8.6 Compute the closure of the following set F of functional dependencies for relation schema $r(A, B, C, D, E)$.

$$A \rightarrow BC$$
$$CD \rightarrow E$$
$$B \rightarrow D$$
$$E \rightarrow A$$

List the candidate keys for R.

$B \rightarrow D \overset{①}{\Rightarrow} \underline{BC} \rightarrow CD \Rightarrow \underline{CD} \rightarrow E \Rightarrow \underline{E} \rightarrow A \Rightarrow \left.\begin{array}{l} \underline{A} \rightarrow BC \\ B \rightarrow D \end{array}\right\} \overset{③}{\Rightarrow} \left.\begin{array}{l} A \rightarrow CD \\ CD \rightarrow E \end{array}\right\} \overset{④}{\Rightarrow} A \rightarrow E$

②

$\left.\begin{array}{l} A \rightarrow BC \\ A \rightarrow CD \\ A \rightarrow E \end{array}\right\} \overset{⑤}{\Longrightarrow} A \rightarrow ABCDE$

① augmentative ② transitive

③ pseduotransitivity ④ transitive ⑤ union

Therefore A, BC, CD and E are the candidate keys

If there exist B only:

$B \rightarrow D$, then nothing else could be concluded.

C and D are of the same.

8.27 Using the functional dependencies of Practice Exercise 8.6, compute B^+.

① result = $\{B\}$

② $B \rightarrow D$ $\quad B^+ = \{B, D\}$

③ No more dependencies in F apply now

Therefore $B^+ = \{B, D\}$.

8.29 Consider the following set F of functional dependencies on the relation schema $r(A, B, C, D, E, F)$:

$$
\begin{array}{l}
① \ A \rightarrow BCD \\
② \ BC \rightarrow DE \\
③ \ B \rightarrow D \\
④ \ D \rightarrow A
\end{array}
$$

a. Compute B^+.

b. Prove (using Armstrong's axioms) that AF is a superkey.

c. Compute a canonical cover for the above set of functional dependencies F; give each step of your derivation with an explanation.

a. ③ $B \rightarrow D \Rightarrow B \rightarrow BD \Rightarrow B^+ = (BD)$.

④ $D \rightarrow A \Rightarrow BD \rightarrow BDA \Rightarrow B^+ = (BDA)$

① $A \rightarrow BCD \Rightarrow BDA \rightarrow ABCD \Rightarrow B^+ = (BDAC)$

② $BC \rightarrow DE \Rightarrow ABCD \rightarrow ABCDE \Rightarrow B^+ = (BDACE)$

Therefore, $B^+ = (ABCDE)$.

b. $AF \rightarrow F$ (reflexivity)
$A \rightarrow BCD$ (condition ①) $\Big\} \Rightarrow AF \rightarrow ABCDF$

$BC \rightarrow DE$ (condition ②) $\Rightarrow ABCDF \rightarrow ADEF$ (augmentation)

$AF \rightarrow ABCDF$, $ABCDF \rightarrow ADEF$ $\Big\} \Rightarrow AF \rightarrow ADEF$ (transitivity)

$AF \rightarrow ABCDF$
$AF \rightarrow ADEF$ $\Big\} \Rightarrow AF \rightarrow ABCDEF$ (union).

Therefore, AF is a superkey

8.29 Consider the following set F of functional dependencies on the relation schema $r(A, B, C, D, E, F)$:

$$
\begin{aligned}
&① \ A \rightarrow BCD \\
&② \ BC \rightarrow DE \\
&③ \ B \rightarrow D \\
&④ \ D \rightarrow A
\end{aligned}
$$

a. Compute B^+.

b. Prove (using Armstrong's axioms) that AF is a superkey.

c. Compute a canonical cover for the above set of functional dependencies F; give each step of your derivation with an explanation.

Since $B \rightarrow D$

$A \rightarrow BC$ is equivalent with $A \rightarrow BCD$

$BC \rightarrow E$ is equivalent with $BC \rightarrow DE$

Therefore, function dependencies could be simplified:

① $A \rightarrow BC$ ② $BC \rightarrow E$ ③ $B \rightarrow D$ ④ $D \rightarrow A$

$$\left.\begin{array}{l}\left.\begin{array}{l}\left.\begin{array}{l} B \rightarrow D \\ D \rightarrow A \end{array}\right\} \Rightarrow B \rightarrow A \\ \quad A \rightarrow BC \end{array}\right\} \Rightarrow B \rightarrow BC \\ \quad BC \rightarrow E \end{array}\right\} \Rightarrow B \rightarrow E \quad B^+ = (ABCDE)$$

$B \rightarrow E$ can be determined from this set

C is extraneous in the dependency ②

Therefore, final caconical cover is:

$F = \{A \rightarrow BC.\ B \rightarrow DE.\ D \rightarrow A\}$.