Mathematical Foundation

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May 30, 2019

Abstract

This Document contain my notes about Axioms, Definitions and basic theories.

1 Real Numbers

1.1 Fields

In rigorous mathematics real number is a set of numbers defined as a complete, ordered field

- DEF. *Field* is a non-empty set on which two $\overline{\text{binary operation}}$ are defined

refer to Group theory and Set theory TODO

- DEF. *Binary Operation* in field $\mathbb{F}$ is a function that "take" an ordered pair of element and "return" an element in $\mathbb{F}$, and it said to be the operation on the set whose both domain and co-domain in the same set.

$$\forall a, b \in \mathbb{F}(\exists c \in \mathbb{F}) : (c = a \circ b)$$

- the 9 golden basic most primitive axioms:

 1. AXI. *Associative law for addition* $(a + b) + c = a + (a + c)$
 2. AXI. *Existence of additive identity* $\exists 0 : a + 0 = 0 + a = a$
 3. AXI. *Existence of additive inverse* $\forall a \in \mathbb{R}\exists(-a) : a + (-a) = (-a) + a = 0$
 4. AXI. *Commutative law of addition* $a + b = b + a$
 5. AXI. *Associative law for multiplication* $(a \cdot b) \cdot c = a \cdot (a \cdot c)$
 6. AXI. *Existence of multiplicative identity* $\exists 1 \neq 0 : a \cdot 1 = 1 \cdot a = a$
 7. AXI. *Existence of multiplicative inverse* $\forall a \neq 0 \in \mathbb{R}\exists(a^{-1}) : a + (a^{-1}) = (a^{-1}) + a = 0$
 8. AXI. *Commutative law of multiplication* $a \cdot b = b \cdot a$
 9. AXI. *Distributive law* $a \cdot (b + c) = a \cdot b + a \cdot c$

- Theorem

Theorem 1. $\forall a \in \mathbb{F} : a \cdot 0 = 0$

Proof. using axiom Num.9

$$\begin{aligned} a \cdot 0 &= a \cdot (0 + 0) \\ &= a \cdot 0 + a \cdot 0 \end{aligned}$$

by adding $-(a \cdot 0)$ to both side

$$a \cdot 0 = 0$$

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