ECE-316 Tutorial 2

Solutions

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P1:

$n = 8$ players

a) Divide into 4 pairs

$$\# \text{ divisions (pairs)} = \binom{8}{2,2,2,2} = \frac{8!}{2^4}$$

Since, there is no ordering of pair ~~[illegible]~~

$$\# \text{ divisions (pairs)} = \frac{8!}{2^4\, 4!}$$

For each pair, there are 2 outcomes and there are 4 games at round 1

$$\therefore \# \text{ outcomes at round 1} = \frac{8!}{2^4\, 4!} \times 2 \times 2 \times 2 \times 2$$

$$= \frac{8!}{4!}$$

b)

$$\# \text{ outcomes at round 2} = \frac{4!}{2!}$$

$$\# \text{ outcomes at round 1} = \frac{2!}{1!}$$

\# outcomes for the entire tournament

$$= \frac{8!}{4!} \times \frac{4!}{2!} \times \frac{2!}{1!} = 8!$$

P2 :

$A = \{1, 2, 3, \dots\}$ $\quad$ $C = \{1, 3, 5, 7, \dots\}$

$B = \{2, 4, 6, 8, \dots\}$ $\quad$ $D = \{2, 3, 5, 7, 11, 17, \dots\}$

i) $A \cap B = \{2, 4, 6, \dots\}$ → even numbers

ii) $A \cap C = \{1, 3, 5, 7, \dots\}$ → odd numbers

iii) $A \cap D = \{2, 3, 5, 7, 11, 17, \dots\}$ ~ set of primes

iv) $B \cap C = \Phi$

v) $B \cap D = \{2\}$

vi) $C \cap D =$ primes except 2

$= \{3, 5, 7, 11, 17, \dots\}$

P3 :

Job	Affiliation
Blue-collar	Republican
White-collar	Democratic
	Independent

\# members = 15

a) \# possibilities for each member = 6

\# outcomes = $\cancel{15^6}$ 6^{15}

b) E = atleast 1 is a blue-collar

= \# outcomes in sample space − none of them is a blue collar

$= \cancel{15^6} \; 6^{15} - 3^{15}$

c) Leaving independent, \# ~~out~~ possibilities for each team member is 4

$\therefore$ # outcomes = 4^{15}

P4:

Sample Space

$:= \{ 6, (1,6), (2,6), \ldots (5,6),$

$(1,1,6), (1,2,6), \ldots (1,5,6),$

$(2,1,6), (2,2,6), \ldots (2,5,6),$

$\ldots\ldots$

$\}$

E_n := # rolls necessary to complete the experiment i.e getting a six

$E_n = \{ (x_1, x_2, \ldots x_{n-1}, x_n) \mid 1 \leq x_i \leq 5,\ x_n = 6 \}$

$\left(\bigcup_{n=1}^{\infty} E_n\right)^c$: event that 6 never appears.

P5:

a) $S = \{ 1, 01, 001, 0001, 00001, \ldots$

$0000\ldots$

b) E = A wins = $\{ 1, 0001, 0000001, \ldots \}$

c) F = B wins = $\{ 01, 00001, 00000001, \ldots \}$

d) $(E \cup F)^c$ = C wins $\cup \{0000\ldots\ldots\}$

$= \{001, 000001, 000000001, \ldots\} \cup \{00000\ldots\}$

P6 :

\# consumers (U) = 1000

\# consumers who like product A = 720
(S)

" product B (T) = 450

$\Rightarrow$ $n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$= 720 + 450 - n(S \cap T)$

$= 1170 - n(S \cap T)$

$\Rightarrow$ we have $S \cup T \subset U$

$\Rightarrow$ $n(S \cup T) \leq n(U)$

$\Rightarrow$ $1170 - n(S \cap T) \leq 1000$

$\Rightarrow$ ~~n(...)~~

$170 \leq n(S \cap T)$

$\therefore$ # consumers who liked both A and B is atleast 170.

P7 :

$P(E \cup F) = P(E) + P(F) - P(E \cap F) \leq 1 \longrightarrow (i)$

$\Rightarrow$ $0.9 + 0.8 - P(E \cap F) \leq 1$

$\Rightarrow$ $P(E \cap F) \geq 0.7$

From (i)

$P(E \cap F) \geq P(E) + P(F) - 1$

P8 :

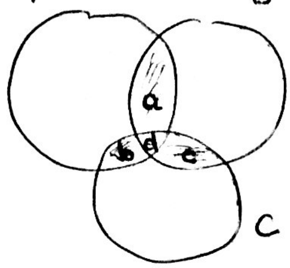
Football − 38 (F)

Basketball − 15 (B)

Cricket − 20 (C)

Total − 58 , 3 got all the medals

Fig:



$n(F) = 38$, $n(B) = 15$, $n(C) = 20$

$n(F \cup B \cup C) = 58$, $n(F \cap B \cap C) = 3$

To find $a + b + c = ?$

$$n(F \cup B \cup C) = n(F) + n(B) + n(C) - n(F \cap B) - n(F \cap C) - n(B \cap C) + n(F \cap B \cap C)$$

$$\Rightarrow 58 = 38 + 15 + 20 - (a + b + c) + 3$$

$$\Rightarrow \boxed{a + b + c = 18}$$

P9:

a) $F^c \cap G^c \cap E$

b) $F^c \cap E \cap G$

c) $E \cup F \cup G$

d) $(E \cap F) \cup (E \cap G) \cup (F \cap G)$

e) $E \cap F \cap G$

f) $(E \cup F \cup G)^c$

g) $EF^cG^c \cup E^cFG^c \cup E^cF^cG \cup E^cF^cG^c$

h) $(EFG)^c$

i) $(E \cap F \cap G^c) \cup EF^cG \cup E^cFG$

j) S

10)

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} A_i$$

$$B_1 = A_1 \qquad B_i = A_i \cap \left(\bigcup_{j=1}^{i-1} A_j\right)^c \quad i > 1$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P\left(\bigcup_{i=1}^{\infty} B_i\right)$$

$$= \sum_{i=1}^{\infty} P(B_i) \quad \text{as } B_i\text{'s are disjoint}$$

$$\leq \sum_{i=1}^{\infty} P(A_i) \quad (\text{as } B_i \subset A_i \text{ for } i > 1)$$