First, we assume P a given value, and we take $p=0$. We consider the case of $p=0$, then we minimize the objective over different P.

With $p=0$. the problem become

$$\left.\begin{aligned}
&\min \ T \\
&\text{s.t.} \ \dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \partial x_1/\partial t \\ \partial x_2/\partial t \end{pmatrix} = \begin{pmatrix} T x_2 \\ T u(t) \end{pmatrix} && (1.a) \\
&x(0) = \begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} && (1.b) \\
&x_1(1) \geq 10 && (1.c) \\
&x_2(t) \leq 4 && (1.d) \\
&x_2(1) \leq 0 && (1.e) \\
&u(t) \in [-10, 10], \quad t \in [0,1] && (1.f)
\end{aligned}\right\} \quad (1)$$

T is a free final time, and we can regard T as one independent variable. We discretize $t \in [0,1]$ into $[t_k, t_{k+1}]$, $k = 0, 1, 2, \cdots, m-1$, with $t_0 = 0$, $t_m = 1$. We introduce initial guess of x, u at each interval. For the first interval, $[t_0, t_1]$, we already know $(x_1(0), x_2(0)) = (0, 0)$, therefore no need to guess these two values. For interval $[t_k, t_{k+1}]$, $k = 1, 2, \cdots, m-1$, we introdue $x(t_k)$, and u_k with $x(t_k) = (x_1(t_k), x_2(t_k))$, $u_k = u(t_k)$. When no confusion arise, we write

$x(t_k) = (x_1(t_k), x_2(t_k)) = (x_1^k, x_2^k)$. $u(t_k) = u_k$. Together with T and u_0, we have the newly introduced (discretized) variable

$$w = (T, u_0, u_1, x_1^1, x_2^1, u_2, x_1^2, x_2^2, \cdots, \underset{:= u(t_k)}{u_k}, \underset{:= x_1(t_k)}{x_1^k}, \underset{:= x_2(t_k)}{x_2^k}, \cdots, u_m, x_1^m, x_2^m)$$

$m: \Rightarrow t_m = 1$

in each interval $[t_k, t_{k+1}]$, we solve equation (1.a). I.e. we have the inital value x_1^k, x_2^k, u_k, u_{k+1}, we ~~would~~ can get a solution of $x^*(t)$, $t \in [t_k, t_{k+1}]$. We would like to enforce the continuity ~~at the~~ at the boundary, we would like our solution at the boundary t_{k+1} matching our guess of the next interval i.e. x_1^{k+1}, x_2^{k+1}. Therefore, the new equality constraints will be

$$\begin{aligned}
x_1^*(t_{k+1}) - x_1^{k+1} &= 0 \\
x_2^*(t_{k+1}) - x_2^{k+1} &= 0
\end{aligned} \qquad k = 0, \cdots, m-1 \qquad (2)$$

numerically solution

... the numerical solution as x^* (x^* ...

A'ZONE

The constraints (1.c), (1.d) (1.e), (1.f) of problem (1) then become inequality constraints of the following form

$$\begin{bmatrix} u_k - 10 \\ -10 - u_k \\ x_2^k - 4 \\ 10 - x_1^m \\ x_2^m \end{bmatrix} \leq 0, \quad k = 0, \cdots, m \qquad (3)$$

In equality constraint (2), the solution $x^*(t_{k+1})$ comes from solving (1.a), with input $x_1^k, x_2^k, u_k, u_{k+1}$. Therefore, it can be ~~written as~~ regarded as function of $x_1^k, x_2^k, u_k, u_{k+1}$, and only depends on them; i.e. $x^*(t_{k+1}) = x^*(t_{k+1}; x^k, u_k, u_{k+1})$

One way to approximate the solution can be as follows:

$$\dot{x} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \partial x_1/\partial t \\ \partial x_2/\partial t \end{pmatrix} = \begin{pmatrix} T x_2 \\ T u(t) \end{pmatrix} \Rightarrow \begin{pmatrix} \int_{t_k}^{t_{k+1}} \dot{x}_1 dt = \int_{t_k}^{t_{k+1}} T x_2 dt \\ \int_{t_k}^{t_{k+1}} \dot{x}_2 dt = \int_{t_k}^{t_{k+1}} T u(t) dt \end{pmatrix} \Rightarrow \begin{aligned} x_2^*(t_{k+1}) - x_2^k &= T\int_{t_k}^{t_{k+1}} u(t) dt \\ &\approx T \frac{u_k + u_{k+1}}{2} \end{aligned}$$

$$\Rightarrow x_2^*(t_{k+1}) = x_2^k + T \frac{u_k + u_{k+1}}{2}$$

and

$$x_1^*(t_{k+1}) - x_1^k = \int_{t_k}^{t_{k+1}} T x_2 dt = \int_{t_k}^{t_{k+1}} T\left(x_2^k + \frac{T(u_k + u(\tau))}{2}\right) d\tau$$

$$\Rightarrow x_1^*(t_{k+1}) \approx x_1^k + T x_2^k \Delta k + \frac{T^2}{2} u_k \Delta k + \frac{T^2}{4}(u_k + u_{k+1}), \text{ where } \Delta_k = t_{k+1} - t_k$$

This is just one of the many numerical method to get the approximate solution.

Then the original problem (1) can be reformulated as

$$\left.\begin{aligned} &\min \; T(w) \\ &\text{s.t.} \quad h(w) = \begin{pmatrix} x_1^*(t_{k+1}) - x_1^{k+1} \\ x_2^*(t_{k+1}) - x_2^{k+1} \end{pmatrix} = 0, \quad k = 0, \cdots, m-1 \\ &\qquad g(w) = \begin{pmatrix} u_k - 10 \\ -10 - u_k \\ x_2^k - 4 \\ 10 - x_1^m \\ x_2^m \end{pmatrix} \leq 0, \quad k = 0, \cdots, m \end{aligned}\right\} \quad (4)$$

where $w = (T, u_0, u_1, x_1^1, x_2^1, u_2, x_1^2, x_2^2, \cdots, u_k, x_1^k, x_2^k, \ldots$

To make it more clear, we write out each sub equations of the ~~equality and~~ inequality constraits.

$$\min \quad T(w)$$

$$\text{s.t.} \quad h(w) = \begin{array}{c} h_1 \\ h_2 \\ h_3 \\ h_4 \\ \vdots \\ h_{2k+1} \\ h_{2k+2} \\ \vdots \\ h_{2m-1} \\ h_{2m} \end{array} \begin{pmatrix} h_1^0 \\ h_2^0 \\ h_1^1 \\ h_2^1 \\ \vdots \\ h_1^k \\ h_2^k \\ \vdots \\ h_1^{m-1} \\ h_2^{m-1} \end{pmatrix} = \begin{pmatrix} x_1^*(t_1) - x_1^1 \\ x_2^*(t_1) - x_2^1 \\ x_1^*(t_2) - x_1^2 \\ x_2^*(t_2) - x_2^2 \\ \vdots \\ x_1^*(t_{k+1}) - x_1^{k+1} \\ x_2^*(t_{k+1}) - x_2^{k+1} \\ \vdots \\ x_1^*(t_m) - x_1^m \\ x_2(t_m) - x_2^m \end{pmatrix} = 0 ,$$

here

$x^*(t_{k+1}) = x^*(t_{k+1}; x_k, u_k, u_{k+1})$

i.e. are functions of $x_k = (x_1^k, x_2^k)$, u_k, u_{k+1}

(5)

Notation $h_1, h_2, \cdots, h_{2m}$ is defined here for easier reference in the matrix location

in total $\underline{\underline{2 \times m}}$ equality constraints

$$g(w) = \begin{array}{c} g_1 \\ g_2 \\ g_3 \\ \vdots \\ g_{3k+1} \\ g_{3k+2} \\ g_{3k+3} \\ \vdots \\ g_{3m+1} \\ g_{3m+2} \\ g_{3m+3} \\ g_{3m+4} \end{array} \begin{pmatrix} g_{\hat{u}_0} \\ g_{\bar{u}_0} \\ g_{x_2^0} \\ \vdots \\ g_{\hat{u}_k} \\ g_{\bar{u}_k} \\ g_{x_2^k} \\ \vdots \\ g_{\hat{u}_m} \\ g_{\bar{u}_m} \\ g_{x_2^m} \\ g_{x_1^m} \end{pmatrix} = \begin{pmatrix} u_0 - 10 \\ -10 - u_0 \\ x_2^0 - 4 \\ \vdots \\ u_k - 10 \\ -10 - u_k \\ x_2^k - 4 \\ \vdots \\ u_m - 10 \\ -10 - u_m \\ x_2^m \\ 10 - x_1^m \end{pmatrix} \leq 0$$

(6)

Notation $g_1, g_2, \cdots g_{3m+4}$ is defined for easier reference in the matrix location.

in (4), there are two inequality constraints related to x_2^m, i.e $x_2^m - 4 \leq 0$ and $x_2^m \leq 0$. obviously $x_2^m - 4 \leq 0$ is redundant if we have $x_2^m \leq 0$. Then we ~~write the~~ only include inequality constraint $g_{x_2^m} = x_2^m \leq 0$

in total $\underline{\underline{3 \times (m+1) + 1}}$ inequality constraints.

Based on KKT condition, we can define a ~~Lamda~~ Lambda function as

$$L(w) = T(w) + \lambda^T \underbrace{h(w)}_{2 \times m \text{ equations}} + \eta^T \underbrace{g(w)}_{3 \times (m+1)+1 = 3m+4 \text{ equations}}.$$

with $\lambda \in R^{2m \times 1}$, i.e $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_{2m})$

$\eta \in R^{3m \times 4}$, i.e. $\eta = (\eta \quad \eta \quad \quad \eta \quad)$

A'ZONE

If w^* is a local minimum, then $\exists\ w^*, \lambda^*, \eta^*$ such that (w^*, λ^*, η^*) satisfy the necessary condition

$$\nabla_w L(w,\lambda,\eta) = \nabla_w T(w) + \lambda^T \nabla_w h(w) + \eta^T \nabla_w g(w) = 0$$

with $w = (T, u_0, u_1, x_1^1, x_2^1, \cdots, u_k, x_1^k, x_2^k, \cdots, u_m, x_1^m, x_2^m)$

Notice $\lambda \in R^{2m}$, i.e $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_{2m})$, $\eta \in R^{3m+4}$, i.e. $\eta = (\eta_1, \eta_2, \cdots, \eta_{3m+3}, \eta_{3m+4})$

We can write $L(w)$ out as

$$L(w) = T(w) + \lambda^T h(w) + \eta^T g(w)$$
$$= T + \lambda_1 h_1^0 + \lambda_2 h_2^0 + \cdots + \lambda_{2m} h_2^{m-1} + \eta_1 g_{\hat{u}_0} + \eta_2 g_{\bar{u}_0} + \eta_3 g_{x_2^0} + \cdots$$
$$+ \eta_{3m+1} g_{\hat{u}_m} + \eta_{3m+2} g_{\bar{u}_m} + \eta_{3m+3} g_{x_2^m} + \eta_{3m+4} g_{x_1^m}$$

~~with the h, g related sub-functions~~ defined in 5 and 6

Then

$$\nabla_w L = \begin{pmatrix} \nabla_T L \\ \nabla_{u_0} L \\ \nabla_{u_1} L \\ \nabla_{x_1^1} L \\ \nabla_{x_2^1} L \\ \vdots \\ \nabla_{u_k} L \\ \nabla_{x_1^k} L \\ \nabla_{x_2^k} L \\ \vdots \\ \nabla_{u_m} L \\ \nabla_{x_1^m} L \\ \nabla_{x_2^m} L \end{pmatrix} \qquad (7)$$

where

$$\nabla_T L = \frac{\partial T}{\partial T} + \lambda_1 \frac{\partial h_1^0}{\partial T} + \cdots + \lambda_{2m} \frac{\partial h_2^{m-1}}{\partial T} + \eta_1 \frac{\partial g_{\hat{u}_0}}{\partial T} + \cdots + \eta_{3m+4} \frac{\partial g_{x_1^m}}{\partial T}$$
$$= 1 + \lambda^T \frac{\partial h(w)}{\partial T} + 0 + \cdots + 0$$

all the sub-functions of $h(w)$: $h_1^0, h_2^0, \cdots, h_1^k, h_2^k, \cdots, h_1^{m-1}, h_2^{m-1}$ are functions of T.

$$\nabla_{u_0} L = \frac{\partial T}{\partial u_0} + \lambda_1 \frac{\partial h_1^0}{\partial u_0} + \lambda_2 \frac{\partial h_2^0}{\partial u_0} + \lambda_3 \frac{\partial h_1^1}{\partial u_0} + \cdots + \lambda_{2m} \frac{\partial h_2^{m-1}}{\partial u_0} + \eta_1 \frac{\partial g_{\hat{u}_0}}{\partial u_0} + \eta_2 \frac{\partial g_{\bar{u}_0}}{\partial u_0} + \cdots + \eta_{3m+4} \frac{\partial g_{x_1^m}}{\partial u_0}$$
$$= 0 + \lambda_1 \frac{\partial h_1^0}{\partial u_0} + \lambda_2 \frac{\partial h_2^0}{\partial u_0} + 0 + \cdots + 0 + \eta_1 + (-1)\eta_2 + 0 + \cdots + 0$$

Similarly, we have

$$\nabla_{u_k} L = \underset{T(w)}{0} + \underbrace{0 + \cdots + \lambda_{2k+1}\frac{\partial h_1^k}{\partial u_k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial u_k} + 0 + \cdots + 0}_{h(w)\ \text{part}} + \underbrace{0 + \cdots + \eta_{3k+1}\cdot 1 + \eta_{3k+2}\cdot(-1) + 0 + 0 + \cdots + 0}_{g(w)\ \text{part}}$$

corresponds to $\eta_{3k+1}\frac{\partial g_{u_k}}{\partial u_k}$ $\quad \eta_{3k+2}\frac{\partial g_{\bar{u}_k}}{\partial u_k}$

$$\cancel{\nabla_{x_1^k} L = 0 + 0 + \cdots \lambda_{2k+1}\frac{\partial h_1^k}{\partial x_1^k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial x_1^k} + \cdots + 0}$$

$$\cancel{+ 0, \quad \text{if } k < m}$$

$$\cancel{\nabla_{x_1^k} L = 0 + 0 + \cdots + \frac{\partial h_1^{k-1}}{\partial x_1^k} + \frac{\partial h_2^{k-1}}{}}$$

Notice
$$\begin{pmatrix} h_1^{k-1} \\ h_2^{k-1} \\ h_1^k \\ h_2^k \end{pmatrix} = \begin{pmatrix} x_1^*(t_k) - x_1^k \\ x_2^*(t_k) - x_2^k \\ x_1^*(t_{k+1}) - x_1^{k+1} \\ x_2^*(t_{k+1}) - x_2^{k+1} \end{pmatrix},$$
where $x^*(t_k)$ are functions of (x^{k-1}, u_{k-1})

$x^*(t_{k+1})$ are functions of (x^k, u_k)

Therefore

$$\nabla_{x_1^k} L = 0 + 0 + \cdots + \lambda_{2k-1}\frac{\partial h_1^{k-1}}{\partial x_1^k} + \lambda_{2k}\frac{\partial h_2^{k-1}}{\partial x_1^k} + \lambda_{2k+1}\frac{\partial h_1^k}{\partial x_1^k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial x_1^k} + 0 + \cdots + 0$$

$$= 0 + 0 + \cdots + (-1)\lambda_{2k-1} + 0 + \lambda_{2k+1}\frac{\partial h_1^k}{\partial x_1^k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial x_1^k} + 0 + \cdots + 0$$

when $k < m$

and
$$\nabla_{x_1^m} L = 0 + 0 + \cdots + \lambda_{2m-1}\frac{\partial h_1^{m-1}}{\partial x_1^m} + \lambda_{2m}\frac{\partial h_2^{m-1}}{\partial x_1^m} + 0 + \cdots + 0 + \eta_{3m+4}\frac{\partial g_{x_1^m}}{\partial x_1^m}$$

$$= 0 + 0 + \cdots + (-1)\lambda_{2m-1} + 0 + 0 + \cdots + (-1)\eta_{3m+4}$$

$$\nabla_{x_2^k} L = 0 + 0 + \cdots + \lambda_{2k-1}\frac{\partial h_1^{k-1}}{\partial x_2^k} + \lambda_{2k}\frac{\partial h_2^{k-1}}{\partial x_2^k} + \lambda_{2k+1}\frac{\partial h_1^k}{\partial x_2^k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial x_2^k} + 0 + \cdots + 0$$

$$= 0 + 0 + \cdots + 0 + \lambda_{2k}(-1) + \lambda_{2k+1}\frac{\partial h_1^k}{\partial x_2^k} + \lambda_{2k+2}\frac{\partial h_2^k}{\partial x_2^k} + 0 + \eta_{3k+3}\frac{\partial g_{x_2^k}}{\partial x_2^k} + 0 + \cdots + 0,$$ when $k < m$

and
$$\nabla_{x_2^m} L = 0 + \cdots + 0 + (-1)\lambda_{2m} + 0 + \cdots + \eta_{3m+3}\cdot(1)$$
$$+ \eta_{3k+3} + 0 + \cdots 0,$$ when $k < m$

i.e. Now we have all the derivatives of L w.r.t w defined, we can then write ① out in a matrix form, this time, we use the index notation $h_1, h_2, \cdots, h_{2m-1}, h_{2m}, g_1, g_2, g_3 \cdots, g_{3m+4}$ directly. Then we have

$$L(w) = T(w) + \lambda^T h(w) + \eta^T g(w)$$

$$= T + \lambda_1 h_1^0 + \lambda_2 h_2^0 + \cdots + \lambda_{2m-1} h_1^{m-1} + \lambda_{2m} h_2^{m-1} + \eta_1 g_{\bar{u}_0} + \eta_2 g_{\bar{u}_0} + \eta_3 g_{x_2^0} + \cdots + \eta_{3m+4} g_{x_1^m}$$

~~Using the index~~ notation for $h(w)$ and $g(w)$

$$= T + \lambda_1 h_1 + \lambda_2 h_2 + \cdots + \lambda_{2m-1} h_{2m-1} + \lambda_{2m} h_{2m} + \eta_1 g_1 + \eta_2 g_2 + \cdots + \eta_{3m+4} g_{3m+4}$$

$$w = (T, u_0, u_1, x_1^1, x_2^1, u_2, x_2^1, x_2^2, \cdots, u_k, x_1^k, x_2^k, \cdots, u_m, x_1^m, x_2^m)$$

$$\nabla_w L = (\nabla_T L, \nabla_{u_0} L, \nabla_{u_1} L, \cdots, \nabla_{u_m} L, \nabla_{x_1^m} L, \nabla_{x_2^m} L)^T$$

corresponds to $T, h_1, h_2, \cdots, h_{2k-1}, h_{2k}, h_{2k+1}, h_{2k+2}, \cdots, h_{2m-1}, h_{2m}, g_1, g_2, g_3, \cdots, g_{3k+1}, g_{3k+2}, g_{3k+3}, \cdots, g_{3m+1}, g_{3m+2}, g_{3m+3}, g_{3m+4}$

$= \nabla_T$: $1, \lambda_1 \frac{\partial h_1}{\partial T}, \lambda_2 \frac{\partial h_2}{\partial T}, \lambda_{2k-1}\frac{\partial h_{2k-1}}{\partial T}, \lambda_{2k}\frac{\partial h_{2k}}{\partial T}, \lambda_{2k+1}\frac{\partial h_{2k+1}}{\partial T}, \lambda_{2k+2}\frac{\partial h_{2k+2}}{\partial T}, \cdots, \lambda_{2m-1}\frac{\partial h_{2m-1}}{\partial T}, \lambda_{2m}\frac{\partial h_{2m}}{\partial T}, 0, 0, 0, \cdots, 0, 0, 0, \cdots, 0, 0, 0, 0$

∇u_0: $0, \lambda_1 \frac{\partial h_1}{\partial u_0}, \lambda_2 \frac{\partial h_2}{\partial u_0}, 0, \cdots, 0, \eta_1, -\eta_2, 0, 0, \cdots, 0$

∇u_1: $0, 0, 0, \lambda_3 \frac{\partial h_3}{\partial u_1}, \lambda_4 \frac{\partial h_4}{\partial u_1}, 0, \cdots, 0, 0, 0, 0, \eta_4, -\eta_5, 0, \cdots$

$\vdots$

∇u_k: $0, 0, \cdots, \lambda_{2k+1}\frac{\partial h_{2k+1}}{\partial u_k}, \lambda_{2k+2}\frac{\partial h_{2k+2}}{\partial u_k}, 0, \cdots, 0, \cdots, \eta_{3k+1}, -\eta_{3k+2}, 0, \cdots, 0$

∇x_1^k: $0, 0, \cdots, 0, -\lambda_{2k-1}, 0, \lambda_{2k+1}\frac{\partial h_{2k+1}}{\partial x_1^k}, \lambda_{2k+2}\frac{\partial h_{2k+2}}{\partial x_1^k}, 0, \cdots, 0, 0, \cdots, 0$

∇x_2^k: $0, 0, \cdots, 0, 0, -\lambda_{2k}, \lambda_{2k+1}\frac{\partial h_{2k+1}}{\partial x_2^k}, \lambda_{2k+2}\frac{\partial h_{2k+2}}{\partial x_2^k}, 0, \cdots, 0, 0, \eta_{3k+3}, 0, \cdots, 0$

$\vdots$

∇x_1^m: $0, 0, \cdots, 0, -\lambda_{2m-1}, 0, \cdots, -\eta_{3m+4}$

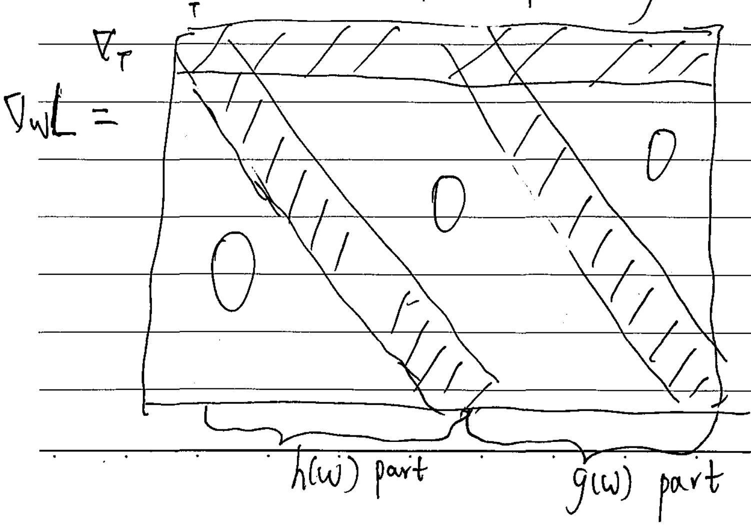
∇x_2^m: $0, 0, \cdots, -\lambda_{2m}, \cdots, \eta_{3m+3}, 0$

(8)

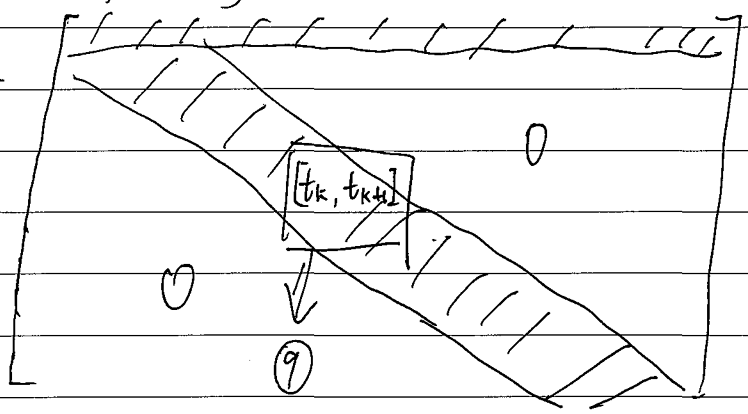
~~i.e.~~

$= 0$

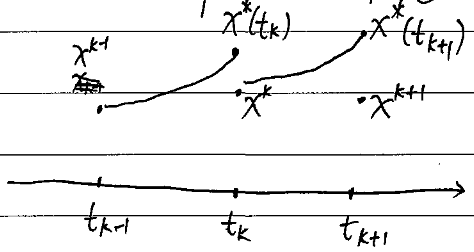
i.e. we have a matrix of the following structure

$\nabla_w L =$  $= 0$;

We can re-arrange the order of $h(w)$ and $g(w)$ so that their corresponding part in interval $[t_k, t_{k+1}]$ can be grouped together, then the $\nabla_w L$ can be written in the following form

$\nabla_w L =$  $= 0$

~~If we take out the component of $[t_k, t_{k+1}]$ out~~



In interval, initial guess x^k, u_k, u_{k+1} is known, the matching condition is

$$\begin{array}{l} h_{2k+1}: \\ h_{2k+2}: \end{array} \begin{pmatrix} h_1^k \\ h_2^k \end{pmatrix} = \begin{pmatrix} x_1^*(t_{k+1}) - x_1^{k+1} \\ x_2^*(t_{k+1}) - x_2^{k+1} \end{pmatrix}$$

with the constraint on u_k, x^k as

$$\begin{array}{l} g_{3k+1}: \\ g_{3k+2}: \\ g_{3k+3}: \end{array} \begin{pmatrix} g_{\hat{u}_k} \\ g_{\bar{u}_k} \\ g_{x_2^k} \end{pmatrix} = \begin{pmatrix} u_k - 10 \\ -10 - u_k \\ x_2^k - 4 \end{pmatrix}$$

Then the box 9 can be written as

$$\cancel{\partial\upsilon} \begin{pmatrix} \nabla u_k \\ \nabla x_1^k \\ \nabla x_2^k \end{pmatrix} = \begin{array}{c} \begin{array}{ccccc} h_{2k+1} & h_{2k+2} & g_{3k+1} & g_{3k+2} & g_{3k+3} \\ h_1^k, & h_2^k & g_{\hat{u}_k} & g_{\bar{u}_k} & g_{x_2^k} \end{array} \\ \begin{pmatrix} \lambda_{2k+1} \frac{\partial h_{2k+1}}{\partial u_k} & \lambda_{2k+2} \frac{\partial h_{2k+2}}{\partial u_k} & \eta_{3k+1} & -\eta_{3k+2} & 0 \\ \lambda_{2k+1} \frac{\partial h_{2k+1}}{\partial x_1^k} & \lambda_{2k+2} \frac{\partial h_{2k+2}}{\partial x_1^k} & 0 & 0 & 0 \\ \lambda_{2k+1} \frac{\partial h_{2k+1}}{\partial x_2^k} & \lambda_{2k+2} \frac{\partial h_{2k+2}}{\partial x_2^k} & 0 & 0 & \eta_{3k+3} \end{pmatrix} \end{array}$$

Details of 9