Black-Litterman Framework to Test the Effectiveness of Technical Indicators in Portfolio Optimization

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17 May 2019

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Abstract

This project explored the Black-Litterman framework to construct a portfolio of stocks listed in the S&P 100 index, and tracked the performance of the eficicient portfolio against the S&P 100 index. Thirteen technical indicators (viz. RSI, Bollinger Bands, MACD etc.) were used to incorporate the investor's personal views into the Bayesian framework. Performance of an equally-weighted linking matrix was compared against the performance of a distributed linking matrix in the training period of January 2009 to December 2013, wherein the constructed portfolios minimized Expected Shortfall under the said probabilistic model. Excluding transaction costs, the resulting optimized portfolio outperformed the benchmark during the testing period of January 2014 to December 2018 on both nominal and risk-adjusted basis as measured by overall performance and Sharpe ratio.

1 Introduction

1.1 Problem Statement

The Markowitz portfolio optimization model is an ubiquitous approach for portfolio management; however, various drawbacks exist in the original methodology. This stems from using only historical data that may not accurately capture how certain assets will perform in the future. For this reason an alternative approach was sought to improve portfolio optimization.

The Black-Litterman framework for constructing the expected return vector of the assets within a portfolio offers a potential rectification. This framework allows the portfolio manager the ability to incorporate his or her views on the returns of the assets into the computation of returns. This Bayesian approach yields the potential to predict the expected return vector more accurately than using solely historical data.

It was thought that the views on these assets could stem from the use of technical indicators that offer additional information as to when an investor should buy or sell an asset. This concept gave rise to the work presented.

1.2 Literature Review

Since the 1990's the Black-Litterman framework developed by Fischer Black and Robert Litterman has been used as an alternative to the typical Markowitz portfolio optimization model. This framework is used to incorporate the views of the portfolio manager to generate a more accurate estimate of the return vector [13]. State Street Corporation attempted to implement a portfolio of global assets using the Black-Litterman framework in 2008; though the framework appeared to increase in-sample performance, the complexity of model also increased due to the need for numerous parameters to accurately create views objectively [13]. Other researchers within the field of portfolio management have modeled efficient frontiers and optimal portfolios using economic indicators and technical indicators on overall indices [1]. These experiments showed the potential of reducing the number of parameters used to construct the manager's views. Similar research done by Chitturi incorporated machine learning and statistical modeling to predict an asset's future returns using factor models, time series modeling, and other techniques on economic data and technical indicators[7].

2 Data Description

For the project, stocks listed in SP100 index were used to construct the portfolio. Specifically 69 stocks were chosen from the SP100 index for which data was available from January 2000 to December 2018. Some of the stocks used were IBM, Verizon (VZ), Boeing Air (BA), Cisco Systems (CSCO), General Motors (GM), etc. Daily Open, High, Low, and Close prices were gathered as some of these data would be used to compute the technical indicators. Further details pertaining to technical indicators are mentioned in Section 3.

Meanwhile, for the measure of risk free rate, US 10 year treasury yields were used. Data were gathered for the same dates as mentioned above. For the construction of the portfolio, monthly returns were computed for the stock prices and for the risk free rate. Since the treasury rates are annualized, this was converted to a monthly measure.

Asset prices, risk free returns, and the associated technical indicators data from January 2000 to December 2008 were used to perform regression analysis. The results of these regression models were used to predict the expected excess returns (trader's view) for first training period of January 2009. For February 2009, data from January 2000 to January 2009 was used to give a measure of expected excess return. Similarly, the analysis was performed until December 2013. These training period expected returns, alongside the uncertainty of views, covariance matrix, and the views linkage matrix were fed into a Black-Litterman framework to yield expected excess returns for each asset and other optimal measures which are tested in the period of January 2014 to December 2018. Details of these measures and the detailed description of the Black Litterman framework are mentioned in Section 4.

3 Trading Strategy

The following 13 technical indicators were used to gather trading views for each stock

1. Accumulation Distribution Index (ADI) is a volume-based indicator which measures the cumulative flow of money into and out of a security. First, a multiplier is calculated based on the relationship of the close to the high-low range. Second, the Money Flow Multiplier is multiplied by the period's volume to come up with a Money Flow Volume. A running total of the Money Flow Volume forms the Accumulation Distribution Line. This indicator is used to affirm a security's underlying trend or anticipate reversals when the indicator diverges from the security price. The specifics regarding the calculation of this index is mentioned in the link in the bibliography [14]. The multiplier ranges from +1 to -1. A high positive multiplier combined with high volume shows strong buying pressure that pushes the indicator higher. Conversely, a low negative number combined with high volume reflects strong selling pressure that pushes the indicator lower.

2. Average Directional Index (ADX), the Plus Directional Indicator (+DI) and Minus Directional Indicator (-DI) are derived from smoothed averages of these differences and measure trend direction over time. The Average Directional Index (ADX) is in turn derived from the smoothed averages of the difference between +DI and -DI; it measures the strength of the trend (regardless of direction) over time. Using these three indicators together, traders can determine both the direction and strength of the trend. Details regarding the construction of this index is mentioned in the linking in the bibliography [9] . A strong trend is present when ADX is above 25 and no trend is present when ADX is below 20. There appears to be a gray zone between 20 and 25. A buy signal occurs when +DI crosses above -DI. A sell signal triggers when -DI crosses above +DI.

3. The Aroon indicators measure the number of periods since price recorded an x-day high or low. Aroon-Up is based on price highs, while Aroon-Down is based on price lows. The Aroon indicators are shown in percentage terms and fluctuate between 0 and 100. View on a particular stock is bullish Aroon-Up is above 50 and Aroon-Down is below 50. This indicates a greater propensity for new x-day highs than lows. The converse is true for a downtrend. The view on a stock is bearish when Aroon-Up is below 50 and Aroon-Down is above 50. The calculation of the Aroon indicator is mentioned in the link in the bibliography[8].

4. Bollinger Bands are volatility bands placed above and below a moving average. Volatility is based on the standard deviation, which changes as volatility increases and decreases. The bands automatically widen when volatility increases and contract when volatility decreases. Prices are relatively high when above the upper band and relatively low when below the lower band. This can be used as a signal for a stock to be undervalued or overvalued. The details pertaining to the calculation of bollinger bands is mentioned in the link in the bibliography[15].

5. Commodity Channel Index (CCI) measures the current price level relative to an average price level over a given period of time. CCI is relatively high when prices are far above their average, but is relatively low when prices are far below their average. Using this interpretation, CCI can be used to identify overbought and oversold levels of a particular stock. Index value above +100 reflects strong price action signalling the start of an uptrend, meanwhile Plunges below -100 reflect weak price action that can signal the start of a downtrend. Details regarding the calculation of this index is mentioned in the link in the bibliography [16].

6. Ease of movement (EMV) is used to relate an asset's price change to its volume, thereby assessing the strength of a trend. High positive values indicate the price is increasing on low volume, strong negative values indicate the price is dropping on low volume. The moving average of the indicator can be added to act as a trigger line, which is similar to other indicators like the MACD. Details regarding the construction of the index is mentioned in the link in the bibliography [10].

7. The Force Index (FI) is used to illustrate how strong the actual buying or selling pressure is. High positive values mean there is a strong rising trend, and low values signify a strong downward trend. The strength of the force is determined by a larger price change or by a larger volume. Further details regarding the index is mentioned in the link in the bibliography [11].

8. Moving Average Convergence Divergence (MACD) is a trend-following momentum indicator that shows the relationship between two moving averages of a security's price. Traders may buy the security when

the MACD crosses above its signal line and sell - or short - the security when the MACD crosses below the signal line. Moving Average Convergence Divergence (MACD) indicators can be interpreted in several ways, but the more common methods are crossovers, divergences, and rapid rises/falls. Further details regarding MACD is mentioned in the link in the bibliography [4].

9. The Money Flow Index (MFI) is used for measuring buying and selling pressure. The MFI's calculation generates a value that is then plotted as a line that moves within a range of 0-100, making it an oscillator. When the MFI rises, this indicates an increase in buying pressure. When it falls, this indicates an increase in selling pressure. Further details regarding the indicator is mentioned in the link in the bibliography [3].

10. The On Balance Volume indicator (OBV) is used to measure buying and selling pressure. On days where prices trend up, that day's volume is added to the cumulative OBV total. If prices trend down, then that day's volume is subtracted from the OBV total. When volume on up days outpaces volume on down days, then OBV rises. When volume on down days outpaces volume on up days, then OBV falls. This suggests that when OBV is up, buying pressure is up and when OBV is down, then selling pressure is up. Further details pertaining to OBV index is mentioned in the link in the bibliography [12].

11. The Relative Strength Index (RSI) is a momentum based oscillator which is used to measure the speed (velocity) as well as the change (magnitude) of directional price movements. It has a reading from 0 to 100. For values of 70 or above, it indicates that a security is becoming overbought or overvalued and may be primed for a trend reversal or corrective pullback in price. An RSI reading of 30 or below indicates an oversold or undervalued condition. Further details pertaining to RSI index is mentioned in the link in the bibliography [5].

12. The True Strength Index (TSI) is a momentum oscillator that shows both trend direction and overbought/oversold conditions. It ranges between limits of -100 and +100 and has a base value of 0. Momentum is positive when the oscillator is positive thereby suggesting a bullish trend and a bearish trend when the indicator is below zero. However, most values generally fall between +25 and 25. Further details pertaining to RSI index is mentioned in the link in the bibliography [6].

13. The Ultimate Oscillator is a momentum oscillator designed to capture momentum across three different time-frames. It is a range-bound indicator with a value that fluctuates between 0 and 100. The Ultimate Oscillator rises when Buying Pressure is strong and falls when Buying Pressure is weak. Levels below 30 are deemed to be oversold, and levels above 70 are deemed to be overbought. Further details pertaining to Ultimate Oscillator is mentioned in the link in the bibliography[17].

4 Model Development and Assumptions

4.1 Black-Litterman Formula

The Black-Litterman framework can be described by the formula presented below:

$$\mathbf{E}[r_{assets} - r_{free}] = [(\tau S)^{-1} + P^T \Omega^{-1} P] \cdot [(\tau S)^{-1} \Pi + P^T \Omega^{-1} Q],$$

where:

- $\mathbf{E}[\cdot]$ is the expectation/expected value function generating estimated excess returns of the assets
- r_{asset} is a vector of the monthly return of the assets
- r_{free} is a constant vector of the monthly risk free return
- τ is a scalar quantity to adjust the sample covariance and uncertainty matrices
- S is the sample covariance matrix of historical returns

- P is the linking matrix that identifies which assets are considered by the portfolio manager's views
- Ω is the uncertainty matrix for the portfolio manager's views
- Π is the implied equilibrium excess return vector of the assets
- Q is the vector containing the portfolio manager's views on the excess returns

For the value of τ, Black and Litterman recommended a value of .025 while other academics and analysts suggest using a value of 1 for convenience. For the purposes of this project, the Black-Litterman estimates obtained were evaluated for various values of τ ranging from .05 to 1 by increments of .05. The value of τ that generated the highest returns for the portfolio in the training period was selected as the optimal τ and used for the test period.

The sample covariance matrix was computed using historical monthly returns with a weighted rate of decay on returns. The rate of decay used was .005. The covariance matrix was constructed using the monthly returns from January 2000 to December 2008 in order to estimate the sample covariance matrix for the first trading period of January 2009 to February 2009. As the trading strategy progressed through time, the covariance matrix was updated with the newly acquired monthly return data. The dimensions of this matrix were 69x69.

The linking matrix was constructed where the rows corresponded to the views based on a given indicator while the columns corresponded to a specific asset. The linking matrix had to satisfy the following properties:

1. The assets for which one anticipates a long position will receive a positive value
2. The assets for which one anticipates a short position will receive a negative value
3. The assets for which one has no view receive a value of 0
4. The sum of the assets with positive views (long) in a row must be equal to 1
5. The sum of the assets with negative views (short) in a row must be equal to -1
6. The sum of the elements across a row must be 0
7. The matrix must be a full rank matrix

Given these properties, two methods were used in construction. The first method gave equal weight to all the assets that had long views and equal weight to all the assets with short views. An example linking matrix is shown in the below table. This construction indicated that the portfolio of all the assets with long views

Indicator	**Asset 1**	**Asset 2**	**Asset 3**	**Asset 4**	**Asset 5**
ADX	0.5	0.5	0	-0.5	-0.5
Aroon	0.333333	0.333333	0.333333	0	-1
RSI	-1	0	0.333333	0.333333	0.333333
MFI	0.5	0.5	-1	0	0
TSI	0	-1	0	.5	.5

Table 1: Equally-Weighted Linking Matrix Example

would outperform a portfolio of all the assets with short views. Rather than simply use equally weighted views as was done above, a second linking matrix was created assigning more weight to assets with stronger indicator values relative to the other assets within the set of long or short views. An example matrix is exhibited below.

This indicated that the ADX indicator signaled a buy for both Asset 1 and Asset 2 and a sell for Asset 4 and Asset 5; however, the indicator itself showed the buy for Asset 1 was stronger than that of Asset 2 while the sell for Asset 4 and Asset 5 were the same. Often certain indicators established the same set of views resulting in the linking matrix being rank deficient. This failed to satisfy the seventh property of the linking matrix. The issue was amended by deleting the duplicated views from the linking matrix. Because of this,

Indicator	Asset 1	Asset 2	Asset 3	Asset 4	Asset 5
ADX	0.7	0.3	0	-0.5	-0.5
Aroon	0.6	0.3	0.1	0	-1
RSI	-1	0	0.4	0.4	0.2
MFI	0.5	0.5	-1	0	0
TSI	0	-0.7	-0.3	1	0

Table 2: Distributed-Weight Linking Matrix Example

the dimensions of the linking matrix ranged between 5x69 to 10x69. It appeared the Commodity Channel Index, Moving Average Convergence Divergence, and Accumulation Distribution Index generated the same set of views as other indicators. The linking matrix was constructed for each day when the portfolio would be rebalanced: every twenty days starting from January 2009 to December 2018. The training period was used as a means of determining whether the equal-weight linking matrix produced higher returns than the distributed-weight linking matrix.

The uncertainy matrix Ω was used to measure the level of uncertainty in the views on expected excess returns. Black and Litterman suggested a formulation for Ω as the following:

$$\Omega = \tau PSP^T$$

This formula was used for the computation of Ω in the analysis.

The implied equilibrium excess return vector was computed using the historical monthly returns of the index (S&P 100) and the historical risk free monthly return and the β of each potential asset in the portfolio. The implied excess return vector was constructed using the monthly returns from January 2000 to December 2008 in order to estimate the vector for the first trading period of January 2009 to February 2009. As the trading strategy progressed through time, the vector was updated with the newly acquired monthly return data on both the index and risk free return as well as an updated β. Implied equilibrium excess returns was calculated by the following formula:

$$\Pi_i = \beta_i(r_{market} - r_{free})$$

where

- Π_i is the implied excess monthly return for the ith asset
- β_i is the beta for the ith asset
- $r_{market} - r_{free}$ is the excess monthly return of the market over the risk free rate (monthly equity risk premium).

The Q vector was constructed using linear regression where the indicator value was the independent value and the excess return over the next month was the dependent variable. The formula below shows the set-up for the regression.

$$returns_{i,t+1} = \beta_{i,t} \cdot Indicator_{i,t} + \alpha_{i,t}$$

where

- $returns_{i,t+1}$ is the next monthly return for the ith asset
- $\beta_{i,t}$ is the slope for the regression of the ith asset at the current time
- $Indicator_{i,t}$ is the value of the indicator for the ith asset at the current time
- $\alpha_{i,t}$ is the intercept for the regression of the ith asset at the current time

The predicted returns for all the assets based on the relevant indicators – those associated to the full rank linking matrix– were temporarily stored in a matrix. To obtain the entries of Q, the dot product between the mth row of the linking matrix and the and the mth row of the predicted returns matrix was taken.

$$Q_m = P_m \cdot predicted\_returns_m$$

This generated the expected excess return for the portfolio of long and shorts with given weights based on the given indicator. As time progressed, the additional values for indicators and monthly returns were used to recompute the slope and intercept of the i^{th} asset to more accurately predict the monthly returns.

Once all these elements were calculated, the estimated excess returns were derived from the Black-Litterman formula and used in optimization.

4.2 Optimization Methods

The following optimization technique was employed to determine the performance of the portfolio using Black-Litterman estimates. It was used to rebalance the portfolio every 20 days (approximately every month). The method is presented below.

$$\begin{aligned}
&\underset{x_0, x_1, \ldots, x_m}{\text{maximize}} \quad x_0 \cdot \mu_0 + \sum_{i=1}^{m} x_i \mu_i \\
&\text{subject to} \\
&\qquad x_0 + \sum_{i=1}^{m} x_i = 1 \\
&\qquad x_0 \geq 0 \\
&\qquad x_0 \leq 1 \\
&\qquad |x_i| \leq .1, \quad i = 1, \ldots, m \\
&\qquad l + \frac{1}{1+\alpha} \sum_{\omega \in P(\omega)} max(-R(\omega)^T x - l, 0) \leq \beta
\end{aligned}$$

where:

- m was the number of assets; in this case m equaled 69
- x_0 was the weight associated with the monthly risk free rate.
- x_i were the weights associated with each asset
- μ_0 was the expected monthly risk-free return
- μ_i was the estimated excess return of the i^{th} asset according to the Black-Litterman framework
- l was the variable loss level
- α was the desired confidence level
- loss(x,ω) was the convex loss function defined as $-R(\omega)^T x$ which signifies the downside risk
- P(ω) was the probability associated with each return
- β was the allowable risk measured in units of wealth

The following assumptions and conditions were used to compute the optimal level of weights for each asset in the portfolio:

1. The optimizer used Expected Shortfall as the risk measure. The maximum allowable loss for the portfolio (β) was set as 0.1 (i.e. the portfolio is allowed to experience a 10% loss on an average.)
2. User defined confidence level to compute losses (α) was set at 95%
3. The portfolio was not allowed to take unlimited leverage by borrowing on the risk-free asset.
4. The portfolio was allowed to take both long and short positions in the risky assets
5. The portfolio was not allowed to put more than the total amount of wealth into savings
6. The portfolio was not allowed to allocate more than 10% weight to any long or short position in order to limit exposure to a single asset

5 Model Testing and Results

5.1 Training Period

The optimization model as discussed in the prior section was implemented on training data for both expected returns matrix generated using equal and distributed weights on views, with a range of τ values. The portfolio was rebalanced every 20 trading days, and the performance was tracked over the training period. For the returns matrix generated using the distributed view, the following portfolio performance was observed over the training period that involved rebalancing the portfolio 62 times at intervals of 20 trading days. The training period ranged from January 2009 to December 2013.

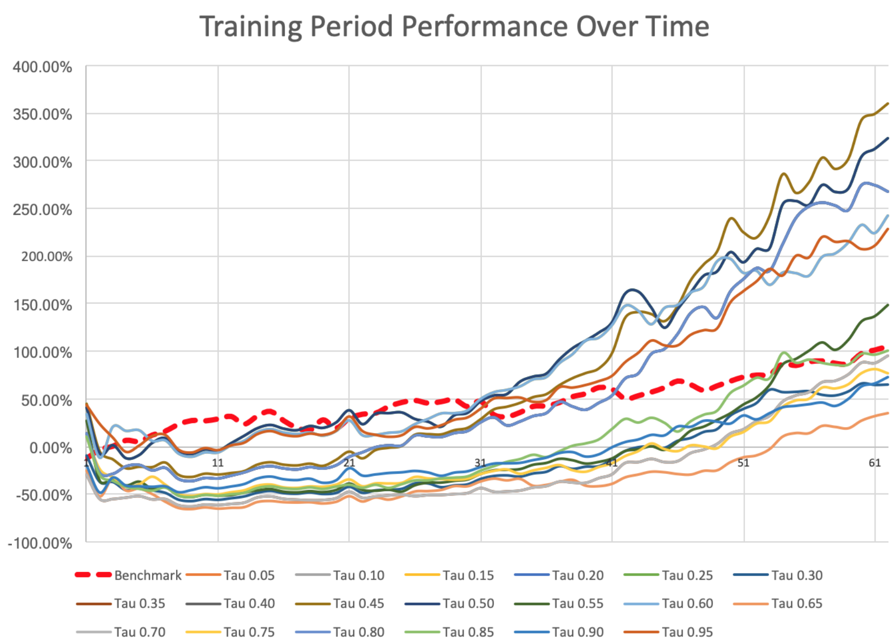

Figure 1: Distributed Views: Training Period Portfolio Performance

Based on Figure 1, it can be seen that the view generated using τ values of 0.5, 0.10, 0.15, 0.20, 0.25, 0.4, 0.45, 0.5, 055, 0.60, 0.8, 0.95 beat the benchmark S&P 100 index on nominal basis. Next the performance was analyzed to determine if the portfolios for various τ values beat the benchmark portfolio on a risk-adjusted basis. Sharpe ratio was used as a performance indicator to measure the performance on a risk-adjusted basis. The returns generated using τ values of less than or equal to 0.25 on average outperformed portfolios for views generated using τ values greater than or equal to 0.55.

The benchmark portfolio had Sharpe ratio of 0.24 and cumulative returns of 104.51% during the training period. The Sharpe ratio and cumulative returns for all the different portfolio can be found in the image below. It was noticed that there was no major correlation between the different τ values and the Sharpe ratio and cumulative returns. The portfolios for τ equal to 0.5, 0.1, 0.15 and 0.40 had highest Sharpe ratio of 0.30 and cumulative returns of 267.95% each. The best performing portfolio in the training period was the one with τ of 0.45 yielding a Sharpe ratio of 0.29 and cumulative return of 359.82%.

Based on the returns and the Sharpe ratio, it was decided to use expected returns generated using τ of 0.10 because that portfolio had the highest Sharpe ratio of 0.30.

To determine whether the returns of the portfolio truly outperformed the benchmark in the given years, a paired student t-test was performed on the returns of the portfolio and benchmark. A one-tailed paired t-Test was performed to see if the returns of the portfolio were significantly greater than those of the benchmark. The null hypothesis was that the difference between the returns of the portfolio and benchmark were less than or equal to zero. The alternative hypothesis was that difference between the returns of the portfolio and benchmark were greater than zero. The results ultimately showed that none of the portfolios ever

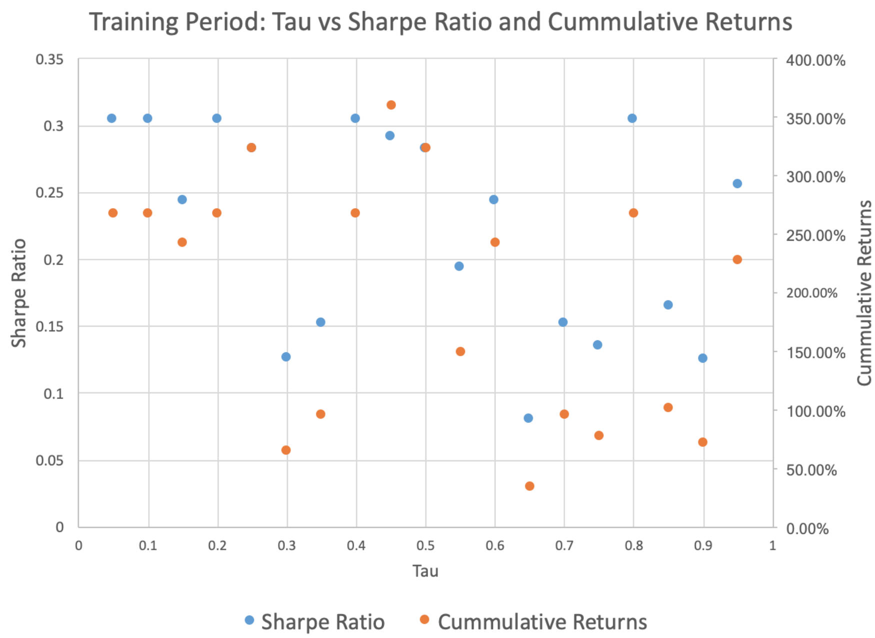

Figure 2: Distributed Views: Tau vs Sharpe Ratio and Cumulative Returns

significantly outperformed the index given that one failed to reject the null hypothesis at the ninety-five percent confidence level.

The same results were observed for the expected returns matrix for equal weights view.

The major issue that discovered with the portfolio was they appeared highly leveraged for all the τ values. The average leverage was around 6.5x. This could have induced a large amount of transaction costs, something that the portfolio rebalancing did not account. If accounted the returns would have been reduced during the training period.

5.2 Testing Period

The optimization model as discussed in the prior section was implemented on the testing data expected returns matrix generated using distributed weights on views, with a τ value of 0.1. The portfolio was rebalanced every 20 trading days, and the performance was tracked over the testing period. For the returns matrix generated using the distributed view, the following portfolio performance was observed over the trading period that involved rebalancing the portfolio 61 times at intervals of 20 trading days. The testing period ranged from January 2014 to December 2018.

As seen in Figure 3, the portfolio outperformed the market at the end of the 61 rebalancing periods. The benchmark had cumulative return of 51.64% with a Sharpe Ratio of 0.18, while the portfolio had cumulative return of 155.03% with a Sharpe Ratio of 0.30.

A one-tailed paired t-Test was performed to see if the monthly returns of the portfolio were significantly greater than those of the benchmark. The t-test results showed that the portfolio did not significantly outperform the index given that one failed to reject the null hypothesis at the ninety-five percent confidence level.

The major issue that found the portfolio was highly leveraged. The average leverage across each of the trading periods was around 6.8x: the same issue that was observed during the training period. As mentioned earlier, this high leverage could have induced higher transaction costs that if accounted would have reduced the overall cumulative returns. Additional constraints could be added to the optimization model in order to account for the leverage: a topic that could be further explored in the future.

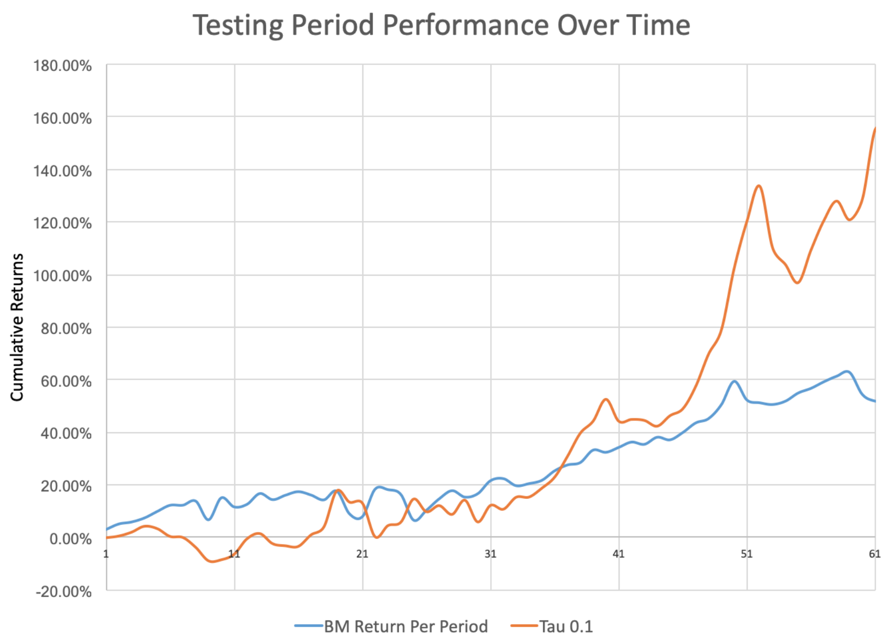

Figure 3: Distributed Views: Test Period Portfolio Performance

6 Conclusions

6.1 Summary

Based on training the portfolio strategy on January 2009 to December 2013 data, it was determined that the expected returns matrix generated using τ value of 0.10 results in the highest return on a risk-adjusted basis. The strategy involved investing in a combination of risky assets and a risk-free asset. The expected returns matrix was generated for the test period using τ of 0.10, and used in the optimization model. The resulting optimized portfolio beat the benchmark during the testing period of January 2014 to December 2018 on both nominal and risk-adjusted basis as measured by overall performance and Sharpe ratio.

Overall, if one were to account for transaction costs associated with the trading strategy, the costs would have rendered the profits generated minuscule to nonexistent when they did occur. The transactions costs would be high due to the highly leveraged nature of the portfolio. Overall, considering the high leverage, the lack of accounting for transaction costs, the portfolio returns, liquidity of certain stocks, costs associated with shorting, and the results of t-test, it can be confidently concluded that the results obtained did not significantly outperform the benchmark.

6.2 Model Improvements

Multiple adjustments could be done to enhance the model's results. First, though performing a simple linear regression on the excess returns and the technical indicator values to compute the expected excess returns proved convenient and easily interpretable, the models often tended to be statistically insignificant. Because the models appeared insignificant, the predict excess returns could be inaccurate. Various alternative techniques could be applied to enhance the prediction capabilities for the views. Machine learning techniques such as polynomial regression on the indicator, multiple regression with additional factors other than the indicator, decision trees, or time series regression with autoregressive moving average (ARMA) and generalized autoregression conditional hetereoscedastic (GARCH) models could generate far more accurate predictions. In addition, controlling for the dependence and correlation between technical indicators should be incorporated into the model. For example, the Moving Average Convergence Divergence(MACD), Accumulation-Distribution Line (ADL), and Commodity Channel Index (CCI) all appeared perfectly correlated with other indicators. This suggested that some indicators are correlated to each other.

In addition, the formulation of the uncertainty matrix (Ω) simply used the approach recommended by Black and Litterman. At times the formulation produced a near singular matrix, which required singular value decomposition to compute its inverse. Instead of using the formula recommended by Black and Litterman, the uncertainty matrix could be created based on the error in the predicted views. This would depend on the machine learning technique employed to predict the views. Each element on the uncertainty matrix would consist of the variance of the predicted value based on the given indicator and asset. An uncertain matrix following the improved approach could more accurately gauge the variability in the estimates than the original computation; however, just as before, this may not guarantee Ω is invertible.

Finally, the existence of substitution bias must be rectified. As the S&P 100 has progressed through time different companies have been added to the index while others have been removed. In order to properly compare the performance of the index to the performance of the model, the model should be adjusted to adapt to these transitions.

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