

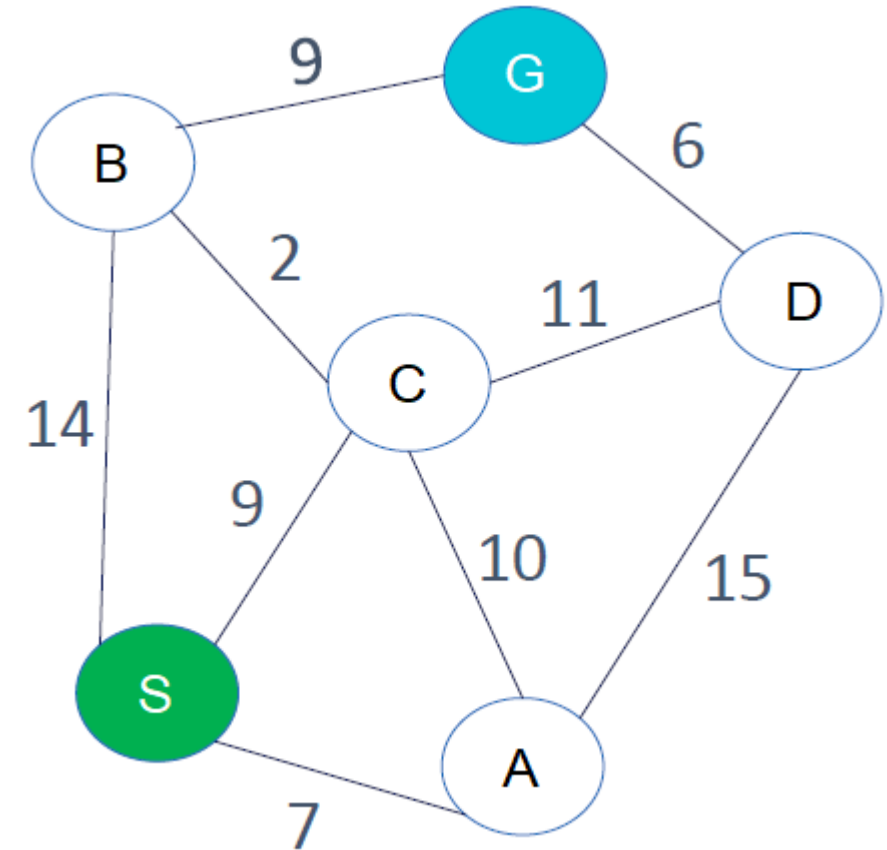
# **Introduction to Artificial Intelligence in Games**

## **Lecture 4**

# Uniform-Cost Search

## Task

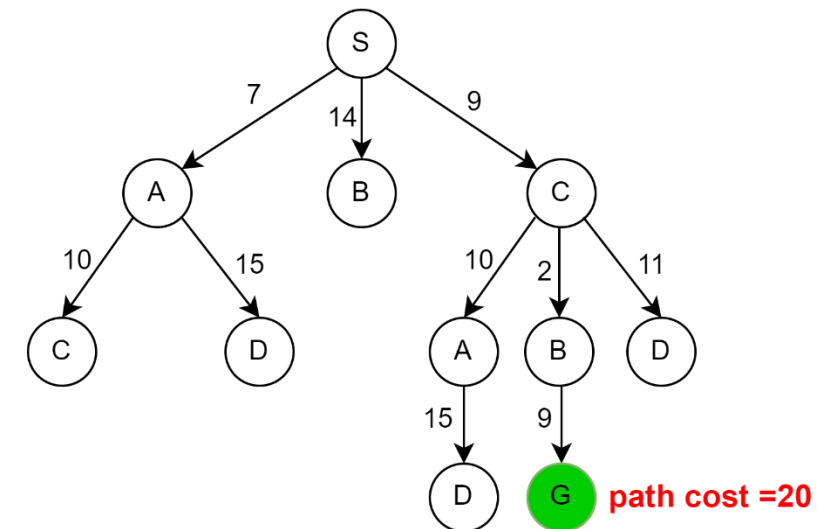
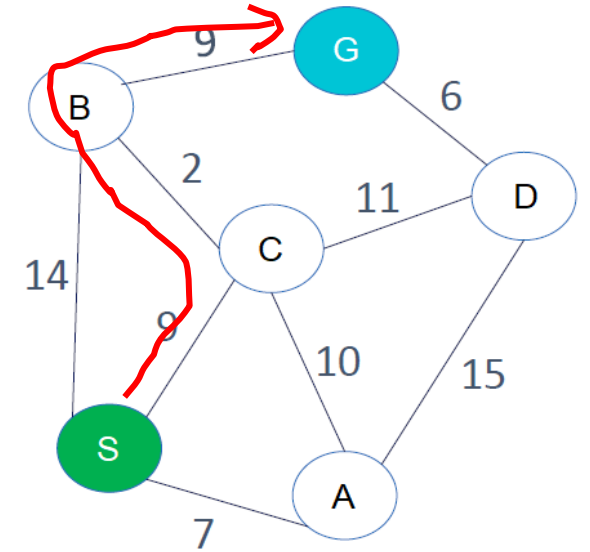
- Construct the search tree and find the optimum path from S to G using the uniform-cost search algorithm.



# Solution

Expanded nodes list (sort then expand)

- S0 { A7, C9, B14 }
- A7 { C9, B14, ~~C~~17, D22 }
- C9 { B11, ~~B~~14, A19, D20, ~~D~~22 }
- B11 { A19, G20, D20 }
- A19 { G20, D20, ~~D~~34 }
- G20
- Solution path found is  $S \rightarrow C \rightarrow B \rightarrow G$ , cost= 20



# Informed Search Methods

# Informed Search Algorithms

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- In informed search algorithm, we have some knowledge about the goal, such as how far we are from the goal (straight line distance), how many wrong-placed tiles in 8-puzzle game, etc.
- This knowledge help agents to explore less to the search space and find more efficiently the goal node.
- Informed search algorithm uses the idea of heuristic, so it is also called **Heuristic search**.
- In a heuristic search, each state is assigned a “heuristic value” (h-value) that the search uses in selecting the “best” next step.
- Heuristic  $h(n)$  gives an estimate of the distance from  $n$  to the goal.
- $h(n)=0$  for goal nodes

# Greedy Best-first Search

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- The basic idea of best first search is similar to uniform cost search.
- The only difference is that the “cost” (or in general, the evaluation function) is estimated based on some heuristics  $h(n)$  which represents how far we are from the goal, rather than the accumulated cost  $g(n)$  calculated during search.
- In the best first search algorithm, we expand the node which is closest to the goal node.
- Similar to the uniform cost search, the greedy best-first algorithm is implemented by the priority queue, with the node that has the lowest heuristic value  $h$  is processed first.

# Greedy Best-first Search

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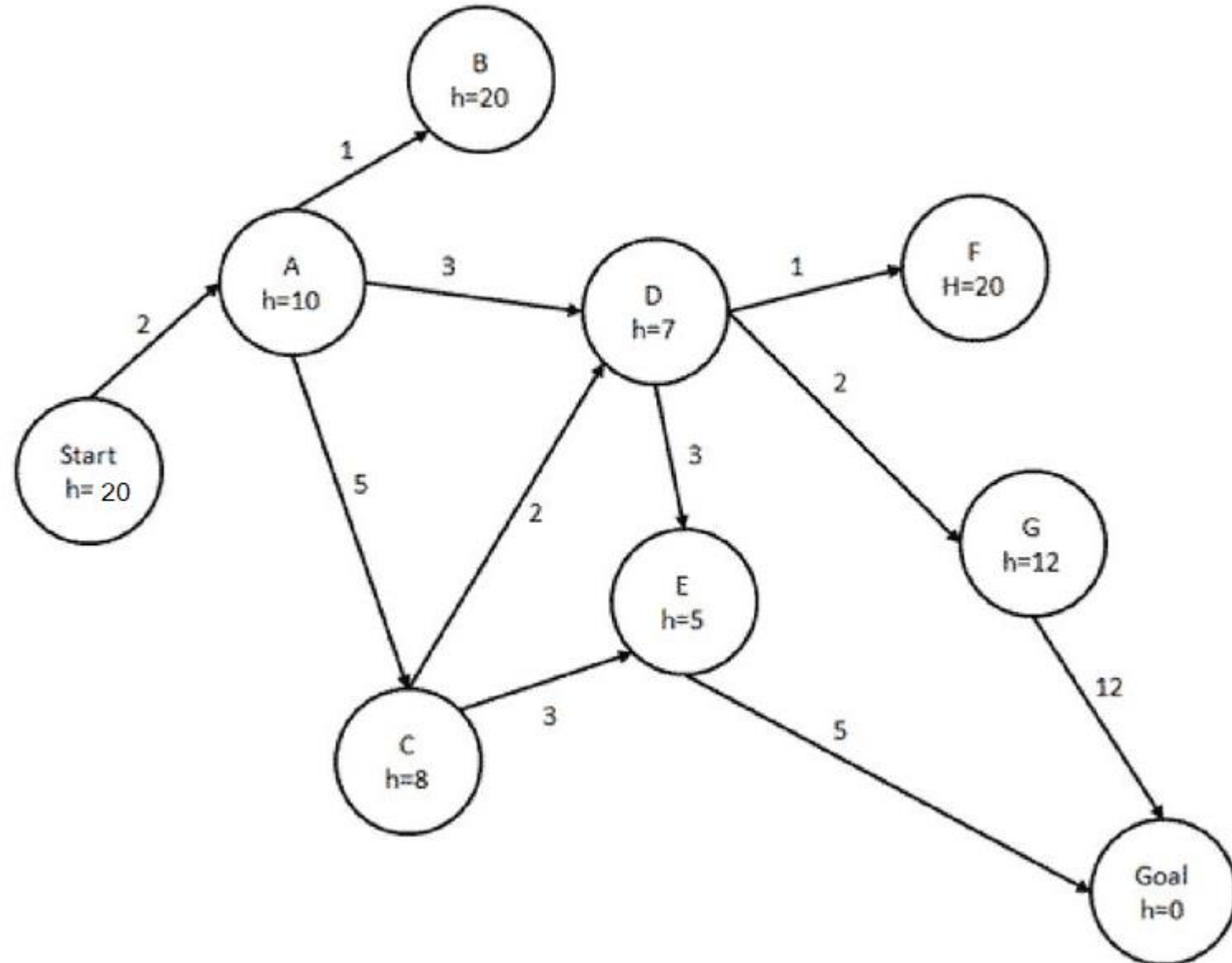
## Steps

- Enqueue nodes by their heuristic value.
- That is, let  $h(n)$  = cost (straight line distance) of the path from the current node  $n$  to the goal node.
- Sort nodes by ascending value of  $h$
- If there is more than one path to the same node, keep only the path with the lowest cost in the queue.
- Stop when the goal is the first node in the queue  
Else, expand nodes starting from the first node in the queue
- Ignore back paths to the start node.
- Ignore back paths to the parent node.

# Greedy Best-first Search

## Example

- Find the path from the start node to the goal node using the greedy best-first search.
- The heuristic value  $h(n)$  for each node is shown on the node.



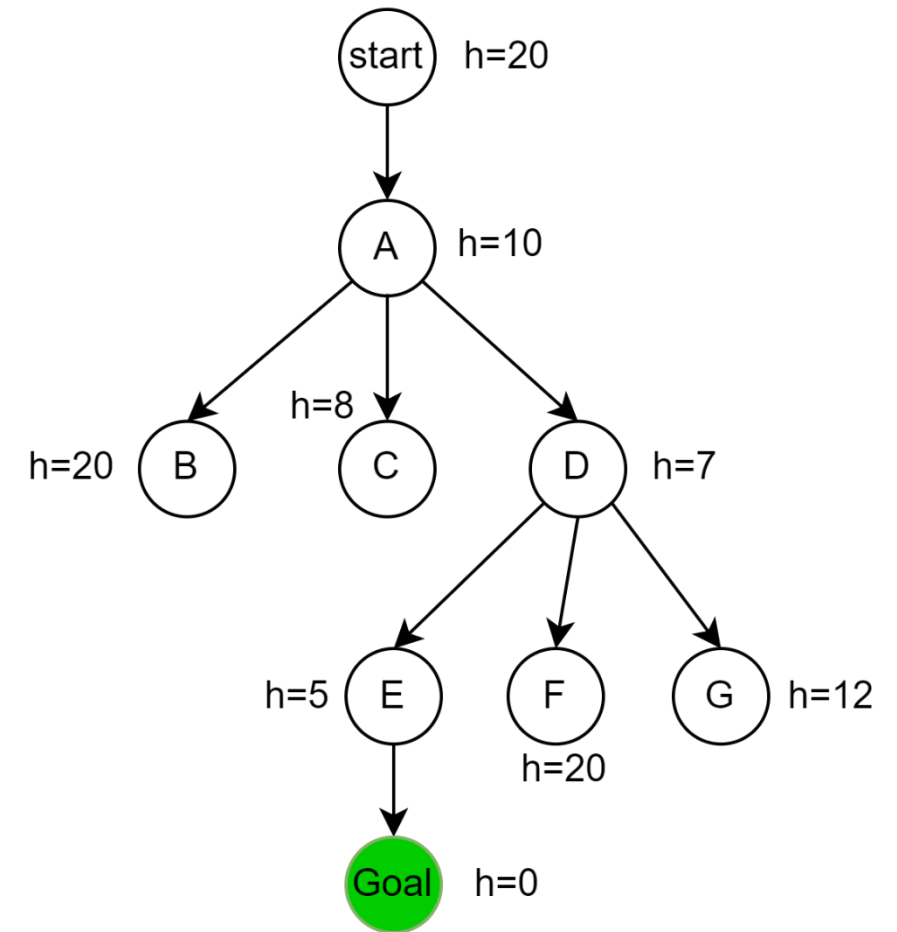


# Greedy Best-first Search

## Solution

Expanded nodes list (sort then expand)

- Start { A10 }
- A10 { D7, C8, B20 }
- D7 { E5, C8, G12, B20, F20 }
- E5 { Goal }
- Goal
- Solution path found is Start  $\rightarrow$  A  $\rightarrow$  D  $\rightarrow$  E  $\rightarrow$  Goal



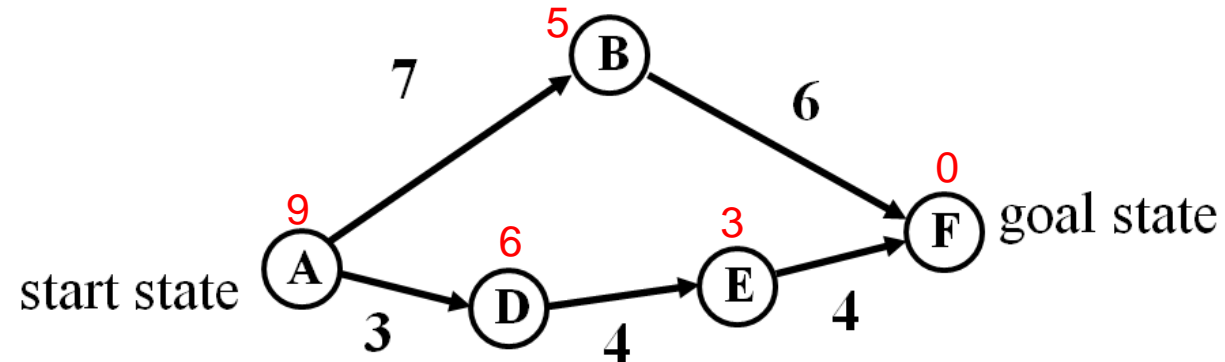
# Greedy Best-first Search

## Completeness

- Greedy best first search algorithm is complete as in worst case scenario it will search the whole space (worst option).

## Optimality

- Greedy best first search algorithm is not optimal.
- A solution can be found in a longer path (lower  $h(n)$  with a higher  $g(n)$  value).

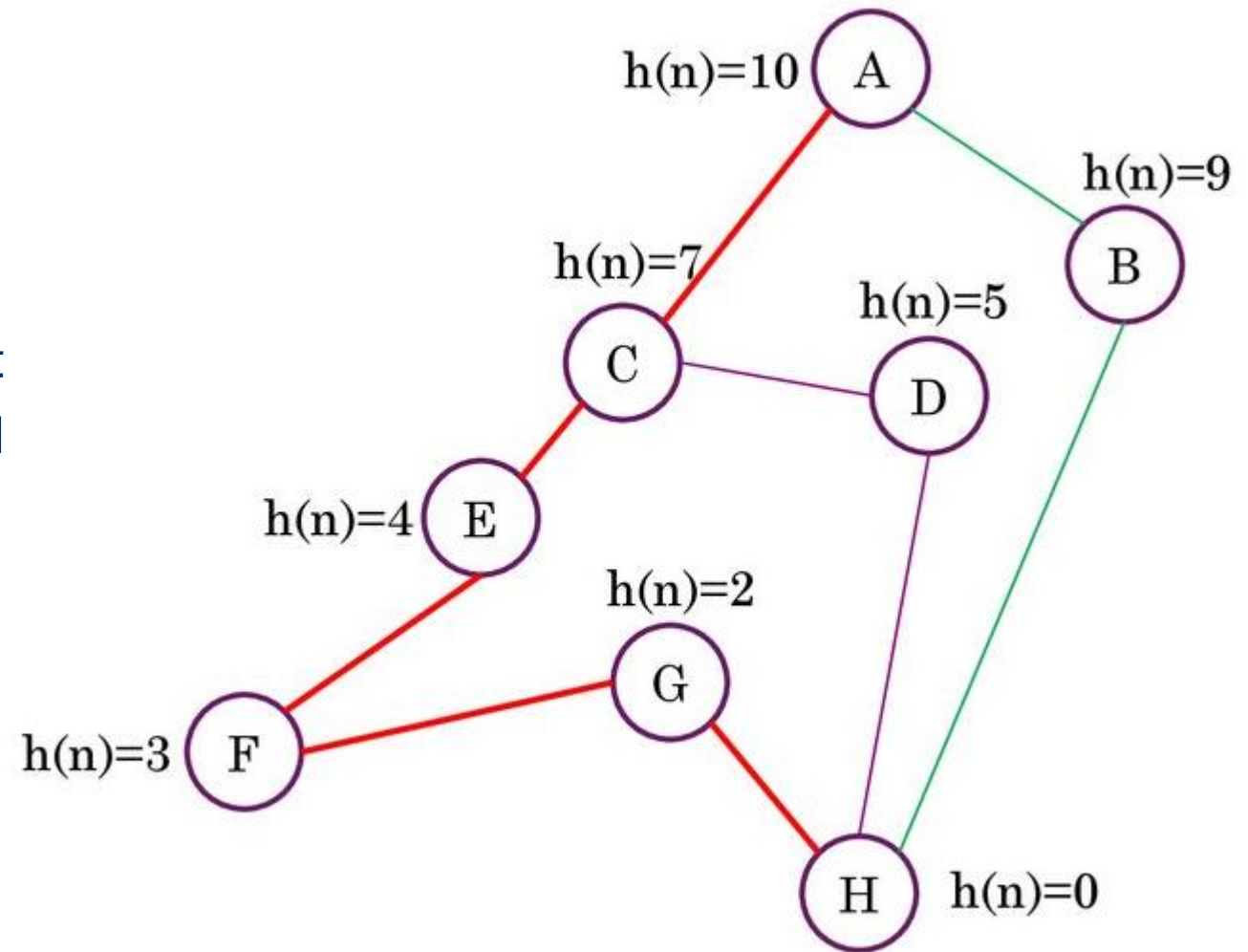


- Say:  $h(A) = 9$ ,  $h(B) = 5$ ,  $h(D) = 6$ ,  $h(E) = 3$ ,  $h(F) = 0$
- It does not take the actual cost into account

# Greedy Best-first Search

## Another example

- The greedy best first search will find the solution path:  
 $A \rightarrow C \rightarrow E \rightarrow F \rightarrow G \rightarrow H(\text{Goal})$
- However, this path is the longest path in the graph to the goal and processes more nodes.



# A\* Search Algorithm

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- A\* Algorithm is one of the best and popular techniques used for path finding and graph traversals.
- A lot of games and web-based maps use this algorithm for finding the shortest path efficiently.
- It combines features of UCS and greedy best-first search, by which it solve the problem efficiently.
- It uses the evaluation function  $f(n) = g(n) + h(n)$ 
  - $g(n)$  is the cumulative cost from the start node to the node  $n$
  - $h(n)$  is the heuristic (estimated) cost from the node  $n$  to the goal
  - $f(n)$  is the estimated total cost of path through  $n$  to the goal

# A\* Search Algorithm

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## Advantages:

- The A\* algorithm can obtain the best solution because it considers both the cost calculated up to now and the estimated future cost.
- A\* search algorithm is optimal and complete.
- This algorithm can solve very complex problems.

## Disadvantages:

- A\* search algorithm has some complexity issues.
- The main drawback of A\* is memory requirement as it keeps all generated nodes in the memory, so it is not practical for various large-scale problems.

# A\* Search Algorithm

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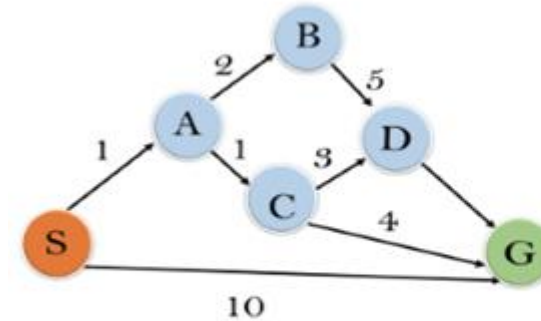
## Steps

- Enqueue nodes by their evaluation function  $f(n) = g(n) + h(n)$ .
- Sort nodes by ascending value of  $f$
- If there is more than one path to the same node, keep only the path with the lowest cost in the queue.
- Stop when the goal is the first node in the queue  
Else, expand nodes starting from the first node in the queue
- Ignore back paths to the start node.
- Ignore back paths to the parent node.

# A\* Search Algorithm

## Example

- In this example, we will traverse the given graph using the A\* algorithm.
- The heuristic value of all states is given in the table.



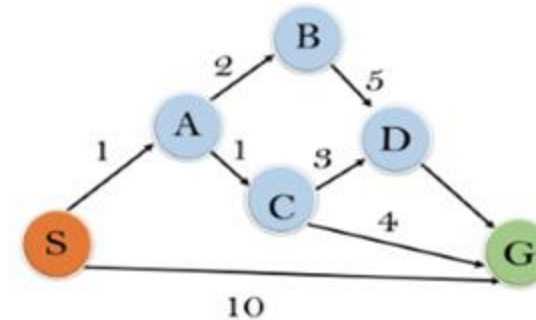
| State | $h(n)$ |
|-------|--------|
| S     | 5      |
| A     | 5      |
| B     | 4      |
| C     | 2      |
| D     | 6      |
| G     | 0      |

# A\* Search Algorithm

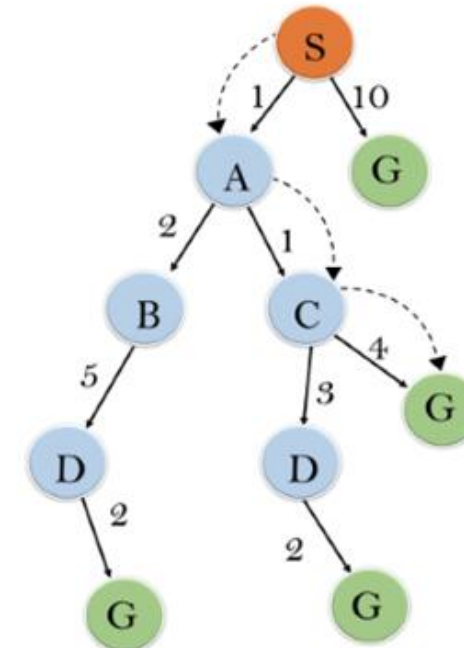
## Solution

Expanded nodes list

- S { f(A), f(G) } not sorted  
 $f(A) = g(A) + h(A) = 1 + 5 = 6$   
 $f(G) = g(G) + h(G) = 10 + 0 = 10$   
 S { A6, G10 } sorted
- A6 { G10, f(B), f(C) } not sorted  
 $f(B) = g(B) + h(B) = 3 + 4 = 7$   
 $f(C) = g(C) + h(C) = 2 + 2 = 4$   
 A6 { C4, B7, G10 } sorted
- C4 { B7, G10, f(D), f(G) } not sorted  
 $f(D) = g(D) + h(D) = 5 + 6 = 11$   
 $f(G) = g(G) + h(G) = 6 + 0 = 6$   
 C4 { **G6**, B7, G10, D11 } sorted
- **G6**
- Solution path found is  $S \rightarrow A \rightarrow C \rightarrow G$



| State | h(n) |
|-------|------|
| S     | 5    |
| A     | 5    |
| B     | 4    |
| C     | 2    |
| D     | 6    |
| G     | 0    |





# A\* Search Algorithm

## Practical Examples: 8-Puzzle

- Given an initial state of an 8-puzzle problem and final state to be reached.
- Find the most cost-effective path to reach the final state from initial state using A\* Algorithm.
- Consider the cost from one state to another = 1 (which will be used to get  $g(n)$ ), and  $h(n)$  = Number of misplaced tiles.

|   |   |   |
|---|---|---|
| 2 | 8 | 3 |
| 1 | 6 | 4 |
| 7 |   | 5 |

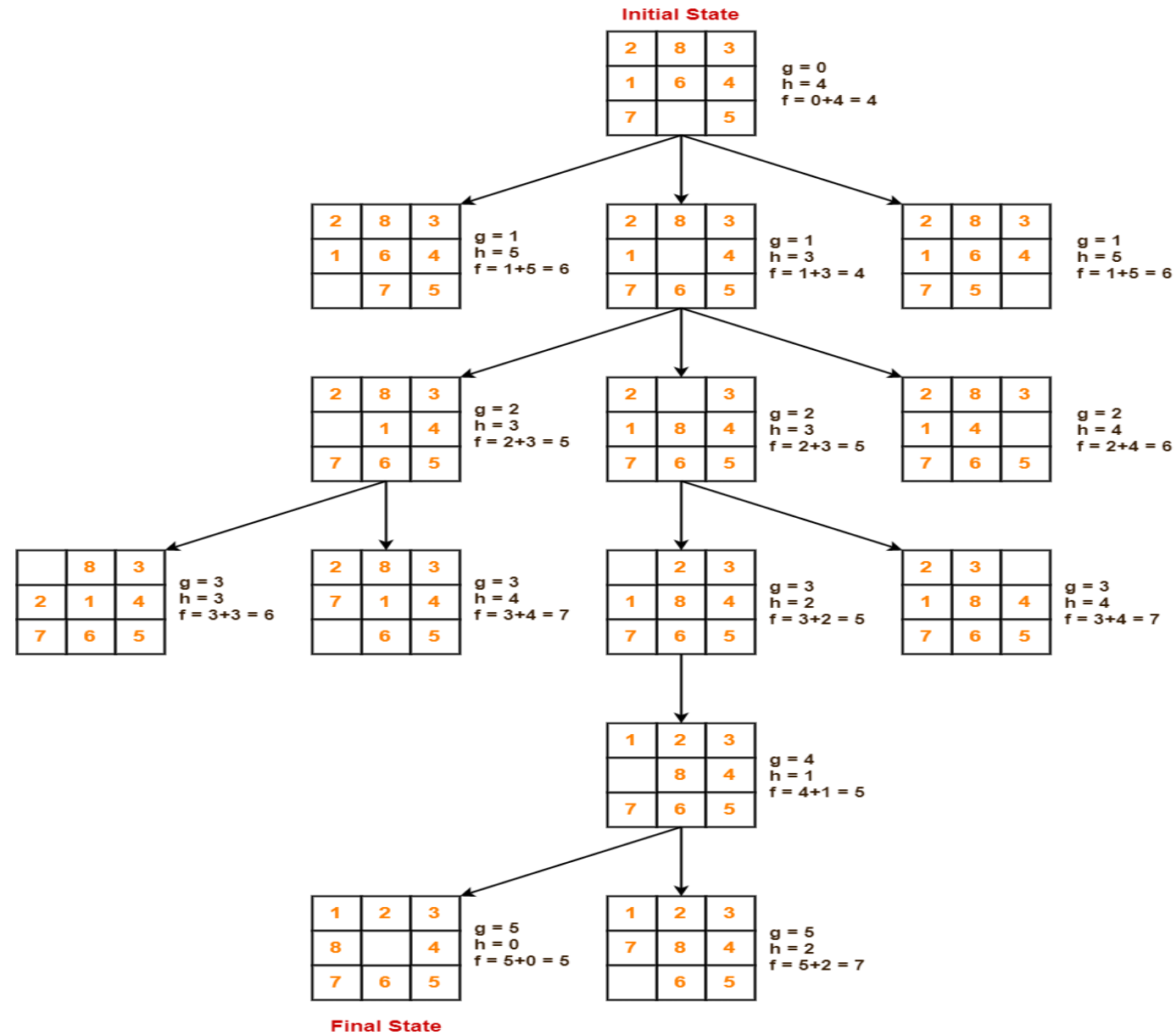
**Initial State**

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 8 |   | 4 |
| 7 | 6 | 5 |

**Final State**

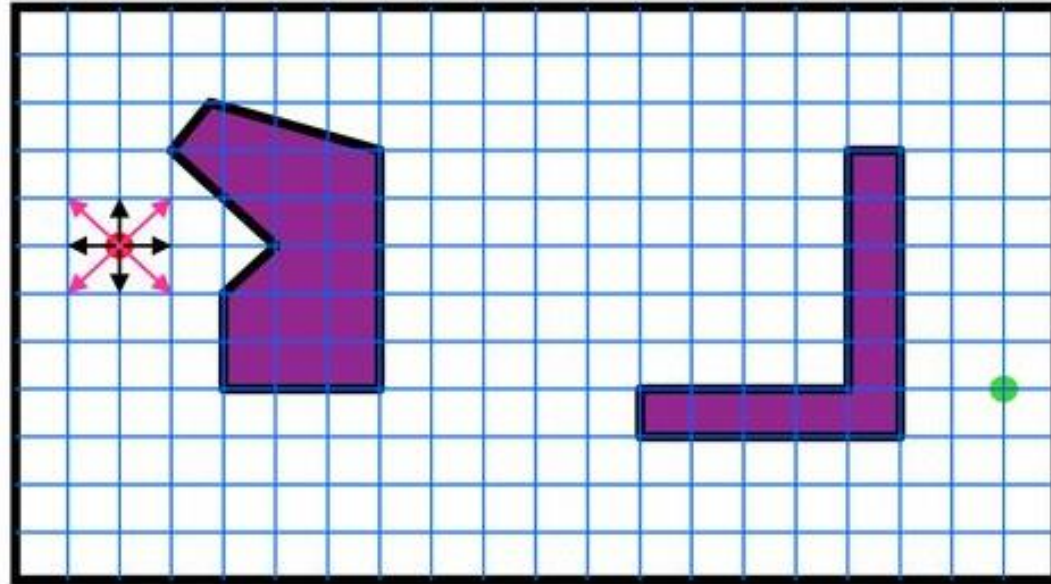
# A\* Search Algorithm

## Solution



# A\* Search Algorithm

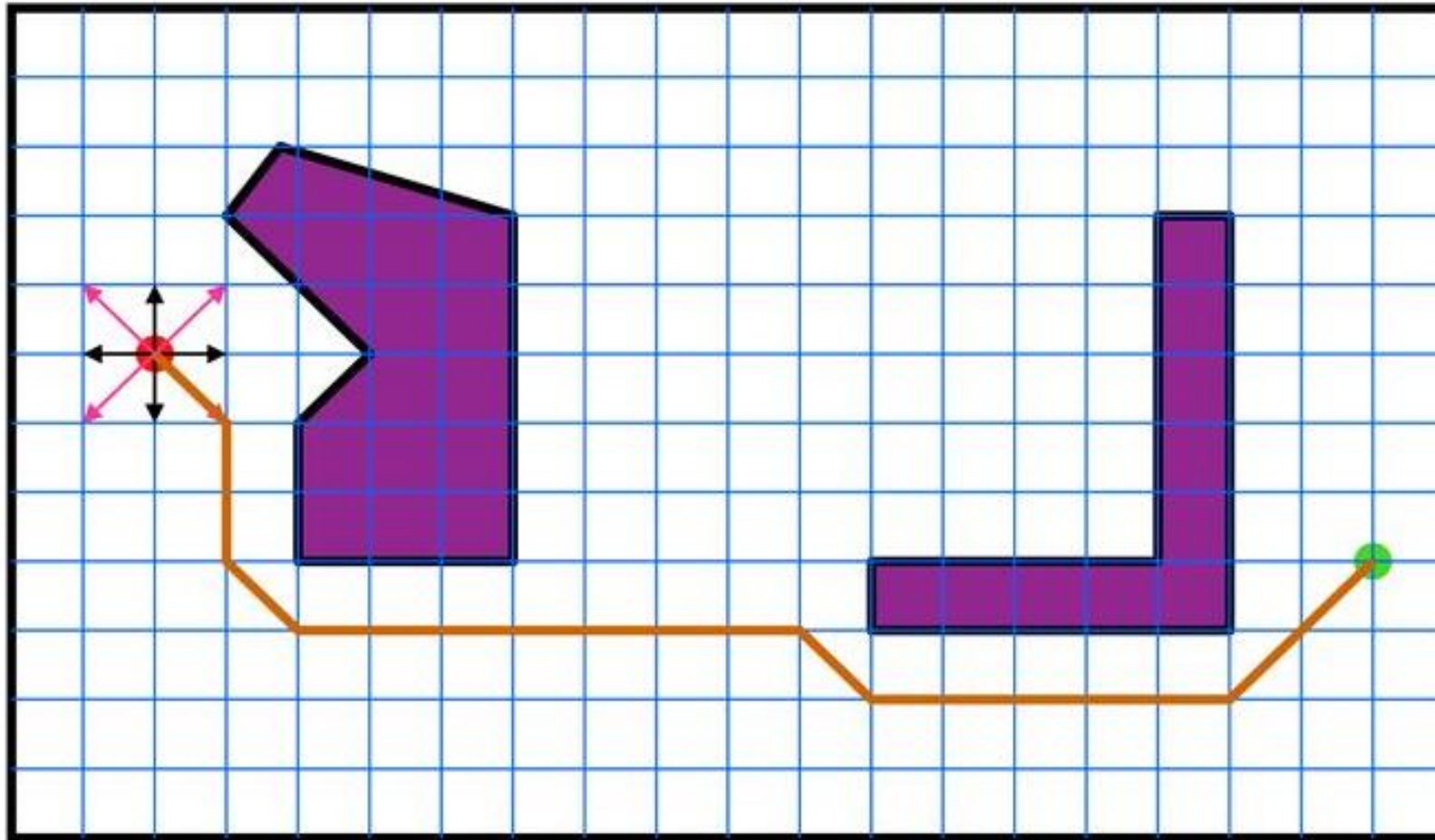
## Practical Examples: Robot Navigation



- $f(n) = g(n) + h(n)$ , with  $h(n)$  = straight-line distance from node  $n$  to goal
- $h(n) = \sqrt{(x_g - x_n)^2 + (y_g - y_n)^2}$
- $g(n)$  : Cost of one horizontal/vertical step = 1  
Cost of one diagonal step =  $\sqrt{2}$

# A\* Search Algorithm

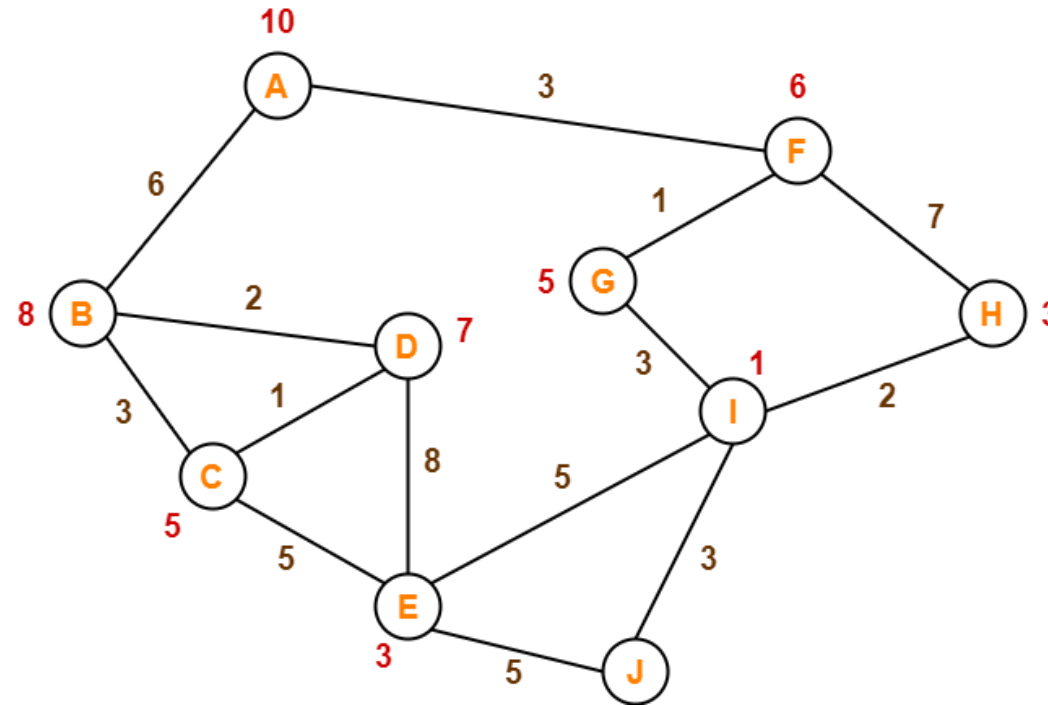
## Solution: Robot Navigation



# A\* Search Algorithm

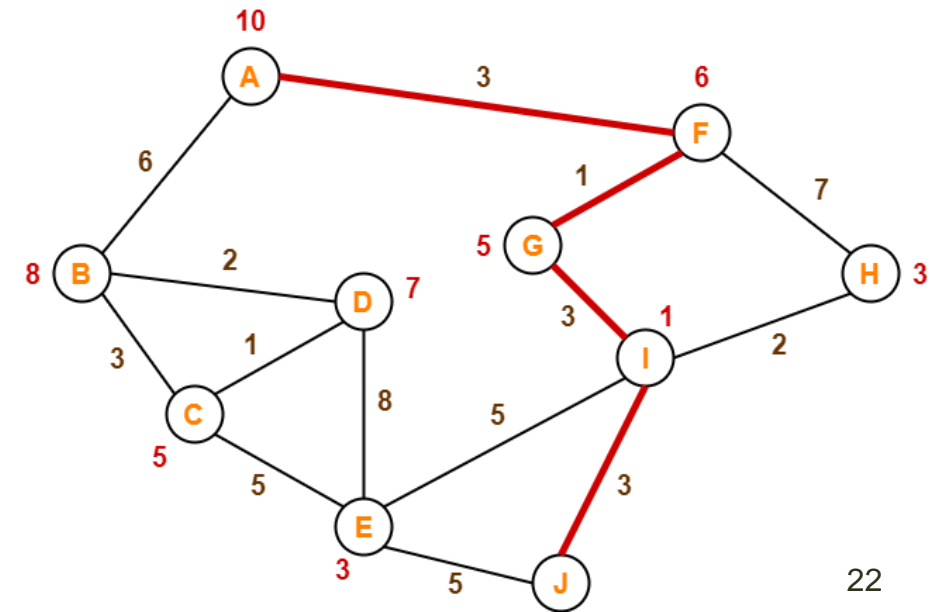
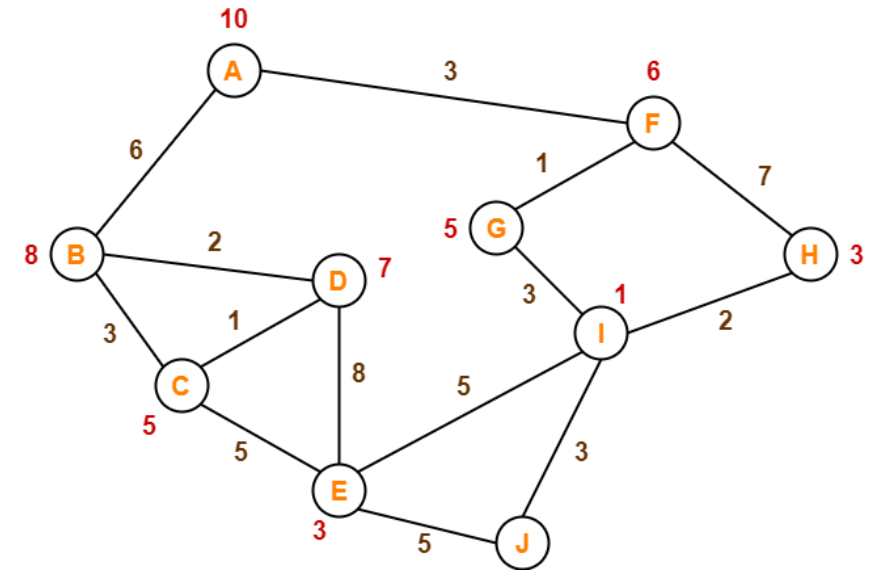
## Practical Examples: Map

- Consider the following map of cities
- The numbers written on edges represent the distance between the cities.
- The numbers written on nodes represent the straight-line distances to the target city.
- Find the most cost-effective path to reach from start city A to target city J using A\* Algorithm.



# A\* Search Algorithm

## Solution



# A\* Search Algorithm

## Practical Examples: 8-Puzzle

- Given an initial state of an 8-puzzle problem and final state to be reached.
- Find the most cost-effective path to reach the final state from initial state using A\* Algorithm. Assume the cost of one move = 1

|   |   |   |
|---|---|---|
| 5 |   | 8 |
| 4 | 2 | 1 |
| 7 | 3 | 6 |

N

|   |   |   |
|---|---|---|
| 1 | 2 | 3 |
| 4 | 5 | 6 |
| 7 | 8 |   |

goal

- You can take the heuristic function  $h(n)$  as:
  - Number of misplaced tiles
  - Sum of distances of each tile to goal



