AI2619 Digital Signal and Image Processing
Lab 2: Signal Sampling

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1 Direct Sampling

Convert the continuous function to a discrete array by creating 1000 frames per second. The sampling period T_s is set to be 500 frames, namely 0.5s. Then time sequence and sampling points are generated as follow

```
time = -5: frame: 15; % time sequence
sample = 1: period: length(time); % sample position
```

Then sampling result and frequency spectrum are obtained by

```
result = signal(sample); % sample result
magnitude = abs(fft(result, 4096)); % frequency result
```

The frequency spectrum is periodic with aliasing between two adjacent periods.

2 Sampling with Time Shift

The signal is shifted $\frac{T_s}{2}$ by modifying the condition as follow

```
0.5 * period * frame <= time(i) && time(i) <= 10 + 0.5 * period * frame
```

Then the approach to obtain sampling result and frequency spectrum is exactly the same with the previous problem. The frequency spectrum is almost the same as the previous one, but the aliasing is more obvious.

3 Sampling with Time Shift and Low-Pass Filtering

Construct a biquad Butterworth filter and apply it to the signal, where the cut-off frequency $f_c \approx 1$Hz, satisfying Nyquist sampling theorem.

```
[p, q] = butter(2, 0.0005); % butterworth design
signal = filter(p, q, signal); % low-pass filter
```

The sampling result and frequency spectrum are obtained similarly. With low-pass filtering, the curve in time domain is more smooth and the aliasing in frequency domain is almost eliminated. The energy of each period is concentrated in the central band.

4 Mathematiccal Derivation

The rectangle window function can be written as

$$x(t) = u(t) - u(t - \tau)$$

where $u(t)$ is the unit step function. The sampling is specified as

$$x_s(t) = x(t) \cdot \delta_{T_s}(t) = x(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_s)$$

In the discrete domain, the sampling result is

$$x_s[n] = \int_{-\infty}^{+\infty} x(t)\delta(t - nT_s)\,\mathrm{d}t = x(nT_s)$$

Frequency spectrum can be obtained by discrete time fourier transform

$$X\left(e^{j\omega}\right) = \sum_{n=-\infty}^{+\infty} x_s[n]e^{-j\omega n} = \sum_{n=-\infty}^{+\infty} x(nT_s)e^{-j\omega n} = \sum_{n=0}^{n_0-1} e^{-j\omega n} = \frac{1 - e^{-j\omega n_0}}{1 - e^{-j\omega}}$$

where $n_0 = \left[\frac{\tau}{T_s}\right]$. Then the amplitude characteristic is

$$\left|X\left(e^{j\omega}\right)\right| = \left|\frac{\sin\frac{n_0\omega}{2}}{\sin\frac{\omega}{2}}\right|$$

which is consistent with the result in the previous problem. Consider a signal with $\frac{T_s}{2}$ time shift

$$x\left(t - \frac{T_s}{2}\right) = u\left(t - \frac{T_s}{2}\right) - u\left(t - \frac{T_s}{2} - \tau\right)$$

which leads to $x_s[0] = 0$. So the frequency spectrum is

$$X\left(e^{j\omega}\right) = \sum_{n=1}^{n_0-1} e^{-j\omega n} = \frac{e^{-j\omega}\left[1 - e^{-j\omega(n_0-1)}\right]}{1 - e^{-j\omega}}$$

Then the amplitude characteristic is

$$\left|X\left(e^{j\omega}\right)\right| = \left|\frac{\sin\frac{(n_0-1)\omega}{2}}{\sin\frac{\omega}{2}}\right|$$

where more energy is distributed to both sides, which is consistent with previous results. Suppose a low-pass filter with impulse resopnse $h[n]$ is applied to the signal. The sampling result is

$$\hat{x}_s[n] = x[n] * h[n]$$

Then the amplitude characteristic is

$$\left|\hat{X}\left(e^{j\omega}\right)\right| = \left|X\left(e^{j\omega}\right)\right| \cdot \left|H\left(e^{j\omega}\right)\right| = \left|\frac{\sin\frac{(n_0-1)\omega}{2}}{\sin\frac{\omega}{2}}\right| \cdot \left|H\left(e^{j\omega}\right)\right|$$

Since $H\left(e^{j\omega}\right)$ has low-pass characteristic, the energy of each period will be concentrated in the central band and the aliasing will be eliminated.

Appendix A Spectrum Result

A.1 Direct Sampling

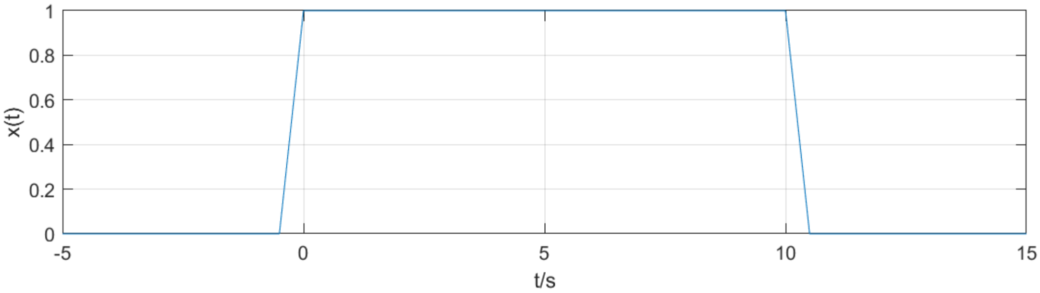

Figure 1: Time Domain

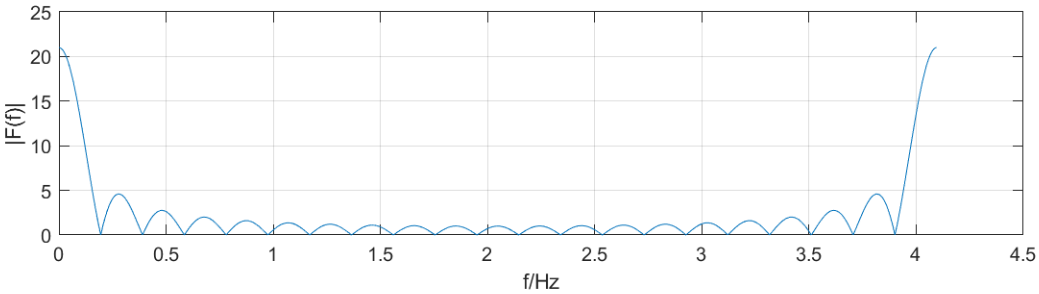

Figure 2: Frequency Domain

A.2 Sampling with Time Shift

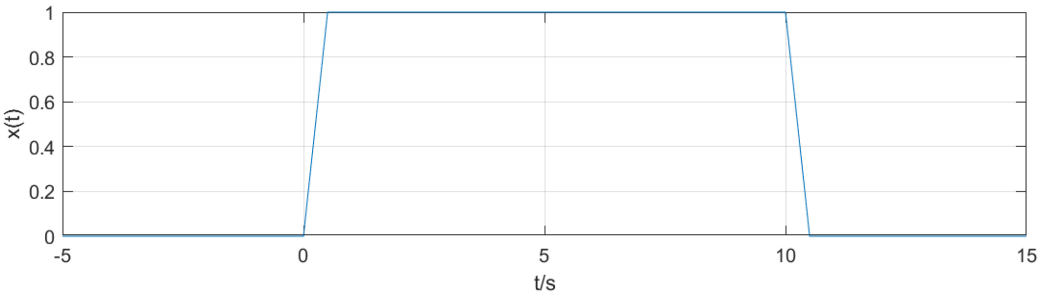

Figure 3: Time Domain

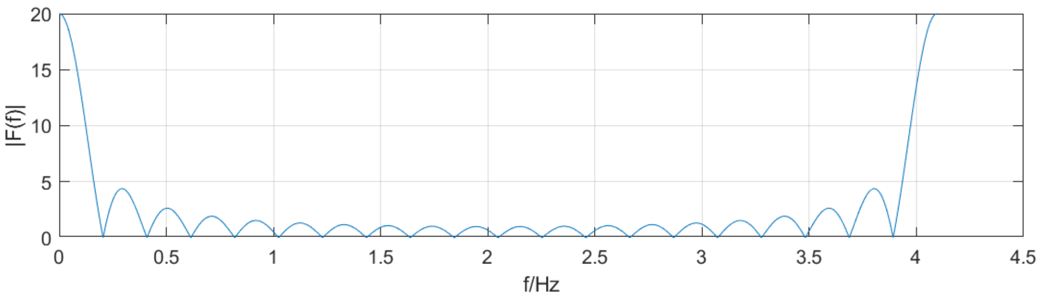

Figure 4: Frequency Domain

A.3 Sampling with Time Shift and Low-Pass Filtering

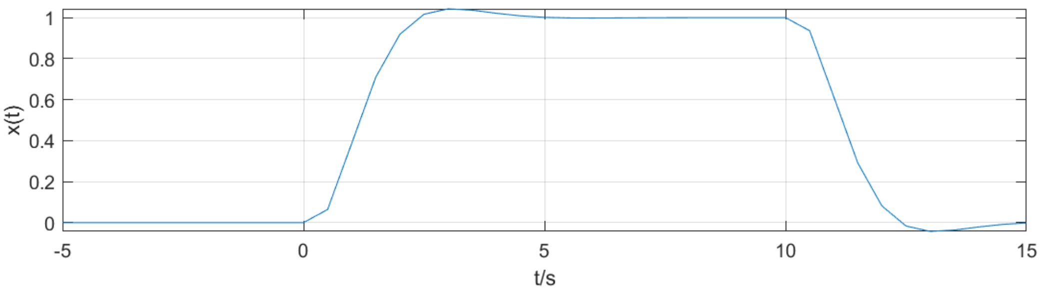

Figure 5: Time Domain

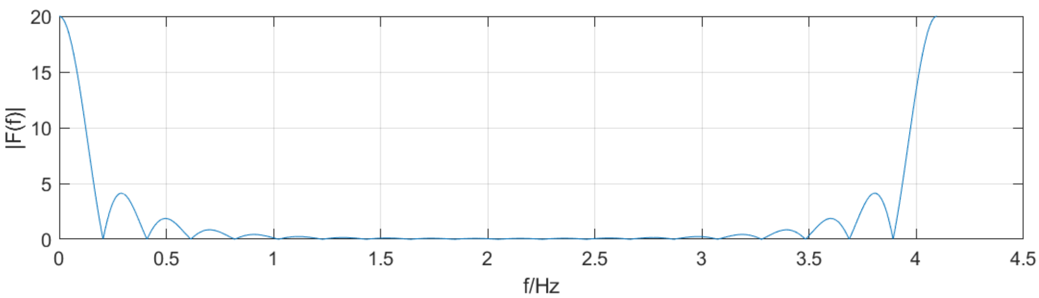

Figure 6: Frequency Domain