ECE316: Tutorial 2 (May 17, 2017)

1 Mutinomial Theorem

1. In the 1st round of a knockout tournament involving $n = 2^m$ players, the n players are divided in $\frac{n}{2}$ pairs. Each pair plays a game. The losers of the games are eliminated while the winners go on to the next round and the process is repeated until only one single player remains. Assume n = 8.
   (a) How many possible outcomes are there for the initial round?
   (b) How many outcomes of the tournaments are possible, where an outcome gives complete information for all rounds?

2 Events and Sample Space

1. If $A = \{x : \text{x is a natural number}\}$, $B = \{x : \text{x is an even natural number}\}$, $C = \{x : \text{x is an odd natural number}\}$ and $D = \{x : \text{x is a prime number}\}$, find the events i) $A \cap B$ ii) $A \cap C$ iii)$A \cap D$ iv) $B \cap C$ v)$B \cap D$ vi) $C \cap D$

2. Consider an experiment that consists of determining the type of job - either blue-collar or white-collar - and the political affiliation - Republican, Democratic, or Independent - of the 15 members of an adult soccer team. How many outcomes are
   (a) in the sample space
   (b) in the event that atleast one of the team members is a blue-collar worker.
   (c) in the event that none of the team members considers himself or herself an Independent?

3. A die is thrown repeatedly untill a six comes up. What is the sample space for this experiment? Let E_n denote the event that n rolls are necessary to complete the experiment. What is E_n and $(\cup_1^n E_n)^c$?

4. A, B, and C take turns flipping a coin. The first one to get a head wins.
   (a) What is the sample space?
   (b) Define the following events: a) $E =$ A wins b) $F = B$ wins c)$(E \cup F)^c$
   Assume that A flips first, then B, then C, then A and so on.

3 Axioms of Probability and Some simple probability propositions

1. A market research group conducted a survey of 1000 consumers and reported that 720 consumers like product A and 450 consumers like product B, what is the least number that must have liked both products?

2. If $P(E) = .9$ and $P(F) = .8$, show that $P(EF) \geq .7$. In general, prove Bonferroni's inequality, namely, $P(EF) \geq P(E) + P(F) - 1$.

3. A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

4. Let E, F, and G be three events. Find expressions for the events so that, of E, F, and G,
   (a) only E occurs
   (b) both E and G, but not F, occur
   (c) at least one of the events occurs
   (d) at least two of the events occurs
   (e) all three events occur

(f) none of the events occurs;
(g) at most one of the events occurs
(h) at most two of the events occur
(i) exactly two of the events occur
(j) at most three of the events occur.

5. Prove that $P(\cup_1^\infty A_i) \leq \sum_1^\infty P(A_i)$