# Tutorial on 2017/05/31

Q1(Self-test Problem 12)

A basketball team consists of 6 frontcourt and 4 backcourt players. If players are divided into roommates at random, what is the probability that there will be exactly two roommate pairs made up of a backcourt and a frontcourt player?

Answer:

There are $(10)!/2^5$ different divisions of the 10 players into a first roommate pair, a second roommate pair, and so on. Hence, there are $(10)!/(5!2^5)$ divisions into 5 roommate pairs. There are $\binom{6}{2}\binom{4}{2}$ ways of choosing the frontcourt and backcourt players to be in the mixed roommate pairs and then 2 ways of pairing them up. As there is then 1 way to pair up the remaining two backcourt players and $4!/(2!2^2) = 3$ ways of making two roommate pairs from the remaining four frontcourt players, the desired probability is

$$P\{2 \text{ mixed pairs}\} = \frac{\binom{6}{2}\binom{4}{2}(2)(3)}{(10)!/(5!2^5)} = .5714$$

Q2（3.4）

Urn *A* contains 2 white balls and 1 black ball, whereas urn *B* contains 1 white ball and 5 black balls. A ball is drawn at random from urn *A* and placed in urn *B*. A ball is then drawn from urn *B*. It happens to be white. What is the probability that the ball transferred was white?

Solutions to Self-Test Problems and Exercises 469

**3.4.** Let $T$ be the event that the transferred ball is white, and let $W$ be the event that a white ball is drawn from urn $B$. Then

$$\begin{aligned} P(T|W) &= \frac{P(W|T)P(T)}{P(W|T)P(T) + P(W|T^c)P(T^c)} \\ &= \frac{(2/7)(2/3)}{(2/7)(2/3) + (1/7)(1/3)} = 4/5 \end{aligned}$$

Q3

**9.** In a certain species of rats, black dominates over brown. Suppose that a black rat with two black parents has a brown sibling.

**(a)** What is the probability that this rat is a pure black rat (as opposed to being a hybrid with one black and one brown gene)?

**(b)** Suppose that when the black rat is mated with a brown rat, all 5 of their offspring are black. Now, what is the probability that the rat is a pure black rat?

**9.** Since the black rat has a brown sibling we can conclude that both its parents have one black and one brown gene.

**(a)** $$P(2\text{ black} \mid \text{at least one}) = \frac{P(2)}{P(\text{at least one})} = \frac{1/4}{3/4} = \frac{1}{3}$$

**(b)** Let $F$ be the event that all 5 offspring are black. Let $B_2$ be the event that the black rat has 2 black genes, and let $B_1$ be the event that it has 1 black and 1 brown gene.

$$P(B_2|F) = \frac{P(F|B_2)P(B_2)}{P(F|B_2)P(B_2) + P(F|B_1)P(B_1)}$$
$$= \frac{(1)(1/3)}{(1)(1/3) + (1/2)^5(2/3)} = \frac{16}{17}$$

Q4（3.6）

An urn contains *b* black balls and *r* red balls. One of the balls is drawn at random, but when it is put back in the urn, *c* additional balls of the same color are put in with it. Now, suppose that we draw another ball. Show that the probability that the first ball was black, given that the second ball drawn was red, is *b/(b + r + c)*.

**3.6.** Let $B_i$ denote the event that ball $i$ is black, and let $R_i = B_i^c$. Then

$$P(B_1|R_2) = \frac{P(R_2|B_1)P(B_1)}{P(R_2|B_1)P(B_1) + P(R_2|R_1)P(R_1)}$$
$$= \frac{[r/[(b+r+c)][b/(b+r)]}{[r/(b+r+c)][b/(b+r)] + [(r+c)/(b+r+c)][r/(b+r)]}$$
$$= \frac{b}{b+r+c}$$

Q5（3.7）

A friend randomly chooses two cards, without replacement, from an ordinary deck of 52 playing cards. In each of the following situations, determine the conditional probability that both cards are aces.

**(a)** You ask your friend if one of the cards is the ace of spades, and your friend answers in the affirmative.

**(b)** You ask your friend if the first card selected is an ace, and your friend answers in the affirmative.

**(c)** You ask your friend if the second card selected is an ace, and your friend answers in the affirmative.

**(d)** You ask your friend if either of the cards selected is an ace, and your friend answers in the affirmative

**3.7.** Let $B$ denote the event that both cards are aces.

**(a)**

$$P\{B|\text{yes to ace of spades}\} = \frac{P\{B, \text{yes to ace of spades}\}}{P\{\text{yes to ace of spades}\}}$$
$$= \frac{\binom{1}{1}\binom{3}{1}}{\binom{52}{2}} \Big/ \frac{\binom{1}{1}\binom{51}{1}}{\binom{52}{2}}$$
$$= 3/51$$

**(b)** Since the second card is equally likely to be any of the remaining 51, of which 3 are aces, we see that the answer in this situation is also 3/51.

**(c)** Because we can always interchange which card is considered first and which is considered second, the result should be the same as in part (b). A more formal argument is as follows:

$$P\{B|\text{second is ace}\} = \frac{P\{B, \text{second is ace}\}}{P\{\text{second is ace}\}}$$
$$= \frac{P(B)}{P(B) + P\{\text{first is not ace, second is ace}\}}$$
$$= \frac{(4/52)(3/51)}{(4/52)(3/51) + (48/52)(4/51)}$$
$$= 3/51$$

**(d)**

$$P\{B|\text{at least one}\} = \frac{P(B)}{P\{\text{at least one}\}}$$
$$= \frac{(4/52)(3/51)}{1 - (48/52)(47/51)}$$
$$= 1/33$$

Q6（3.9）

You ask your neighbor to water a sickly plant while you are on vacation. Without water, it will die with probability .8; with water, it will die with probability .15. You are 90 percent certain that your neighbor will remember to water the plant.

**(a)** What is the probability that the plant will be alive when you return?

**(b)** If the plant is dead upon your return, what is the probability that your neighbor forgot to water it?

**3.9.** Let $A$ denote the event that the plant is alive and let $W$ be the event that it was watered.

**(a)**

$$P(A) = P(A|W)P(W) + P(A|W^c)P(W^c)$$
$$= (.85)(.9) + (.2)(.1) = .785$$

**(b)**

$$P(W^c|A^c) = \frac{P(A^c|W^c)P(W^c)}{P(A^c)}$$
$$= \frac{(.8)(.1)}{.215} = \frac{16}{43}$$

Q7 (3.10)

Six balls are to be randomly chosen from an urn containing 8 red, 10 green, and 12 blue balls.

**(a)** What is the probability at least one red ball is chosen?

**(b)** Given that no red balls are chosen, what is the conditional probability that there are exactly 2 green balls among the 6 chosen?

**3.10. (a)** $1 - P(\text{no red balls}) = 1 - \frac{\binom{22}{6}}{\binom{30}{6}}$

**(b)** Given that no red balls are chosen, the 6 chosen are equally likely to be any of the 22 nonred balls. Thus,

$$P(2 \text{ green}|\text{no red}) = \frac{\binom{10}{2}\binom{12}{4}}{\binom{22}{6}}$$

Q8 (3.11)

A type C battery is in working condition with probability .7, whereas a type D battery is in working condition with probability .4. A battery is randomly chosen from a bin consisting of 8 type C and 6 type D batteries.

**(a)** What is the probability that the battery works?

**(b)** Given that the battery does not work, what is the conditional probability that it was a type C battery?

**3.11.** Let $W$ be the event that the battery works, and let $C$ and $D$ denote the events that the battery is a type $C$ and that it is a type $D$ battery, respectively.

**(a)** $P(W) = P(W|C)P(C) + P(W|D)P(D) = .7(8/14) + .4(6/14) = 4/7$

**(b)**

$$P(C|W^c) = \frac{P(CW^c)}{P(W^c)} = \frac{P(W^c|C)P(C)}{3/7} = \frac{.3(8/14)}{3/7} = .4$$