Homework Assignment 3

Yi Liu djl174
ZhaoYang Xu chm564
Jingzheng Li bnd559

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1 A bit of modelling

1.1 Write for a generic pixel, light source and normal, Lambert's law in its simplest form.

Linearised Lambertian model: $I(\mathbf{p}) = \rho(\mathbf{x})\mathbf{s}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x})$ where ρ is the albedo at $\mathbf{x}$, light source $\mathbf{s}(\mathbf{x})$, surface normal $\mathbf{n}(\mathbf{x})$, generic pixel I.

1.2 How is Lambert's law modified to deal with self shadows.

Lambert's law and self-shadows: $I = \rho \max(\mathbf{s} \cdot \mathbf{n}, \mathbf{0})$.

1.3 What about cast shadows? Comment on the difference between the two.

Since cast shadows has non local phenomenon, but Lambert's law is local so cast shadows does not obey the Lambert's law.
In our notes, self shadow is the surface behind the light source and it has a softer edge to it than a cast shadow. Cast shadow is part of the scene occludes another part and is the adjacent shadow of the distorted object outline.

1.4 Comment on the modelling limits of Lambert's law.

In FP textbook on page 34 it states that the Lambert's cosine law states the brightness of a diffuse patch illuminated by a distant point light source.
So there are two conditions:
1.Lambert's law only valid for matte materials: specularities.
2.By a distant point light source.
Besides, when the angle between the normal and the light source exceeds 90 degrees, the result will become 0, which is not based on the law of physics, and the surface will look too flat.

1.5 How can we obtain an estimate of albedo and normals in Woodham's approach to Photometric Stereo. Write the equation.

Woodham idea: Normal + Albedo Simultaneously

$$\mathbf{m} := \rho\mathbf{n}, \rho = \|\mathbf{m}\|, \mathbf{n} = \frac{\mathbf{n}}{\|\mathbf{m}\|}$$

1.6 What should be done if one uses RANSAC. Please describe. It will help you when implementing the RANSAC based estimation.

Ransac is a robust approach. To prepare data, we should extract the non-zero parts of image data and reshape it. Then run Ransac on each pixel. The specific algorithm is as follows,

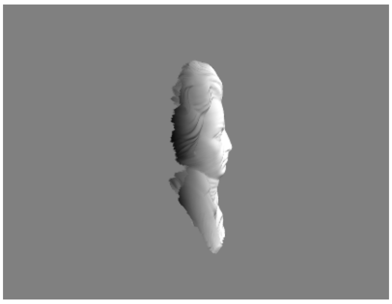
1. Select a few points in the data randomly and set them as the inliers set.
2. Calculate the model suitable for the inliers set.
3. Select all the unused points for the model and calculate whether they are inliers.
4. Write down the number of inliers.
5. Repeat the above steps for several times, and compare which calculation has the largest number of inliers, and the model built with the most inliers is the solution we want.

2 Beethoven Dataset

Figure 1: Albedo of Beethoven

Below are depth maps from three different viewpoints with simchony and unbiased integrate

(a) Depth by Poisson: View 1



(b) Depth by Poisson: View 2

(c) Depth by Poisson: View 3

(d) Depth by Fourier: View 1

(e) Depth by Fourier: View 2

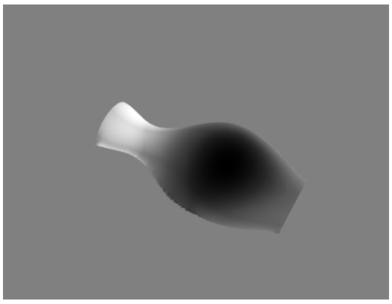
(f) Depth by Fourier: View 3

Figure 2: Depth maps of Beethoven at different viewpoints

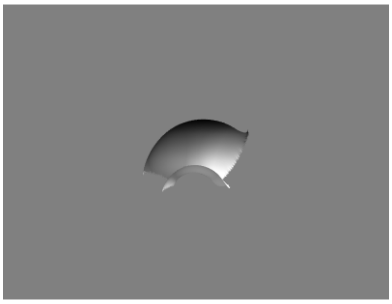
3 mat_vase Dataset

Figure 3: Albedo of Mat_vase

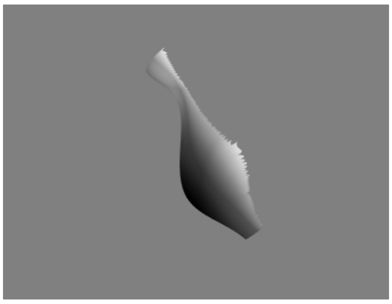
Below are depth maps from three different viewpoints with simchony and unbiased integrate



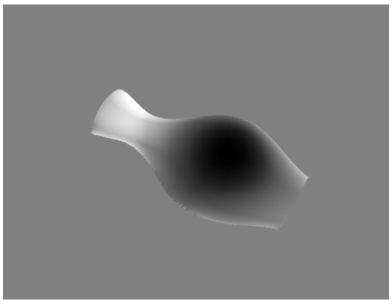
(a) Depth by Poisson: View 1



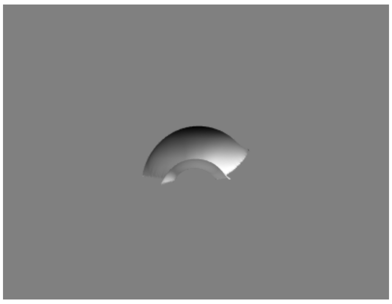
(b) Depth by Poisson: View 2



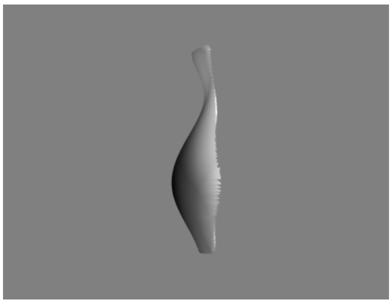
(c) Depth by Poisson: View 3



(d) Depth by Fourier: View 1



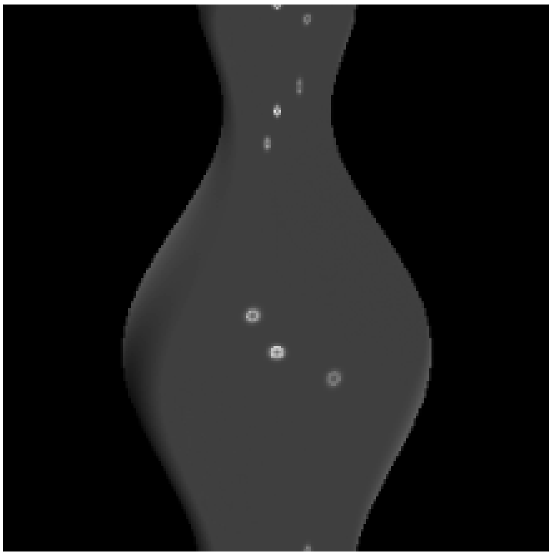
(e) Depth by Fourier: View 2



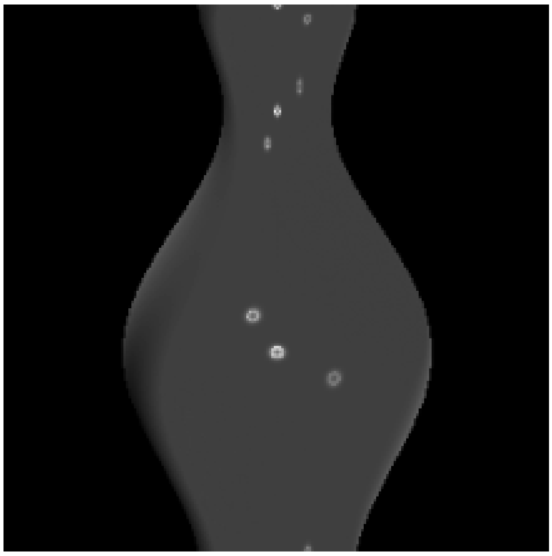
(f) Depth by Fourier: View 3

Figure 4: Depth maps of mat_vase at different viewpoints

4 shiny_vase Dataset



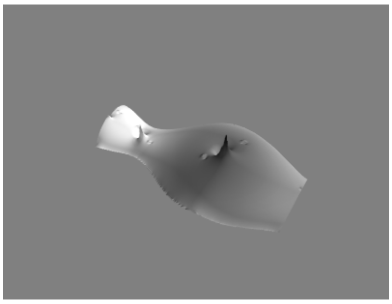
(a) Obtain by Woodham's approach



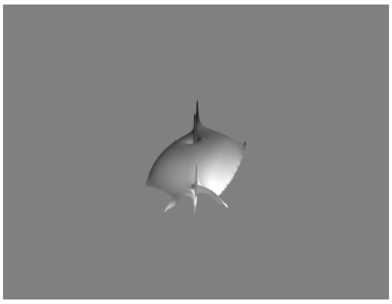
(b) Obtain by Ransac estimation

Figure 5: Albedo of shiny_vase

Below are depth maps from three different viewpoints with simchony and unbiased integrate.

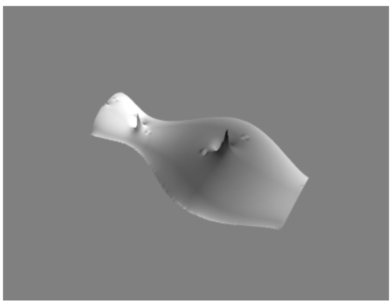


(a) Depth by Poisson: View 1

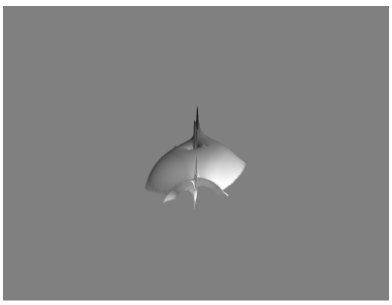


(b) Depth by Poisson: View 2

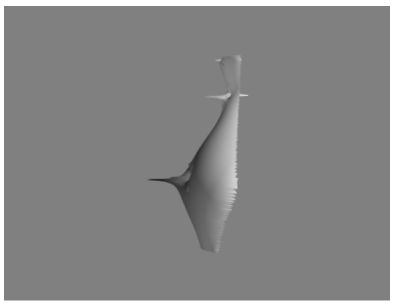
(c) Depth by Poisson: View 3



(d) Depth by Fourier: View 1

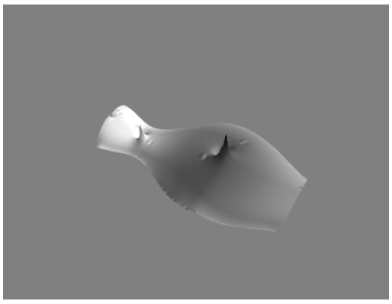


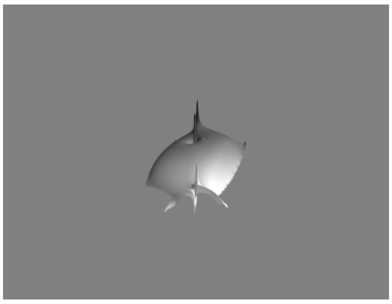
(e) figure Depth by Fourier: View 2

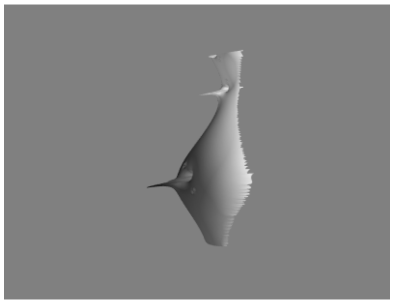


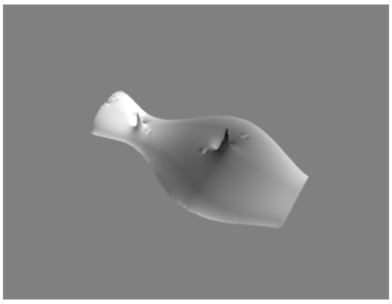
(f) figure 6Depth by Fourier: View 3

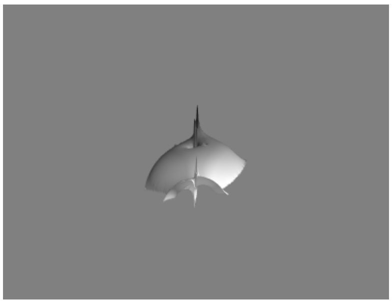
Figure 6: Depth maps of shiny_vase at different viewpoints by Woodham's approach


(a) Depth by Poisson: View 1


(b) Depth by Poisson: View 2


(c) Depth by Poisson: View 3


(d) Depth by Fourier: View 1


(e) Depth by Fourier: View 2

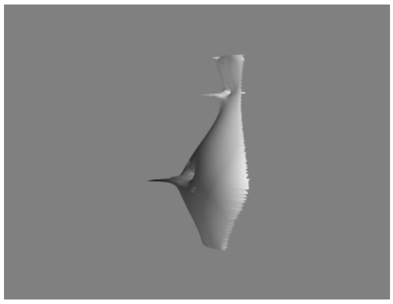
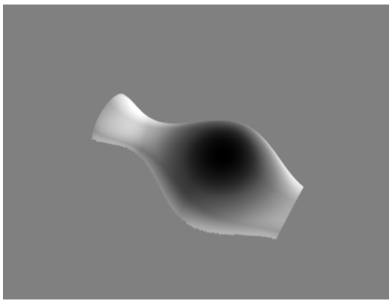
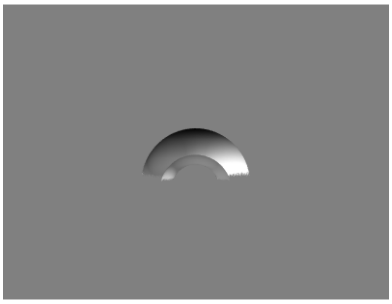
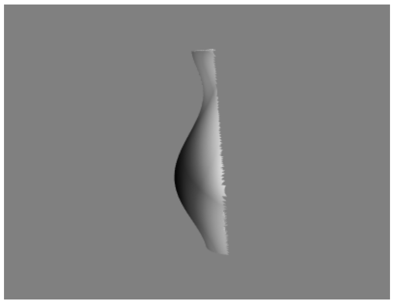
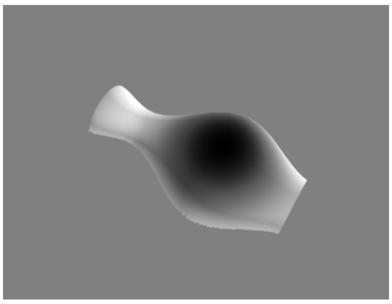

(f) Depth by Fourier: View 3

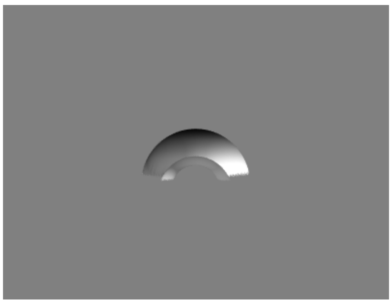
Figure 7: Depth maps of shiny_vase at different viewpoints by RANSAC of threshold 1


(a) Depth by Poisson: View 1


(b) Depth by Poisson: View 2


(c) Depth by Poisson: View 3


(d) Depth by Fourier: View 1


(e) Depth by Fourier: View 2

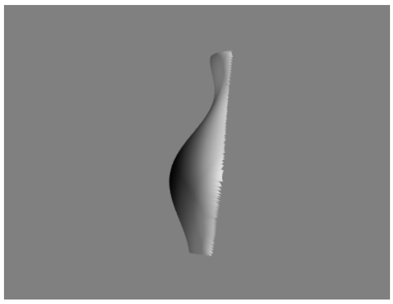

(f) Depth by Fourier: View 3

Figure 8: Depth maps of shiny_vase at different viewpoints by RANSAC of threshold 1 after smooth

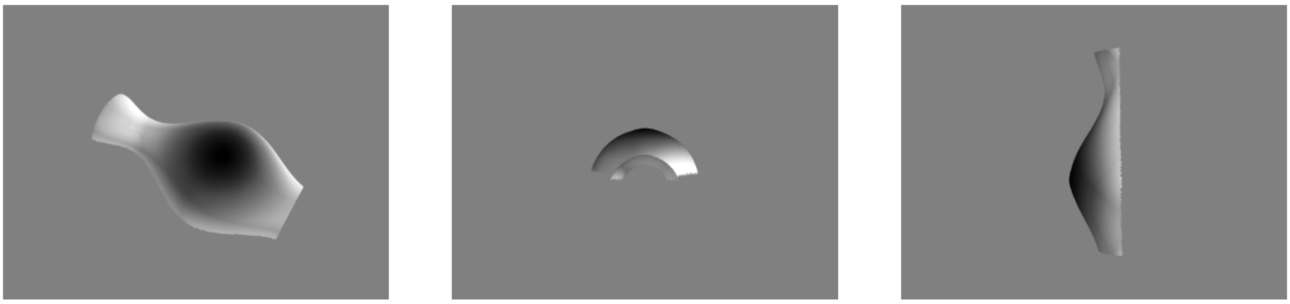
From Fig6, the surface of the vase is not smooth, the depth of highlights are much higher. The reason is Woodham's approach is effected by non-Lambertian factors. It cannot remove the strong specular on vase.
From Fig7, we can see that RANSAC doesn't provide a better result, it looks as same as Fig6. RANSAC algorithm is random. It selects inliers randomly each time. In this data, there are some odd points. So RANSAC randomly selects some points, thus it's possible that RANSAC classified

the outliers as inliers. Therefore, the result it not good as it's expect. But after smooth function, the picture is better.

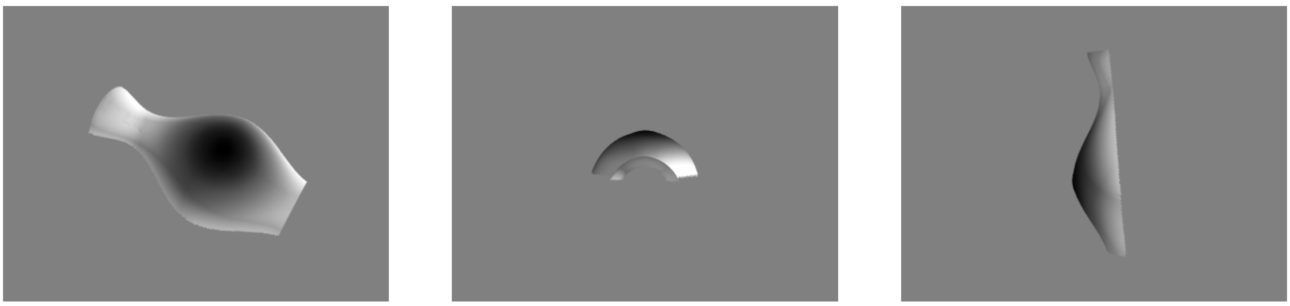
5 shiny_vase2 Dataset

(a) Obtain by Woodham's approach　　(b) Obtain by Ransac estimation

Figure 9: Albedo of shiny_vase2

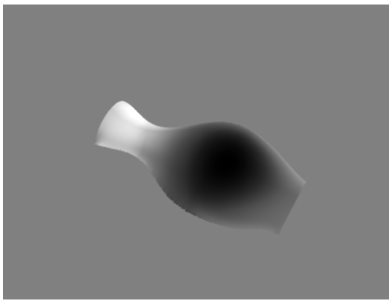


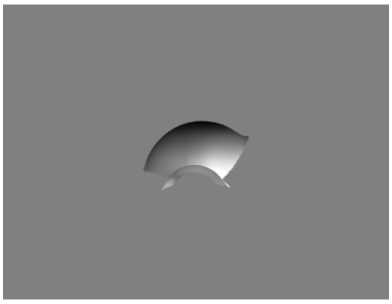
(a) Depth by Poisson: View1　　(b) Depth by Poisson: View2　　(c) Depth by Poisson: View3

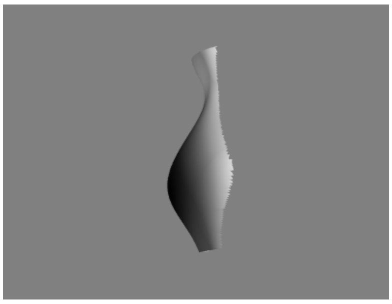


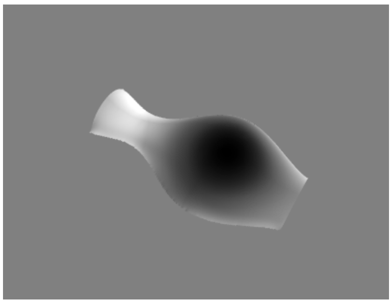
(d) Depth by Fourier: View1　　(e) Depth by Fourier: View2　　(f) Depth by Fourier: View3

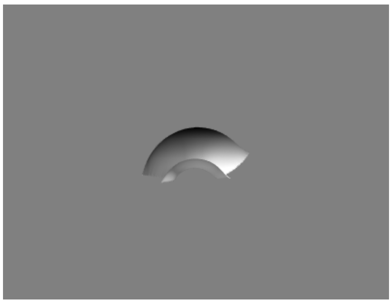
Figure 10: Depth maps of shiny_vase2 at different viewpoints by Woodham's approach


(a) Depth by Poisson: View1


(b) Depth by Poisson: View2


(c) Depth by Poisson: View3


(d) Depth by Fourier: View1


(e) Depth by Fourier: View2

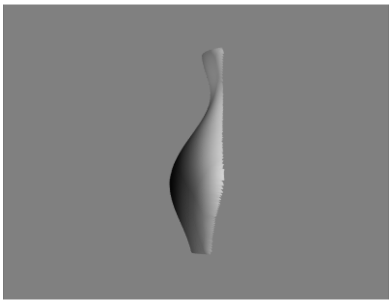
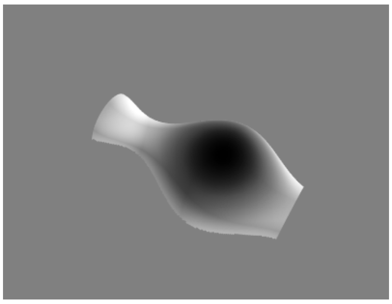
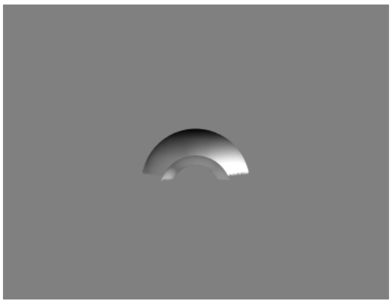
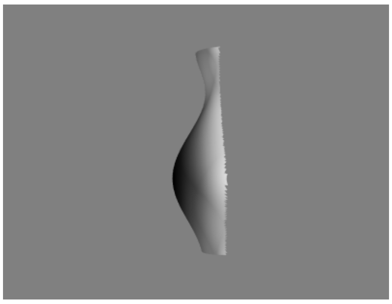
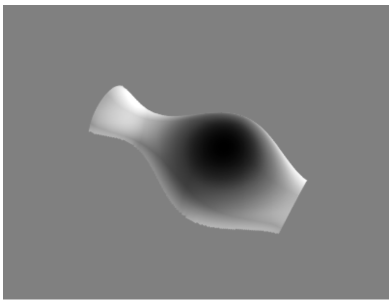

(f) Depth by Fourier: View3

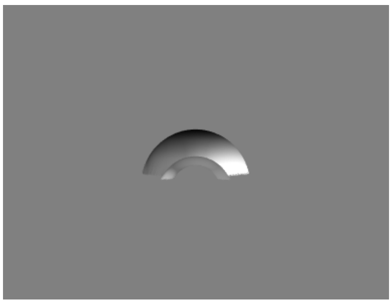
Figure 11: Depth maps of shiny_vase2 at different viewpoints with threshold 1 by RANSAC


(a) Depth by Poisson: View1


(b) Depth by Poisson: View2


(c) Depth by Poisson: View3


(d) Depth by Fourier: View1


(e) Depth by Fourier: View2

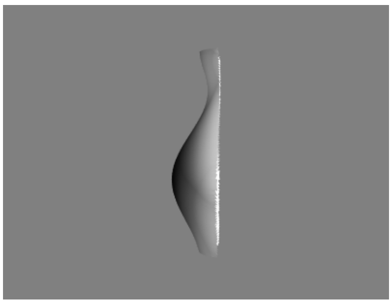

(f) Depth by Fourier: View3

Figure 12: Depth maps of shiny_vase2 at different viewpoints by RANSAC with threshold 1 after smooth

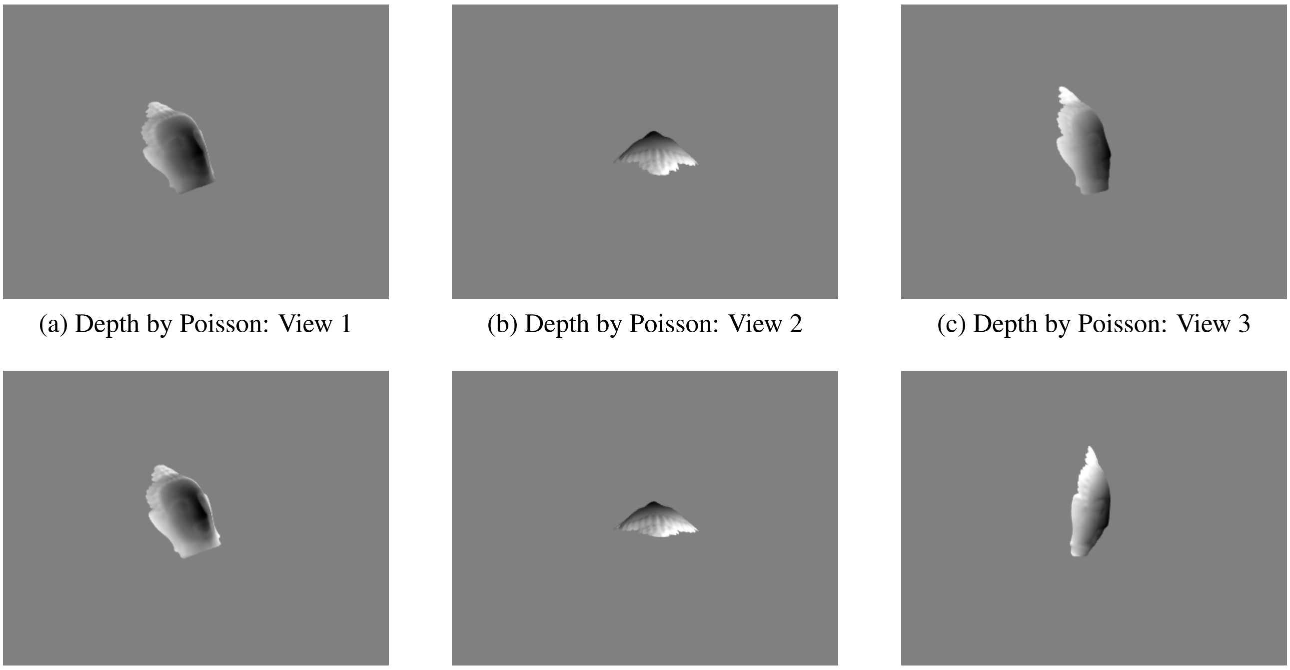
The difference between the albedo by Woodham's and Ransac is quite obvious, not only on the color, but also we can see albedo by Ransac doesn't contain any specularities. RANSAC should provide a better result than Woodham's. Unfortunately, we don't see a big difference in the depth maps above.

6 Buddha Dataset

(a) Obtain by Woodham's approach

(b) Obtain by Ransac estimation

Figure 13: Albedo of Buddha



(a) Depth by Poisson: View 1

(b) Depth by Poisson: View 2

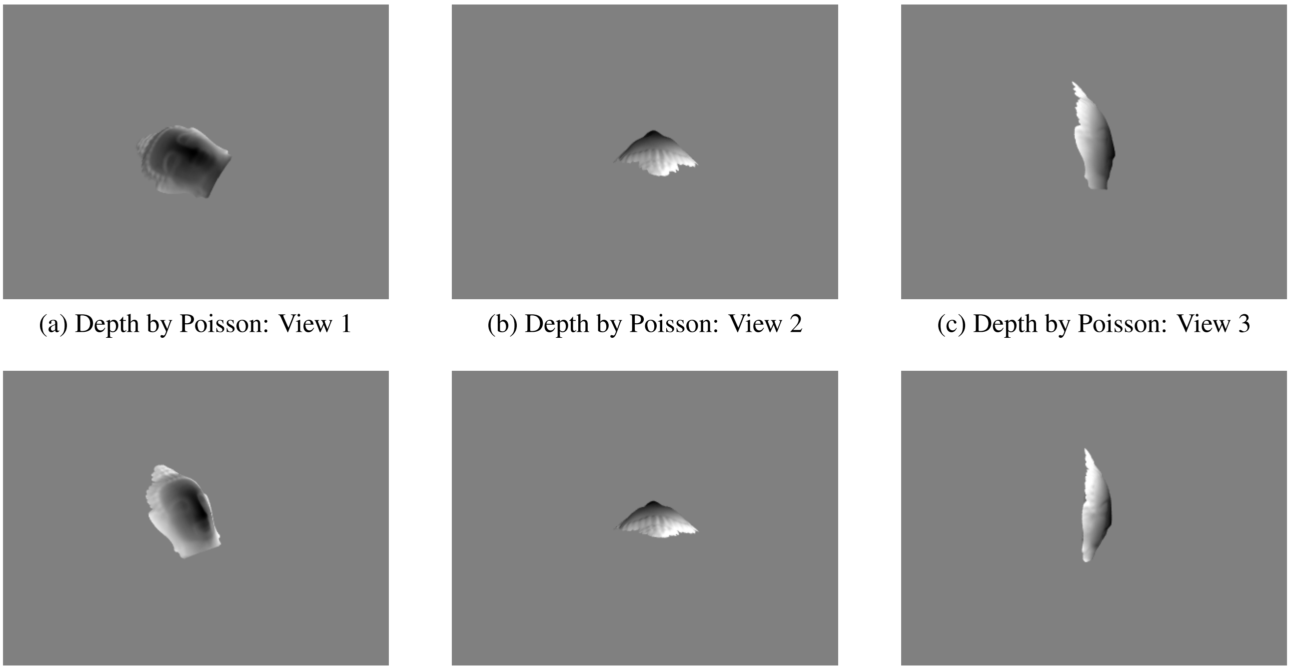
(c) Depth by Poisson: View 3

(d) Depth by Fourier: View 1

(e) Depth by Fourier: View 2

(f) Depth by Fourier: View 3

Figure 14: Depth maps of Buddha at different viewpoints by RANSAC of threshold 125 after smooth



(a) Depth by Poisson: View 1

(b) Depth by Poisson: View 2

(c) Depth by Poisson: View 3

(d) Depth by Fourier: View 1

(e) Depth by Fourier: View 2

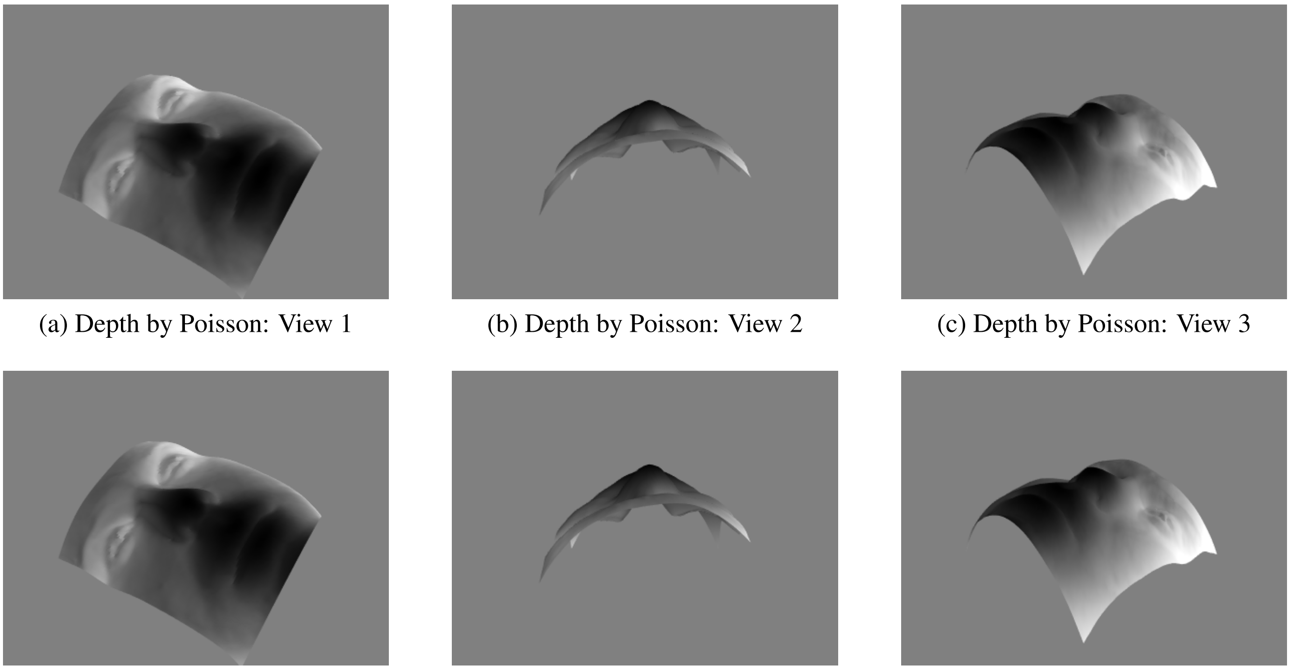
(f) Depth by Fourier: View 3

Figure 15: Depth maps of Buddha at different viewpoints by RANSAC of threshold 200 after smooth

Fig 13b is the albedo of Buddha by RANSAC with threshold value at 25. It shows darker than Woodham's approach. Because Woodham's estimation was always disrupted by some non-Lambertian effects, like shadows, image noise, whereas RANSAC treated them as outliers.
When the threshold value is at 25, then all the depth maps by Direct Poisson and Fourier-Transform solver of Woodham's, Ransac and Ransac's smooth functions algorithm looks like a pure white plank in grey-scale, we think that's happens because all the data are NaN. If we increase the threshold value to 125, then we can have a 3D depth map by the smooth normal field function. Also, if we change the threshold value from 125 to 200, we can see the 3D depth map and albedo images looks slightly brighter.

7 face Dataset

The difference between the following depth maps by RANSAC and the smooth normal field function is the smoothness level. If we use the smooth normal field function, then it becomes smoother clearly and reduce the noise and details of the image.
Below are depth maps from three different viewpoints with simchony and unbiased integrate.

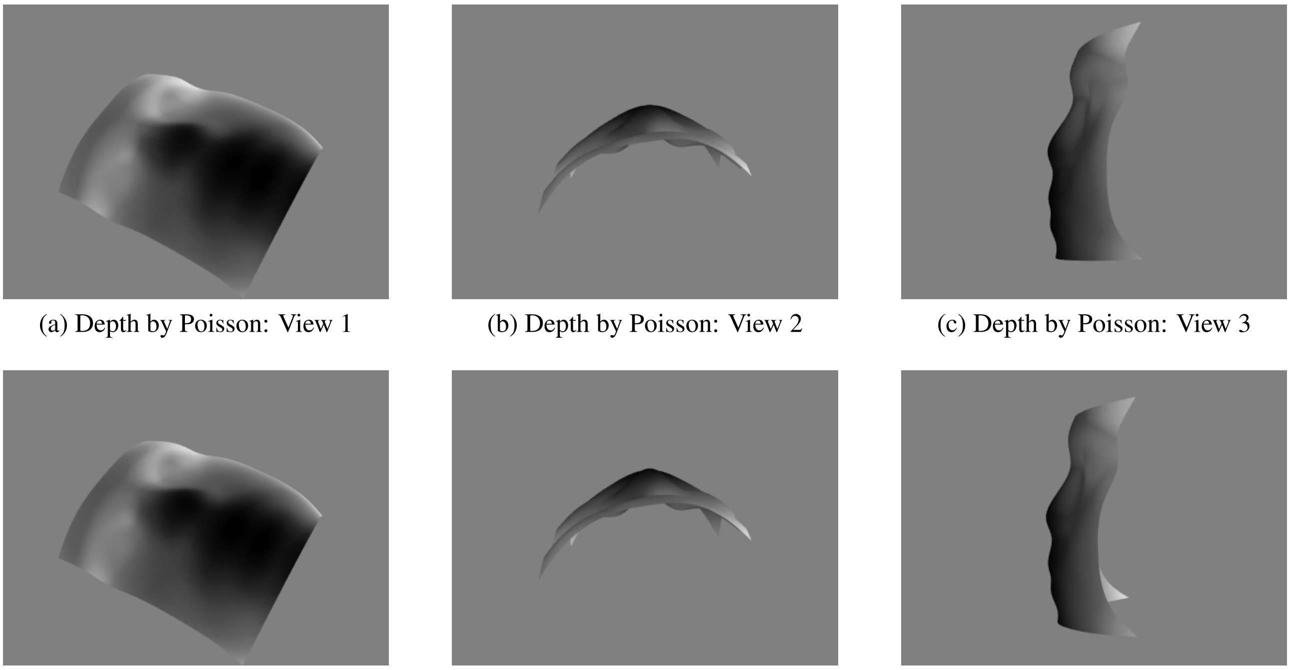


(a) Depth by Poisson: View 1 (b) Depth by Poisson: View 2 (c) Depth by Poisson: View 3

(d) Depth by Fourier: View 1 (e) Depth by Fourier: View 2 (f) Depth by Fourier: View 3

Figure 16: Depth maps of face at different viewpoints by RANSAC of threshold 10

Below are depth maps from three different viewpoints with simchony and unbiased integrate by the smooth normal function.



(a) Depth by Poisson: View 1 (b) Depth by Poisson: View 2 (c) Depth by Poisson: View 3

(d) Depth by Fourier: View 1 (e) Depth by Fourier: View 2 (f) Depth by Fourier: View 3

Figure 17: Depth maps of face at different viewpoints by RANSAC of threshold 10 after smooth