CS2601 Linear and Convex Optimization

Homework 2

Due: 2022.10.10

1. Find the stationary points of the following functions, and determine if they are local minima, local maxima, or neither.

(a). $f(x_1, x_2, x_3) = 2x_1^2 + \frac{5}{2}x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_2x_3 + 2x_1 - 3x_2 + 2x_3$

(b). $f(x_1, x_2, x_3) = \frac{1}{2}x_1^2 + x_2^2 - \frac{3}{2}x_3^2 + 2x_1x_2 - x_2x_3 + x_2 - 3x_3$

2. For what value of α is the following matrix $\boldsymbol{A}$ positive semidefinite?

$$\boldsymbol{A} = \begin{pmatrix} 3 & -1 & 2 \\ -1 & 1 & \alpha \\ 2 & \alpha & 2 \end{pmatrix}$$

3. Let $f(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}$ be an affine function from $\mathbb{R}^n$ to $\mathbb{R}^m$, where $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ and $\boldsymbol{b} \in \mathbb{R}^m$. Show that if $C \subset \mathbb{R}^m$ is convex, so is its inverse image $f^{-1}(C) \triangleq \{\boldsymbol{x} : f(\boldsymbol{x}) \in C\}$.

4. Suppose C_1 and C_2 are convex sets. Show that $C = C_1 + C_2$ is also a convex set, where $C_1 + C_2 = \{\boldsymbol{x}_1 + \boldsymbol{x}_2 : \boldsymbol{x}_1 \in C_1, \boldsymbol{x}_2 \in C_2\}$.

5. Suppose C is a convex set.

(a). Show that its interior $\operatorname{int} C$ is convex. Hint: $B(\boldsymbol{x}_0, r) = \{\boldsymbol{x}_0 + r\boldsymbol{u} : \boldsymbol{u} \in B(\boldsymbol{0}, 1)\}$.

(b). Show that its closure $\bar{C}$ is convex.