AI2619 Digital Signal and Image Processing
Lab 3: Discrete Fourier Transform

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1 Naïve Iteration

The formula of DFT is $\tilde{X}[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$, where $W_N = e^{-j\frac{2\pi}{N}}$ is the unity root. The naive implementation of DFT is to iterate the summation, whose time complexity is $O(N^2)$.

```
for k = 1: N % for each output element
    for n = 1: N % for each input element
        X(k) = X(k) + x(n) * (W(n) ^ k); % accumulate term
```

When $N = 2^{12}$, it takes about 5 seconds for computation. Larger N will take too much time to compute for our experiment.

2 Matrix Computation

Let $\boldsymbol{W}_N = \left(W_N^0, W_N^1, \ldots, W_N^n\right)^T$ and $\boldsymbol{F}$ be the Vandermonde matrix generated by $\boldsymbol{W}_N$, then the formula of DFT can be specified as $\tilde{\boldsymbol{X}} = \boldsymbol{F}\boldsymbol{x}$.

```
W = exp(-1j * 2 * pi * (0: N-1) / N); % unit root
F = fliplr(vander(W)); % vandermonde matrix
X = F * x; % matrix computation
```

Its time complexity is still $O(N^2)$, but it turns out much faster with optimization for matrix computation in Matlab. Due to its large space complexity, it does not work for $N > 2^{12}$.

3 Builtin Function

The builtin function in Matlab is implemented with FFT algorithm, whose time complexity is $O(N \log N)$, which supports the DFT up to $N = 2^{24}$ within acceptable time and memory.

4 GPU Acceleration

With parallel computing toolbox in Matlab, we can accelerate the computation with GPU.

```
X = fft(gpuArray(x)); % GPU acceleration
```

GPU allows more parallel computation for FFT algorithm. For small N, it is not that fast, but for $N \geq 2^{20}$, it brings a considerable speedup.

Appendix A Running Time

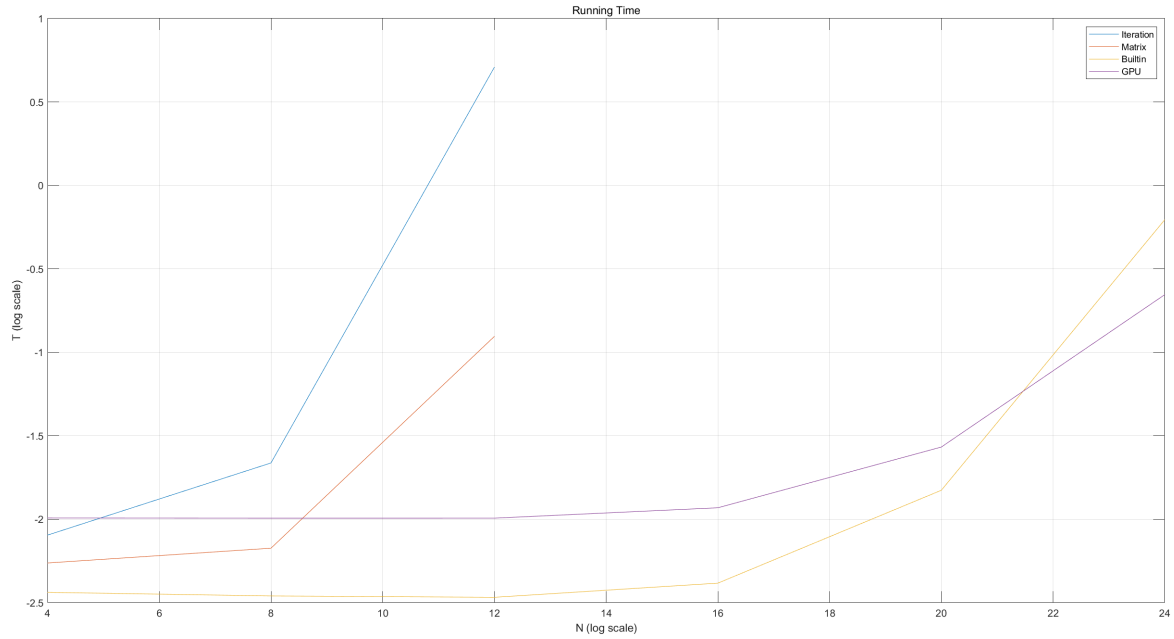

Figure 1: Running Time

The running time of the four methods are shown in the above figure. Note that N and T are in log scale. Naïve iteration is extremely slow. Matrix computation is slightly faster but limited by the memory. Builtin function runs much faster and needs less memory. GPU acceleration is significantly faster than builtin function when N becomes very large.