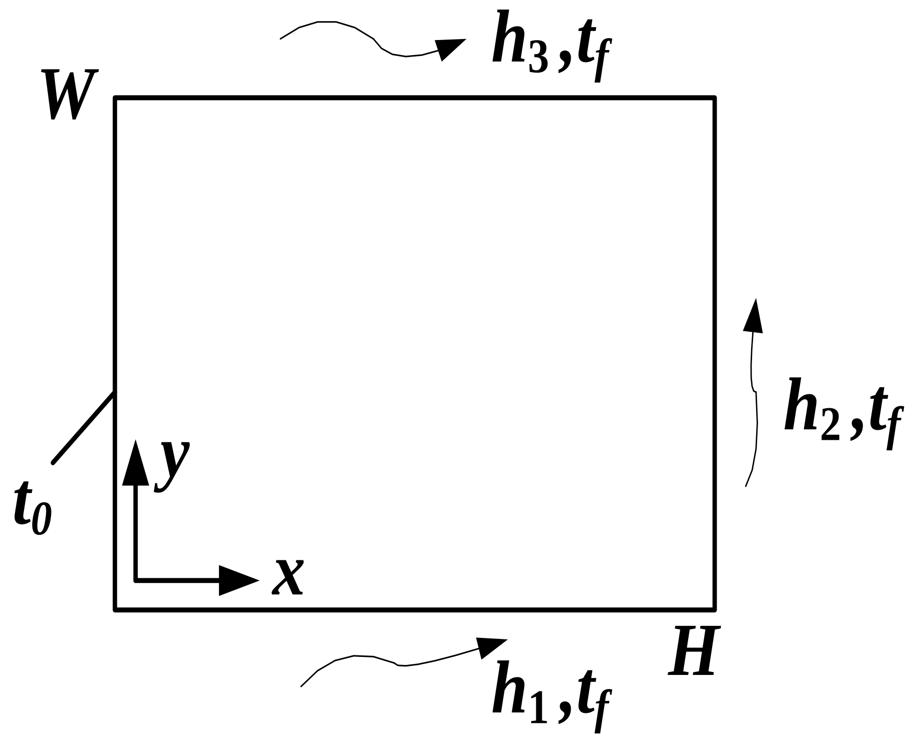
4.5 一维非稳态导热问题的计算

物理问题

二维矩形域内，稳态，无内热源，常物性的导热问题，第一、三类边界条件



（1）建立控制方程及定解条件



导热微分方程

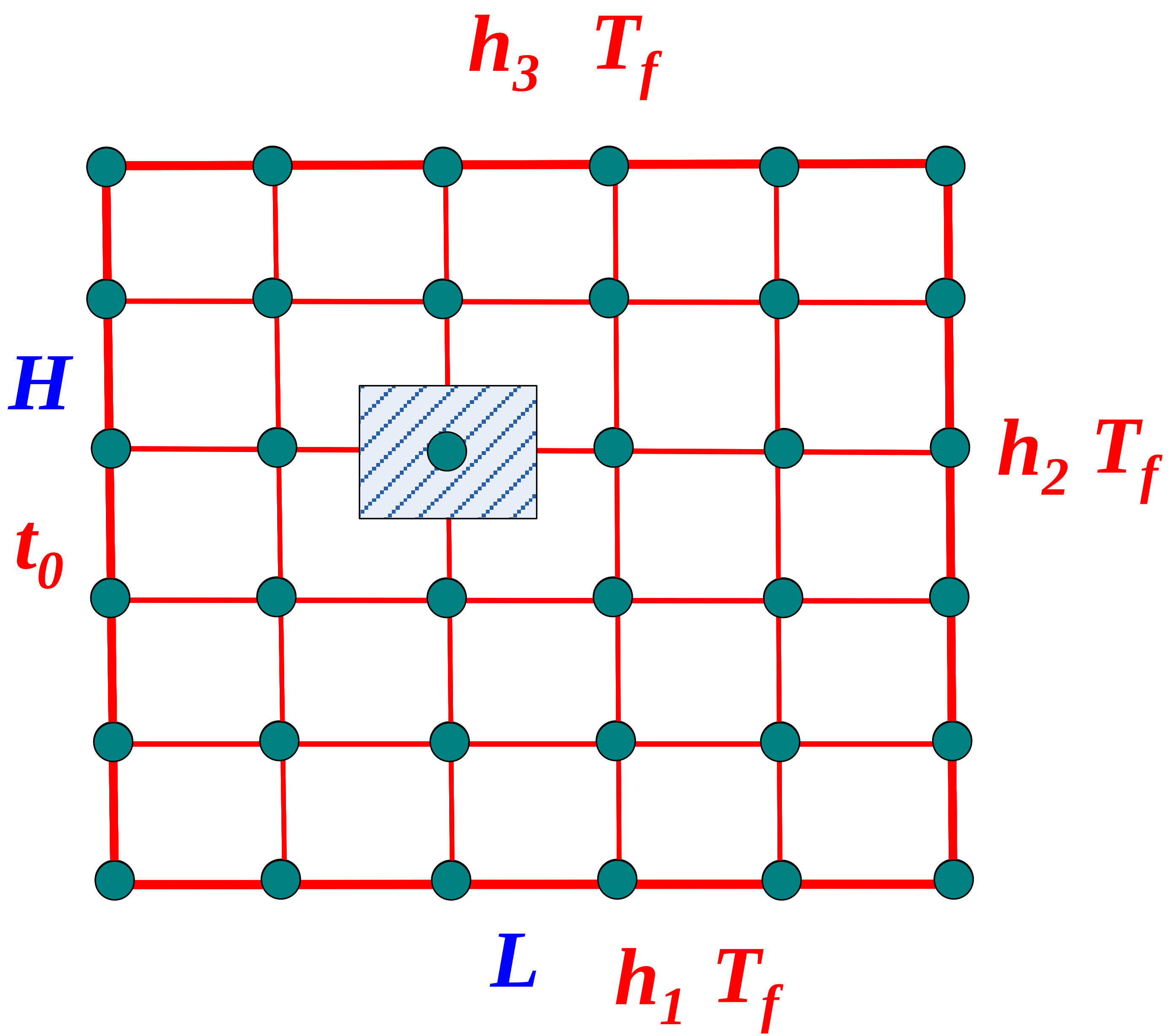
$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}=0$$

边界条件

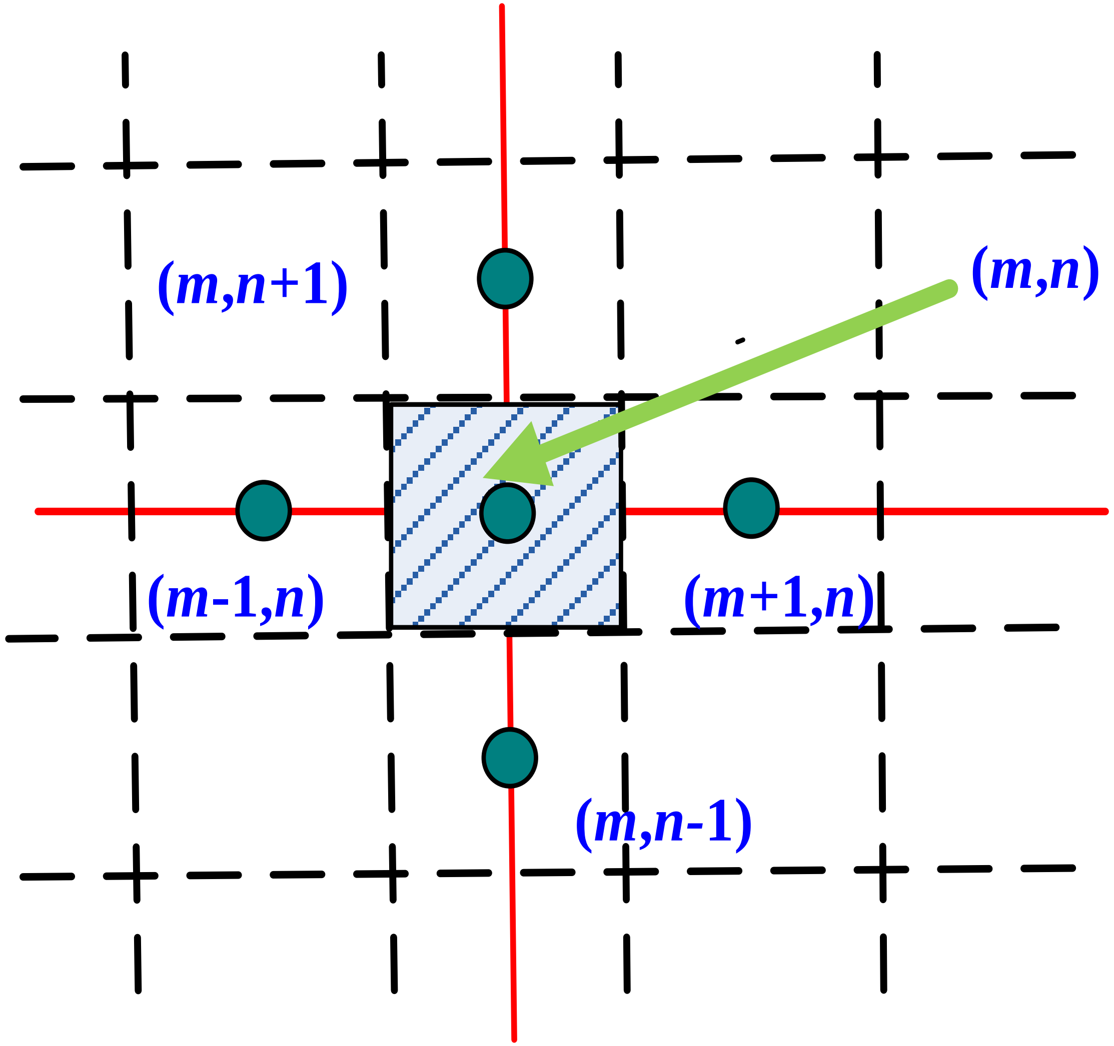
$$x=0,\quad t=t_0 \qquad\qquad x=H,\quad -\lambda\frac{\partial t}{\partial x}=h_2\left(t-t_f\right)$$

$$y=0,\quad -\lambda\frac{\partial t}{\partial y}=h_1\left(t-t_f\right) \qquad y=W,\quad -\lambda\frac{\partial t}{\partial y}=h_3\left(t-t_f\right)$$

（2）区域离散化



(3) 建立节点物理量的代数方程



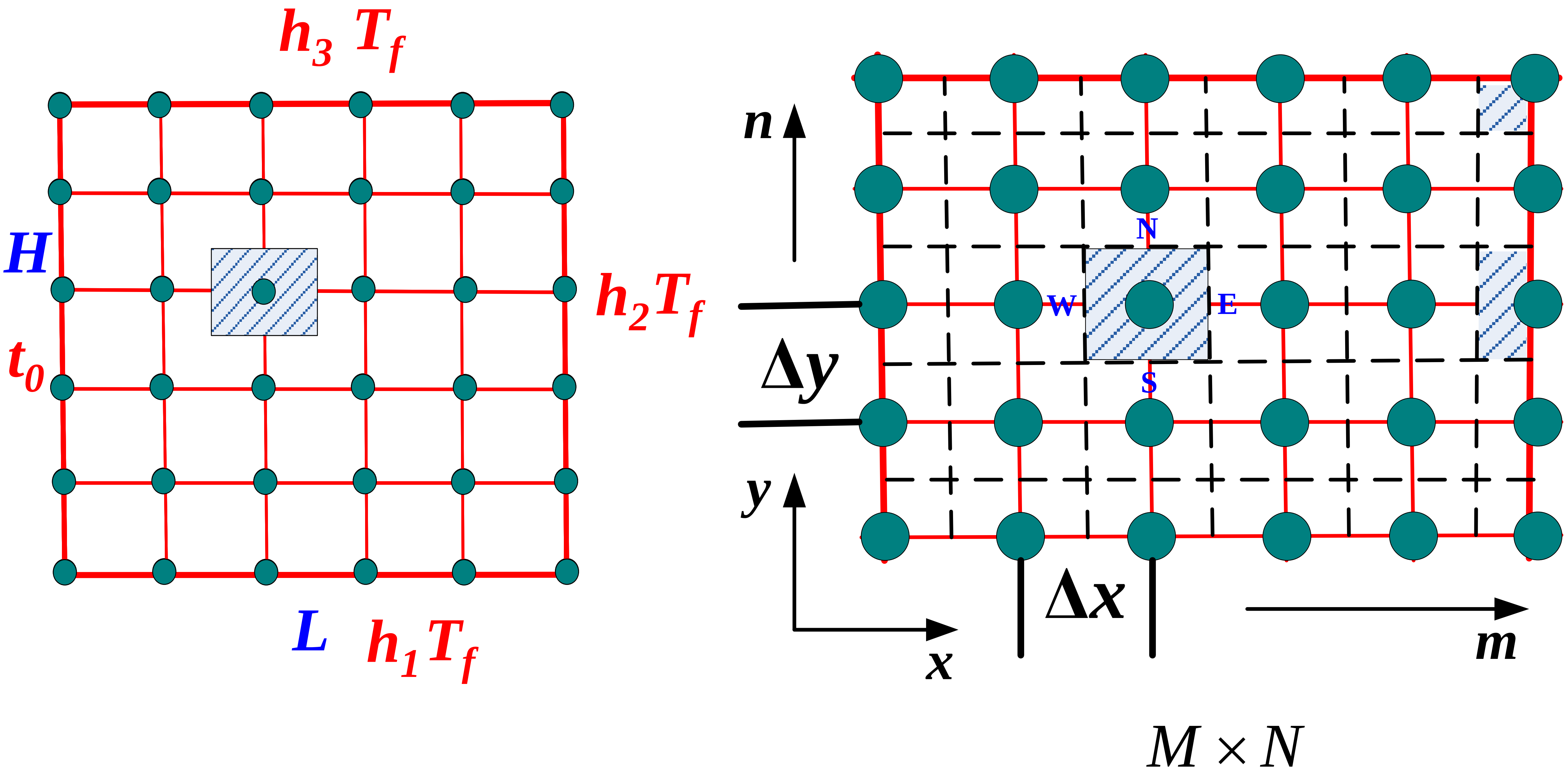
$$\frac{\partial^2 t}{\partial x^2}+\frac{\partial^2 t}{\partial y^2}=0$$

内节点

$$t_{m,n}=\frac{1}{4}\left(t_{m+1,n}+t_{m-1,n}+t_{m,n+1}+t_{m,n-1}\right)$$

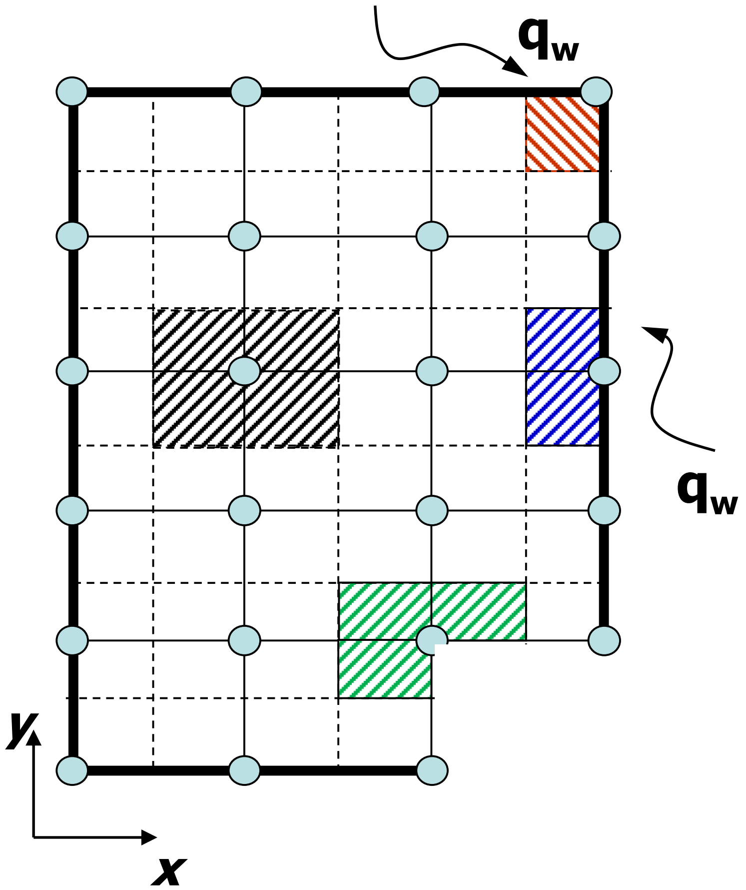
$$m=2,\cdots,M-1$$

$$n=2,\cdots,N-1$$



h3 Tf
H
t0
h2Tf
L
h1Tf
n
N
W
E
S
Δy
y
x
Δx
m
M × N

(3) 建立边界节点物理量的代数方程



- **第一类边界条件**

边界节点温度给定，可将其以数值的形式加入内节点的离散方程中，组成**封闭的代数方程组**，可直接求解，因此处理较简单。

- **第二类边界条件和第三类边界条件**

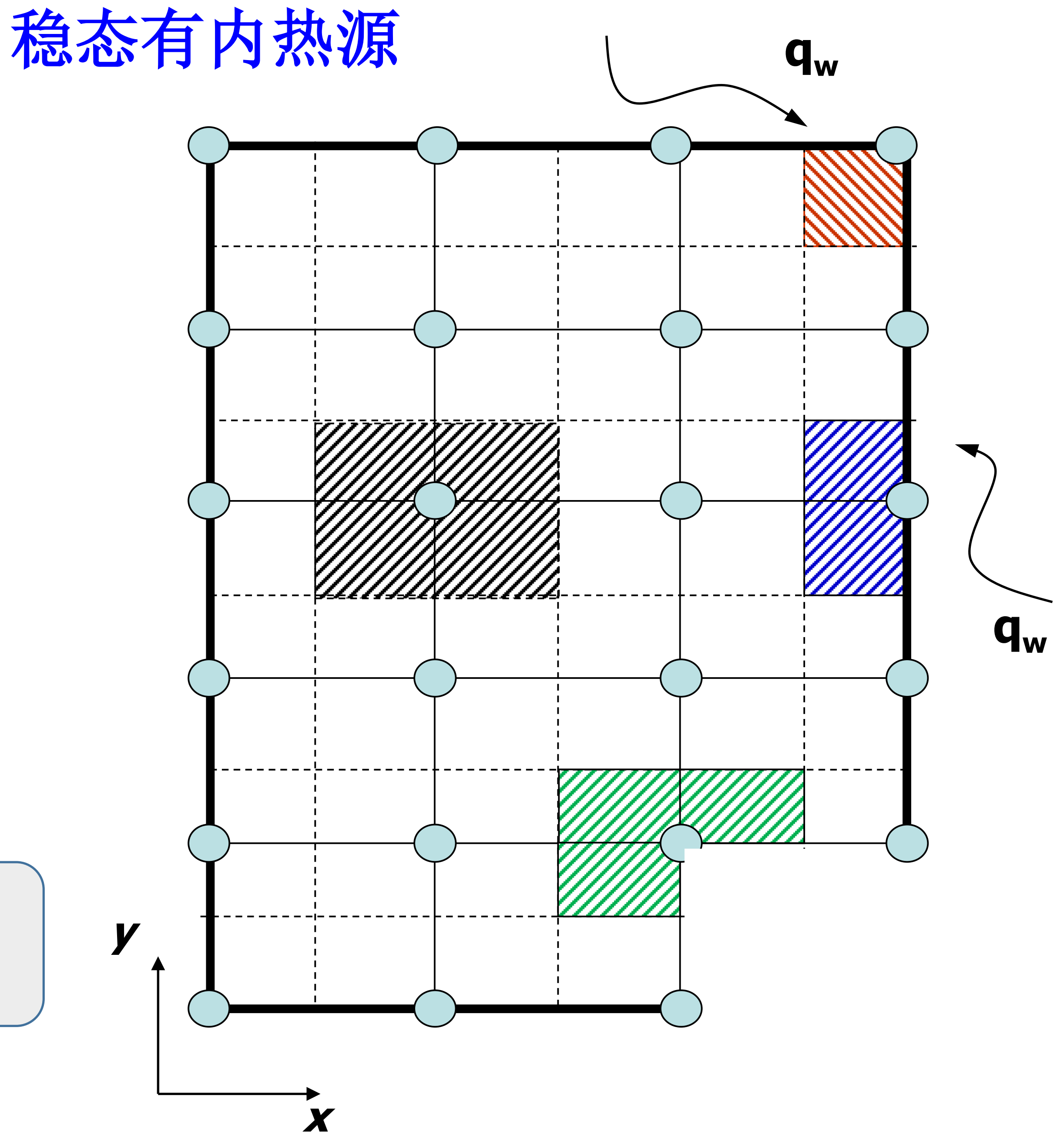
用**热平衡**的方法，建立边界节点的离散方程，边界节点与内节点的离散方程一起组成**封闭的代数方程组**，才能求解。

边界节点的类型：

(1) 平直边界上的节点

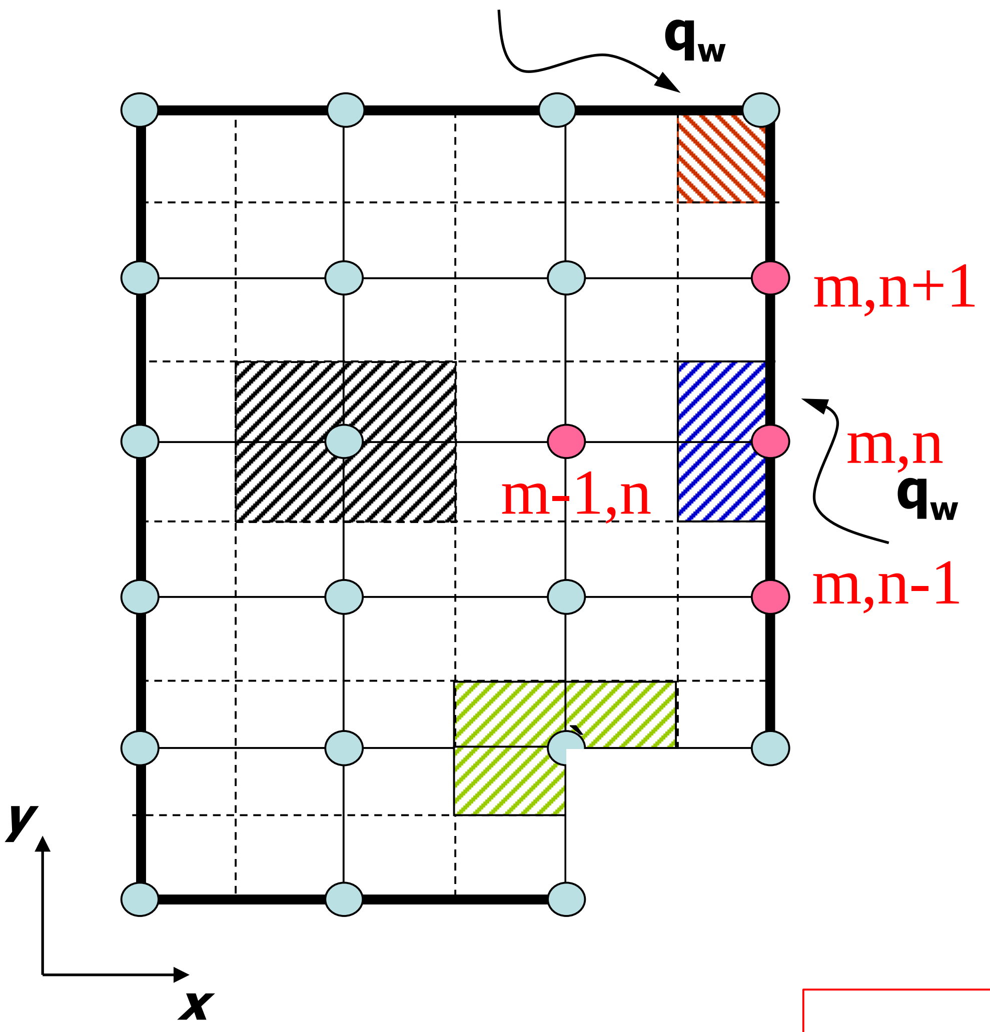
(2) 外部角点

(3) 内部角点



注：用q_w表示边界上的热流密度
用Φ表示内热源强度。

(1) 平直边界上的节点



稳态有内热源

从所有方向流入控制体的总热量
＋ 控制体内热源生成热＝ 0

$$\lambda \Delta y \frac{t_{m-1,n} - t_{m,n}}{\Delta x} + \Delta y q_w$$

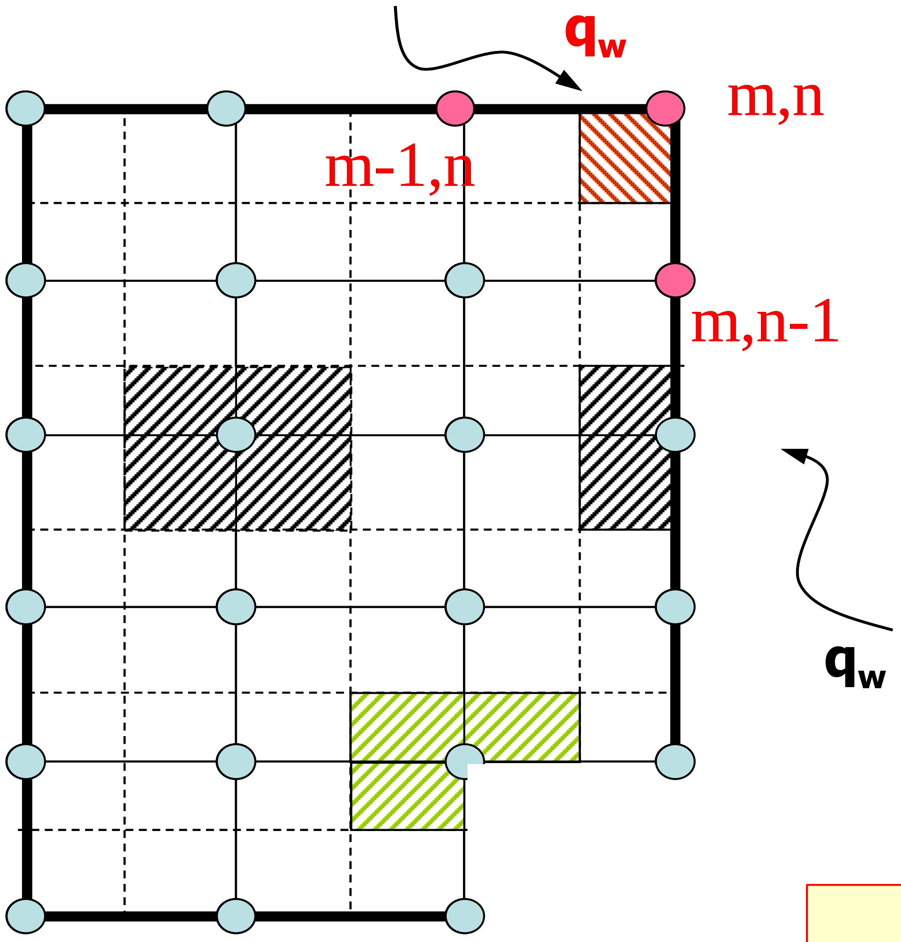
$$+\lambda \frac{\Delta x}{2} \frac{t_{m,n+1} - t_{m,n}}{\Delta y} + \lambda \frac{\Delta x}{2} \frac{t_{m,n-1} - t_{m,n}}{\Delta y}$$

$$+\dot{\Phi}_{m,n} \frac{\Delta x}{2} \Delta y = 0$$

$$\Delta x = \Delta y \quad \Rightarrow$$

$$\boxed{t_{m,n} = \frac{1}{4}\left(2t_{m-1,n} + \frac{2\Delta x}{\lambda} q_w + t_{m,n+1} + t_{m,n-1} + \dot{\Phi}_{m,n} \frac{\Delta x^2}{\lambda}\right)}$$

（2）外部角点



$$\lambda\frac{\Delta y}{2}\frac{t_{m-1,n}-t_{m,n}}{\Delta x}+\frac{\Delta y}{2}q_w+\frac{\Delta x}{2}q_w$$

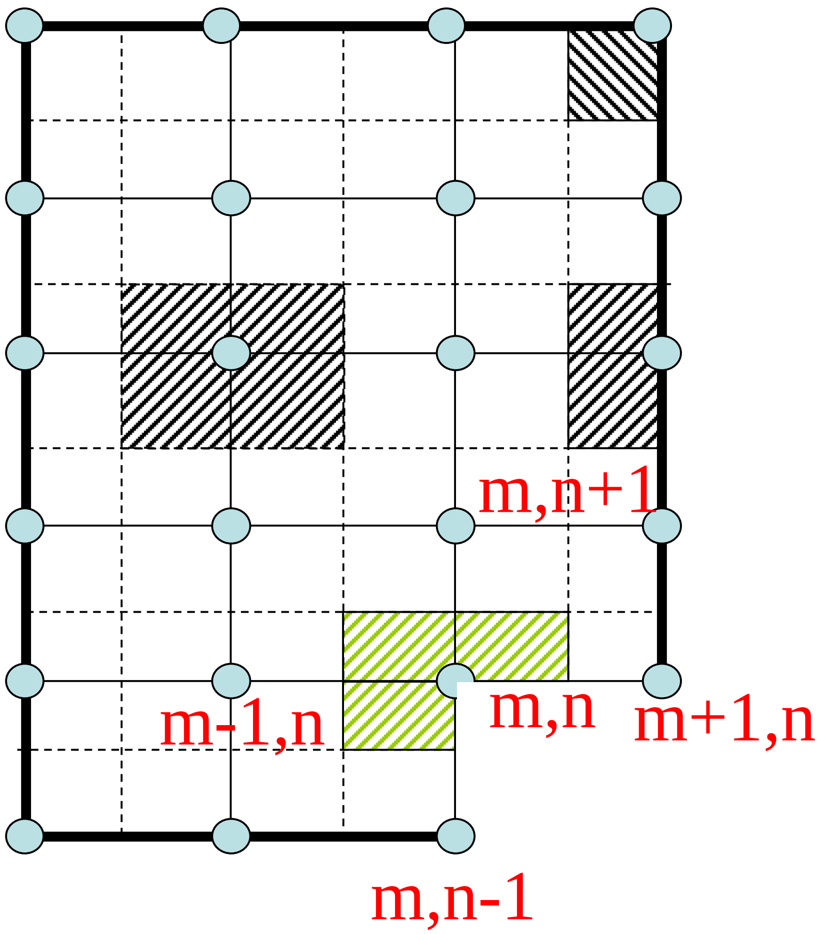
$$+\lambda\frac{\Delta x}{2}\frac{t_{m,n-1}-t_{m,n}}{\Delta y}+\dot{\Phi}_{m,n}\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}=0$$

$$\Delta x=\Delta y\Rightarrow$$

$$\boxed{t_{m,n}=\frac{1}{2}\left(t_{m\text{-}1,n}+t_{m,n\text{-}1}+\frac{2\Delta x}{\lambda}q_w+\dot{\Phi}_{m,n}\frac{\Delta x^2}{2\lambda}\right)}$$

稳态有内热源

3）内部角点



$$\lambda\Delta y\frac{t_{m-1,n}-t_{m,n}}{\Delta x}+\left(\lambda\frac{\Delta y}{2}\frac{t_{m+1,n}-t_{m,n}}{\Delta x}+\frac{\Delta y}{2}q_w\right)$$

$$+\lambda\Delta x\frac{t_{m,n+1}-t_{m,n}}{\Delta y}+\left(\lambda\frac{\Delta x}{2}\frac{t_{m,n-1}-t_{m,n}}{\Delta y}+\frac{\Delta x}{2}q_w\right)$$

$$+\dot{\Phi}_{m,n}\frac{3\Delta x\Delta y}{4}=0$$

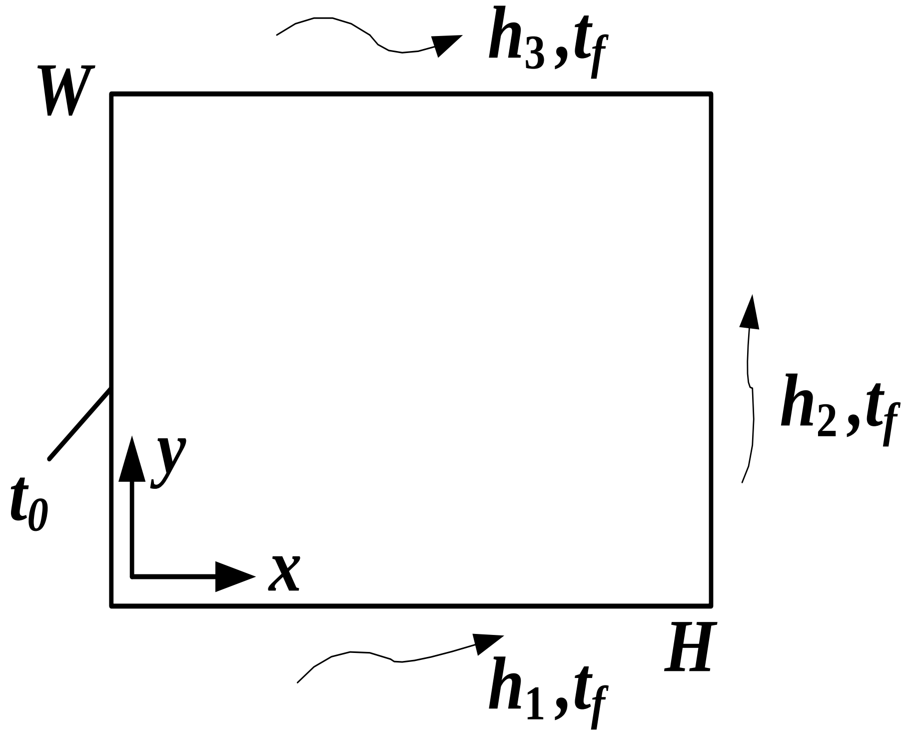
$$\Delta x=\Delta y\quad\Rightarrow$$

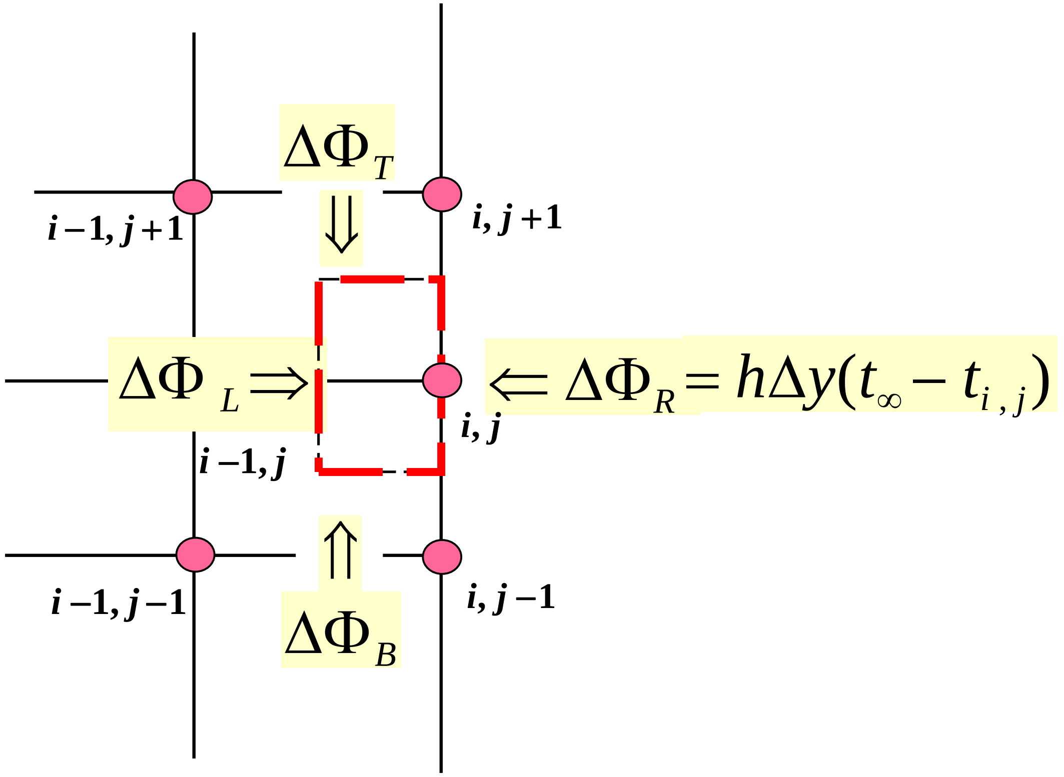
$$t_{m,n}=\frac{1}{6}(2t_{m-1,n}+2t_{m,n+1}+t_{m,n-1}+t_{m+1,n}+\frac{3\Delta x^2}{2\lambda}\dot{\Phi}_{m,n}+\frac{2\Delta x}{\lambda}q_w)$$

稳态有内热源

稳态无内热源条件

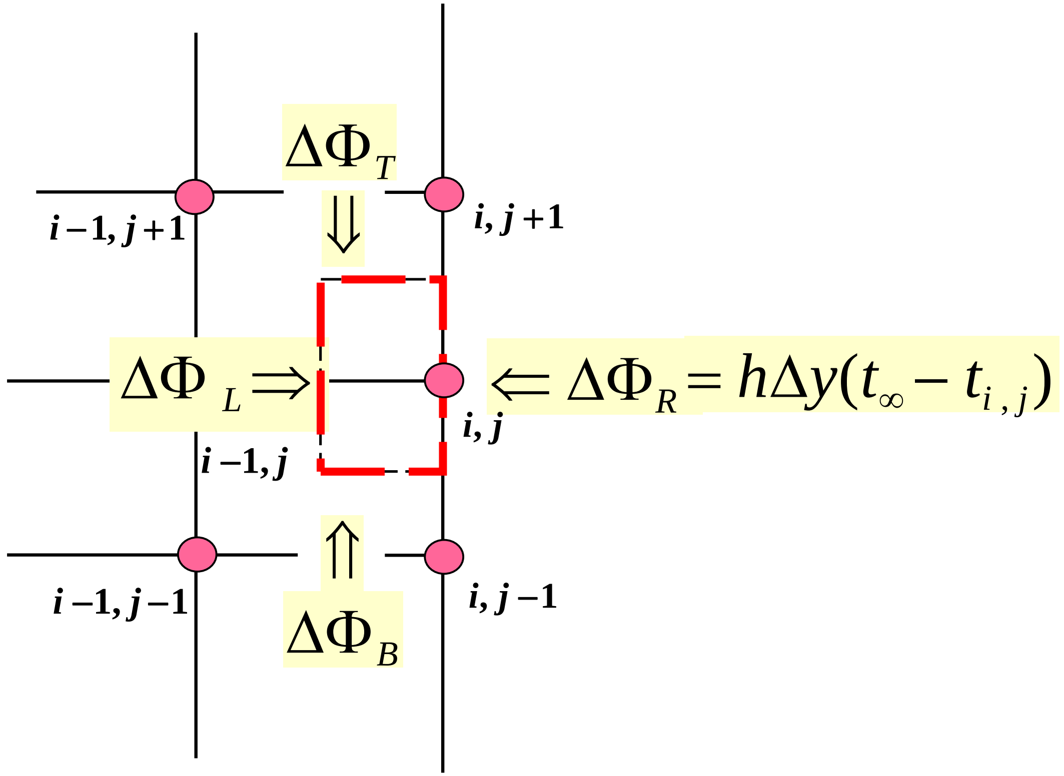
$$\Delta\Phi_L + \Delta\Phi_R + \Delta\Phi_B + \Delta\Phi_T = 0$$





假设$\Delta x = \Delta y$

试计算（i,j）点温度$t_{i,j}$



$$\begin{cases} \Delta\Phi_L = -\lambda\Delta y\left(\dfrac{t_{i,j}-t_{i-1,j}}{\Delta x}\right), \\ \Delta\Phi_R = -h\Delta y\left(t_{i,j}-t_\infty\right) \\ \Delta\Phi_B = -\lambda\dfrac{\Delta x}{2}\left(\dfrac{t_{i,j}-t_{i,j-1}}{\Delta y}\right), \\ \Delta\Phi_T = \lambda\dfrac{\Delta x}{2}\left(\dfrac{t_{i,j+1}-t_{i,j}}{\Delta y}\right) \end{cases}$$

$$-\lambda\Delta y\left(\frac{t_{i,j}-t_{i-1,j}}{\Delta x}\right)+h\Delta y\left(t_\infty-t_{i,j}\right)-\lambda\frac{\Delta x}{2}\left(\frac{t_{i,j}-t_{i,j-1}}{\Delta y}\right)+\lambda\frac{\Delta x}{2}\left(\frac{t_{i,j+1}-t_{i,j}}{\Delta y}\right)=0$$

若 $\Delta x=\Delta y$

$$\boxed{-\left(4+\frac{2h\Delta x}{\lambda}\right)t_{i,j}+2t_{i-1,j}+t_{i,j-1}+t_{i,j+1}+\frac{2h\Delta x}{\lambda}t_\infty=0}$$